Contents

Volume 1

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Numbers and Operations Paper 1

1   (a)   Calculate the value of 0.1×0.06.   *Answer*   (a) .... [1]

      (b)   Find the decimal number exactly halfway between 1.01 and 1.02.

                                      (b) .... [1]

J02/1/Q1

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2   Giving the answer as simply as possible, calculate

      (a) $\dfrac{3}{4} - \dfrac{1}{3}$,   *Answer*   (a) .... [1]

      (b) $\dfrac{2}{5}$ of $\dfrac{15}{16}$.   *(b)* .... [1]

J02/1/Q2

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3   (a)   Evaluate $\dfrac{7}{8} - \dfrac{7}{10}$, giving your answer as a fraction in its lowest terms.

                        Answer (a) .... [1]

      (b)   Evaluate $2\frac{1}{3} \times 3\frac{1}{2}$, giving your answer as a mixed number.

                        (b) .... [1]

J03/1/Q2

Answers: (a) $\dfrac{3}{100}$; (b) 30%.

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4   Evaluate

      (a) $24 \div 6 + 2 \times 9$,   *Answer (a)* .... [1]

      (b) 0.4×0.02 .   *(b)* .... [1]

J03/1/Q3

Answers: (a) 22; (b) 0.008.

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5   (a)   Express $\dfrac{72}{108}$ as a fraction in its lowest terms.   *Answer (a)* .... [1]

      (b)   Evaluate $\dfrac{1}{3} + \dfrac{4}{7}$.   *(b)* .... [1]

J04/1/Q2

Answers: (a) $\dfrac{2}{3}$; (b) $\dfrac{19}{21}$.

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6   (a)   Evaluate $63 \div 0.9$.   *Answer (a)* .... [1]

      (b)   Add brackets to the expression in the answer space to make it correct.

                     Answer (b) $1 \;+\; 72 \;\div\; 4 \;\times\; 2 \;=\; 10$ [1]

7   Evaluate

(a) $2\frac{2}{3} \times \frac{1}{7}$,

(b) $\frac{2}{5} \div \frac{7}{12}$.

Answer (a) .... [1]

(b) .... [1]

8   It is given that $N = 87 \times 132$.

(a)   Complete the statements in the answer space.

Answer (a) $88 \times 132 = N +$ .... [1]

$87 \times 131 = N -$ .... [1]

(b)   Hence evaluate $88 \times 132 - 87 \times 131$.

Answer (b) .... [1]

9   Evaluate

(a) $\frac{6}{7} - \frac{1}{3}$,

(b) $\frac{2}{5} \times \frac{4}{9}$.

Answer (a) ....[1]

(b) ....[1]

10   (a)   Evaluate $3 + 25 \div 2$.

(b)   Express $17\frac{1}{2}\%$ as a decimal.

Answer (a) .... [1]

(b) ....[1]

11   Evaluate

(a) $\frac{1}{4} + \frac{1}{7}$,

(b) $1\frac{7}{8} \div \frac{3}{16}$.

Answer (a) ....[1]

(b) ....[1]

12 It is given that $\frac{2}{3}$, $\frac{8}{d}$ and $\frac{n}{39}$ are equivalent fractions.

Find the value of d and the value of n.　　Answer $d = $ ....[1]

$n = $ ....[1]

　　　　　　　　　　　　J07/1/Q3

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13 (a) Write the following in order of size, starting with the smallest.

$$\frac{66}{100} \qquad 0.\dot{6} \qquad 0.67 \qquad \frac{666}{1000}$$

Answer (a) .... , ... , ... , ... [1]
　　　　　　　　　　smallest

J07/1/Q5a

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14 Evaluate

(a) $\frac{1}{2} - \frac{3}{7}$,　　　　　　Answer (a) ....[1]

(b) $2\frac{2}{3} \times 1\frac{3}{4}$.　　　　　　(b) ....[1]

J08/1/Q1

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15 Evaluate

(a) $25 - 18.3$,　　　　Answer (a) ....[1]

(b) 1.7×0.03 .　　　　(b) ....[1]

J08/1/Q2

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16 It is given that $68.2 \times 0.235 = 16.027$.

Hence evaluate

(a) 0.0682×2350,　　　　Answer (a) ....[1]

(b) $160.27 \div 0.0235$.　　　　(b) ....[1]

　　　　J08/1/Q5

17 Express as a single fraction in its lowest terms,

(a) $\frac{8}{9} \times \frac{3}{4}$,

Answer (*a*) .... [1]

(b) $\frac{3}{4} - \frac{2}{3}$.

Answer (*b*) .... [1]

Answer: (a) $\frac{2}{3}$ (b) $\frac{1}{12}$

---

18 Evaluate

(a) $\frac{1}{2} + \frac{2}{9}$,

Answer (*a*) ....[1]

(b) $\frac{2}{3} \div \frac{9}{11}$.

Answer (*b*) .... [1]

Answer: (a) $\frac{13}{18}$ (b) $\frac{22}{27}$

---

19 Evaluate

(a) $1.5 - 0.2 \times 4$,

Answer (*a*) .... [1]

(b) $4.2 \div 0.07$.

Answer (*b*) .... [1]

Answer: (a) 0.7 (b) 60

---

20 Express as a single fraction

Answer (*a*) .... [1]

(a) $\frac{5}{7} - \frac{2}{5}$,

(b) $1\frac{1}{5} \div 2\frac{1}{3}$.

Answer (*b*) .... [1]

Answer: (a) $\frac{11}{35}$ (b) $\frac{18}{35}$

---

21 Evaluate

(a) $52.3 \times 10 - 3.76 \times 100$,

Answer .... [1]

(b) $20 - 8 \div 2 + 1$.

Answer .... [1]

Answer: (a) 147 (b) 17

---

22 Evaluate

(a) $\frac{2}{3} + \frac{3}{10}$,

Answer .... [1]

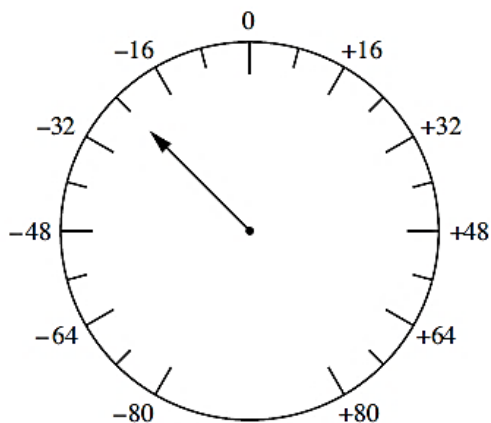(b) $1\frac{3}{5} \div 3$.

Answer .... [1]

Answers: (a) $\frac{29}{30}$ (b) $\frac{8}{15}$

23 An instrument is used to measure the height of an object above sea level.
The height, in metres, is shown on the dial.

(a) What is the reading on the dial?

Answer .... m [1]

(b) The object moves from position A, where the dial reads −54, to position B, where the dial reads +48.

What is the difference in height between A and B?   Answer .... m [1]

Answers: **(a)** −24   **(b)** 102   **J11/11/Q5**

24

(a) Evaluate $12 + 6 \div 2 - 8$.   Answer .... [1]

(b) Evaluate 2.6×0.2 .   Answer .... [1]

Answer: **(a)** 7   **(b)** 0.52   **J11/12/Q1**

25

(a) Evaluate $\frac{2}{3} - \frac{3}{8}$.   Answer .... [1]

(b) Evaluate $1\frac{3}{4} \times \frac{2}{9}$, giving your answer as a fraction in its lowest terms.

Answer .... [1]

Answer: **(a)** $\frac{7}{24}$ **(b)** $\frac{7}{18}$ **J11/12/Q3**

26

Given that $\pi = 3.141592654$, find the difference between $\frac{22}{7}$ and π, correct to two significant figures.

Show your working.   Answer ....[2]

Answer: 0.0013   **J12/11/Q6**

27 Evaluate

(a) $\frac{3}{5} - \frac{2}{7}$,

Answer ....[1]

(b) $1\frac{2}{3} \div 1\frac{3}{4}$.

Answer ....[2]

Answer: (a) $\frac{11}{35}$ (b) $\frac{20}{21}$

28

| 0.2 | 2 | $\sqrt{2}$ | $\frac{1}{3}$ | 0.83 | 8 | 81 |

From the numbers listed above, write down

(a) a prime number,

Answer ....[1]

(b) a cube number,

Answer ....[1]

(c) an irrational number.

Answer ....[1]

Answer: (a) 2 (b) 8 (c) $\sqrt{2}$

29 Evaluate

(a) 0.3×0.2,

Answer ....[1]

(b) $3.5 \div 0.07$.

Answer ....[1]

Answer: (a) 0.06 (b) 50

30 A bag contains red, yellow and green sweets.
$\frac{2}{5}$ of the sweets are red and $\frac{1}{4}$ of the sweets are yellow.

What fraction of the sweets are green?

Answer ....[2]

Answer: $\frac{7}{20}$

31

| 6 | 9 | 1 |

The three cards above can be rearranged to make three-digit numbers, for example 916.

Arrange the three cards to make

(a) the three-digit number that is closest to 650,

Answer ....[1]

(b) the three-digit number that is a multiple of 7, *Answer* ....[1]

(c) a three-digit number that is a square number. *Answer* ....[1]

Answer: **(a)** 619 **(b)** 196 **(c)** 169, 196 or 961   **J13/11/Q12**

32

(a) Evaluate $5 + 1 \times 0.3$. *Answer* ....[1]

(b) Evaluate $18 \div 0.2$. *Answer* ....[1]

Answers: **(a)** 5.3 **(b)** 90   **J14/11/Q2**

33

15 (a) Find an integer r such that $r > 5$ and $5r - 1$ is a square number. *Answer* $r =$ ....
[1]

(b) Find the value of s which makes $8s + 2$ a prime number. *Answer* $s =$ ....[1]

(c) Write down an irrational number between 7 and 8. *Answer* ....[1]

Answers: **(a)** e.g. 10 **(b)** e.g. 0 **(c)** e.g. $\sqrt{50}$ **J14/11/Q15**

34

(a) Evaluate $\frac{3}{8} + 1\frac{1}{3}$. *Answer* ....[1]

(b) Evaluate $5 - 3(2 - 1.4)$. *Answer* ....[1]

Answers: **(a)** $1\frac{17}{24}$ **(b)** 3.2   **J15/11/Q1**

35

(a) Express $3\frac{3}{4}\%$ as a fraction in its simplest form. *Answer* ....[1]

(b) Arrange these fractions in order, beginning with the smallest.

$$\frac{4}{5} \qquad \frac{3}{4} \qquad \frac{31}{40}$$

Answer .... , ... , ... [1]
smallest

Answers: **(a)** $\frac{3}{80}$ **(b)** $\frac{3}{4}$, $\frac{31}{40}$, $\frac{4}{5}$ **J15/11/Q3**

36

(a) Evaluate $12 - 6 \div 3 + 4$. *Answer* ....[1]

(b) Evaluate 0.3×1.5 . *Answer* ....[1]

Answers: **(a)** 14 **(b)** (0).45   **J16/11/Q1**

37

  (a) Evaluate $\frac{2}{3} - \frac{5}{8}$.

                                   Answer .... [1]

  (b) Evaluate $\frac{1}{3} \div \frac{7}{9}$, giving your answer as a fraction in its lowest terms.

                                   Answer .... [1]

Answers: (a) $\frac{1}{24}$ (b) $\frac{3}{7}$ **J16/11/Q2**

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38   **(a)** Evaluate 0.2×0.08.

                                   Answer .... [1]

  (b) Add one pair of brackets to make the statement below true.

        $2 \ \times \ 3 \ + \ 4 \ \times \ 5 \ = \ 70$ [1]

Answers: (a) (0).016   (b) $2 \times (3 + 4) \times 5$ **J17/11/Q1**

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39   **(a)** Evaluate $\frac{3}{5} - \frac{1}{8}$.   *Answer* .... [1]

    (b) Find A where $A \times - = -$.   *Answer* $A =$ .... [1]

    (c) Find the fraction which is exactly halfway between $\frac{5}{8}$ and $\frac{2}{3}$.

                                     Answer .... [1]

Answers: (a) $\frac{19}{40}$ (b) $\frac{14}{15}$ (c) $\frac{31}{48}$ **J17/11/Q13**

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40 Find the value of

  (a) 0.2×0.45,   *Answer (a)* .... [1]

  (b) $1\frac{3}{5} \div 2\frac{1}{3}$.   *(b)* .... [1]

Answers: (a) 0.09; (b) $\frac{24}{35}$.   **N01/Q1**

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41 Evaluate

  (a) $12\frac{1}{3} - 9\frac{3}{5}$,   *Answer (a)* .... [1]

  (b) $8.4 \div 0.02$.   *(b)* .... [1]

Answers: (a) $2\frac{11}{15}$; (b) 420.   **N03/Q3**

42

    (a) Write down the square root of $6\frac{1}{4}$.

    (b) State which of the following numbers are irrational

$$\sqrt{2} \times \sqrt{8}, \qquad \frac{22}{7}, \qquad \pi, \qquad 2\sqrt{3}.$$

Answer (a).. [1]

(b) .... [1]

Answers: **(a)** $2\frac{1}{2}$; **(b)** π and $2\sqrt{3}$.

43

    Evaluate

    (a) 0.4×0.06,

    (b) $\sqrt[3]{0.008}$.

Answer (a) .... [1]

(b) .... [1]

Answers: **(a)** 0.024; **(b)** 0.2.

44

    Evaluate

    (a) $1\frac{2}{3} \div 5$,

    (b) $4\frac{1}{4} - 1.43$, giving your answer as a decimal.

Answer (a) .... [1]

(b) .... [1]

Answers: **(a)** $\frac{1}{3}$; **(b)** 2.82.

45

    Evaluate

    (a) $10 - 7.56$,

    (b) 0.105×0.2 .

Answer (a) .... [1]

(b) .... [1]

Answers: **(a)** 2.44; **(b)** 0.021.

46

    (a) Evaluate $3\frac{1}{5} - 2\frac{3}{4}$.

    (b) $\frac{2}{3}$ of a plot of land is garden.

    $\frac{1}{5}$ of the garden is lawn.

    Find the fraction of the plot of land which is lawn.

Answer (a) .... [1]

(b) .... [1]

Answers: **(a)** $\frac{9}{20}$; **(b)** $\frac{2}{15}$.

47   Evaluate

     (a) $3 + 2(4 - 5)$,   *Answer (a)* ....[1]

     (b) $1\frac{1}{3} \div 2\frac{1}{2}$.   *(b)* ....[1]

Answer: **(a)** 1;   **(b)** $\frac{8}{15}$.   **N06/1/Q1**

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48   Express as a single fraction in its lowest terms

     (a) $3\frac{5}{9} - 2\frac{2}{3}$,   *Answer (a)* ....[1]

     (b) $\frac{3}{8} \div 2\frac{1}{4}$.   *(b)* ....[1]

Answer: **(a)** $\frac{8}{9}$ **(b)** $\frac{1}{6}$ **N07/1/Q2**

---

49   Evaluate

     (a) 0.3×0.06,   *Answer (a)* ....[1]

     (b) $0.4 + 0.3 \times 5$.   *(b)* ....[1]

Answer: **(a)** 0.018,   **(b)** 1.9.   **N08/1/Q1**

50

     (a)   Evaluate $\frac{2}{3} - \frac{4}{7}$.   *Answer (a)* ....[1]

     (b)   Evaluate $1\frac{1}{3} \times \frac{5}{8}$,   giving your answer in its simplest form.

                                       Answer (b) ....[1]

Answers: **(a)** $\frac{2}{21}$ **(b)** $\frac{5}{6}$ **N09/1/Q1**

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51   **(a)**   Add brackets to the equation in the answer space to make it correct.

                                 Answer (a) $4 + 6 \times 7 - 5 = 16$ [1]

     (b)   Find the value of 27×0.002.   *Answer (b)* ....[1]

Answers: **(a)** $4 + 6 \times (7 - 5) = 16$ **(b)**   0.054   **N09/1/Q2**

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52 Arrange these values in order of size, starting with the smallest.

$$\frac{9}{20} \qquad 0.39 \qquad 46\% \qquad \frac{2}{5}$$

Answer ....[2]

smallest

Answer: 0.39, $\frac{2}{5}$, $\frac{9}{20}$, 46%

53 (a) Evaluate $35 - 27.3$.

Answer (a) ....[1]

(b) Evaluate 1.3×0.03 .

Answer (b) ....[1]

Answers: (a) 7.7 (b) 0.039

54 (a) Evaluate $\frac{1}{3} + \frac{3}{7}$.

Answer (a) ....[1]

(b) Evaluate $2 \div 2\frac{2}{3}$.

Answer (b) ....[1]

Answers: (a) 16/21 (b) 3/4

55 (a) Evaluate $3\frac{1}{7} - 2\frac{1}{3}$.

Answer (a) ....[1]

(b) Evaluate $\frac{2}{9} \times 1\frac{7}{8}$, giving your answer as a fraction in its lowest terms.

Answer (b) ....[1]

Answer: (a) $\frac{17}{21}$ (b) $\frac{5}{12}$

56 (a) Evaluate $6.3 \div 0.09$.

Answer (a) ....[1]

(b) Find the decimal number that is exactly halfway between 3.8 and 4.3 .

Answer (b) ....[1]

Answer: (a) 70 (b) 4.05

57 (a) Evaluate $3 + 5(3 - 1.4)$.

Answer ....[1]

(b) Evaluate 0.2×0.07 .

Answer ....[1]

Answers: (a) 11 (b) 0.014

58 (a) Evaluate $3\frac{2}{3} - 2\frac{4}{5}$.

Answer .... [1]

(b) Express $\frac{48}{84}$ in its lowest terms.

Answer .... [1]

Answers: (a) $\frac{13}{15}$ (b) $\frac{4}{7}$

N11/11/Q2

59 (a) Evaluate $2\frac{3}{4} - 1\frac{7}{9}$.

Answer .... [1]

(b) Evaluate $0.7 - 0.1 \times 3$.

Answer .... [1]

Answer: (a) $\frac{35}{36}$ (b) 0.4

N11/12/Q1

60 (a) Evaluate $3\frac{2}{5} - 2\frac{5}{6}$.

Answer .... [1]

(b) Evaluate $\frac{2}{3} \div 3\frac{3}{4}$.

Answer .... [1]

Answers: (a) $\frac{17}{30}$ (b) $\frac{8}{45}$

N12/11/Q1

61 (a) Evaluate $0.7 + 0.2 \times 0.3$.

Answer .... [1]

(b) Evaluate $\frac{0.9}{0.06}$.

Answer .... [1]

Answers: (a) 0.76 (b) 15

N12/11/Q2

62 (a) Evaluate $2\frac{3}{4} - 1\frac{13}{16}$.

Answer .... [1]

(b) Evaluate $5 + 3 \times 2 + 2(2 - 3)$.

Answer .... [1]

Answers: (a) $\frac{15}{16}$ (b) 9

N13/11/Q1

63 (a) Evaluate 0.02×1.2.

Answer .... [1]

(b) Arrange these values in order of size, starting with the smallest.

$$22\% \qquad \frac{2}{9} \qquad 0.2$$

Answer .... , ... , ... [1]
smallest

Answers: (a) 2:9 (b) 4.8

N13/11/Q2

16

64　(a)　Evaluate $10 + 2n^2$ when $n = -1$.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

　　(b)　Evaluate 0.4×0.2.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

Answers: **(a)** 12; **(b)** 0.08.　　　　　　　　　**N14/11/Q2**

65　(a)　Write 3% as a fraction.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

　　(b)　Work out $90 - 16 \div 2$.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

Answers: **(a)** $\dfrac{3}{100}$; **(b)** 82.　　　　　　**N14/11/Q3**

66　x is an integer between 50 and 70.

Write down the value of x when

　　(a)　x is a cube number,

　　　　　　　　　　　　　　　　　　　Answer ....[1]

　　(b)　x is a prime factor of 268.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

Answers: **(a)** 64; **(b)** 67.　　　　　　　　　**N14/11/Q4**

67　(a)　Work out $12 + 6 \div 3 + 1 \times 5$.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

　　(b)　Work out $\dfrac{7}{9} - \dfrac{3}{5}$.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

Answers: **(a)** 19 **(b)** $\dfrac{8}{45}$　　　　　　　　**N15/11/Q1**

68　(a)　Evaluate $3\dfrac{1}{6} - 2\dfrac{3}{5}$.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

　　(b)　Evaluate 0.03×0.11.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

Answers: **(a)** $\dfrac{17}{30}$ **(b)** 0.0033　　　　　　**N16/11/Q1**

69　(a)　Express $32\dfrac{1}{2}$% as a fraction in its simplest form.

　　　　　　　　　　　　　　　　　　　Answer ....[1]

　　(b)　Arrange these values in order of size, starting with the smallest.

　　　　　0.38　　　　　$\dfrac{9}{25}$　　　0.4　　　$\dfrac{7}{20}$

　　　　　　　Answer .... , ... , ... , ... [1]
　　　　　　　　　　　smallest

Answers: **(a)** $\dfrac{13}{40}$ **(b)** $\dfrac{7}{20}, \dfrac{9}{25}$, 0.38, 0.4　　　**N16/11/Q3**

70

(a) Evaluate $1\frac{3}{8} - \frac{2}{3}$.

Answer .... [1]

(b) Evaluate 0.4×1.3.

Answer .... [1]

Answers: (a) $\frac{17}{24}$ (b) 0.52

71

The table shows the square roots, given correct to 4 significant figures, of some numbers from 31.0 to 32.9.

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
31	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648
32	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736

For example, the square root of 32.5 is 5.701.

Use the table to find

(a) the difference between the square root of 32.5 and the square root of 31.3,

Answer .... [1]

(b) an estimate of the square root of 32.25,

Answer .... [1]

(c) the number which has a square root of 56.39.

Answer .... [1]

Answers: (a) 0.106 (b) 5.678 to 5.68(0) (c) 3180

Simultaneous Equations Paper 1

1   Solve the simultaneous equations

$$2y = 3x - 13,$$
$$5x - 6y = 23.$$

Answer $x = $ ....

$y = $ .... [3]

2   Solve the simultaneous equations

$$4x - y = 9,$$
$$2x - 3y = -23.$$

Answer $x = $ ....

$y = $ .... [3]

Answers: $x = 5$, $y = 11$.

3   Solve the simultaneous equations

$$3x + y = 95,$$
$$x + y = 29.$$

Answer $x = $ ....

$y = $ ....[2]

Answer: $x = 33$, $y = -4$.

4   Solve the simultaneous equations

$$3x = 7y,$$
$$12y = 5x - 1.$$

Answer $x = $ ....

$y = $ ....[3]

Answer: $x = -7$, $y = -3$

5   **(b)**   Solve the simultaneous equations $x + y = 29,$
$$4x = 95 - 2y.$$

(b) $x = $ .... $y = $ ....[3]

(b) $x = 18\frac{1}{2}$ $y = 10\frac{1}{2}$

6   Solve the simultaneous equations

$$2x - 3y = 13,$$
$$3x + y = 3.$$

Answer $x = $ ....

$y = $ ....[3]

Answer: $x = 2$ $y = -3$

7   **(b)** Solve the simultaneous equations

$$5x - 2y = 16,$$
$$2x - 3y = 13.$$

Answer   *(b)* $x =$ ....

$y =$ .... [3]

(b) $x = 2$ $y = -3$

8   Solve the simultaneous equations.

$$3x + 2y = 7$$
$$x - 3y = 17$$

Answer $x =$ ....

$y =$ ....[3]

Answer: 5   - 4

9   **(a)** Solve $10 - 3(2x - 1) = 3x + 1$.

Answer $x =$ ....[2]

   (b) Solve the simultaneous equations.

$$4x + 3y = 11$$
$$2x - 5y = 25$$

Answer $x =$ ....

$y =$ ....[3]

Answer: **(a)** $1\frac{1}{3}$ **(b)** $x = 5, y = -3$

10   Solve the simultaneous equations.

$$3x + 5y = 0$$
$$2x - 3y = 19$$

Answer $x =$ ....

$y =$ .... [3]

Answer: $x = 5$ $y = -3$

11   Solve the simultaneous equations.

$$3x = 4y$$
$$1 + 5x = 6y$$

Answer $x =$ ....

$y =$ .... [3]

Answers: $x = -2, y = -1.5$

12   Solve the simultaneous equations

$$x + 2y = 8,$$
$$y = \frac{1}{3}x + 9.$$

Answer $x =$ ....

$y =$ ....[3]

Answers: $x = -6, y = 7$

13   Solve the simultaneous equations.

$$3y = 2x$$
$$x + 2y = 21$$

Answer $x =$ ....

$y =$ .... [3]

Answer: $x = 9$, $y = 6$

14) Solve the simultaneous equations.

$$2x + 3y = 0$$
$$x + 4y = -15$$

Answer $x = $ ....

$y = $ .... [3]

Answer: $x = 9$, $y = -6$

15 Solve the simultaneous equations.

$$2x - 3y = 11$$
$$5x - 4y = 24$$

Answer $x = $ ....

$y = $ .... [3]

Answer: $x = 4$, $y = -1$.

16 Solve the simultaneous equations.

$$2x + 5y = 2$$
$$3x + 4y = -4$$

Answer $x = $ ....

$y = $ .... [4]

Answers: $x = -4$, $y = 2$

17 Solve the simultaneous equations.

$$2y = x + 12$$
$$3y = 11 - 2x$$

Answer $x = $ ....

$y = $ .... [3]

Answer: $x = -2$, $y = 5$

Algebra Paper 1

1 (a) Factorise completely $\qquad 18rc - 3rd - 6tc + td.$ $\qquad$ *Answer* (a) .... [2]

(b) Solve the equation $\qquad \dfrac{4}{x+3} = \dfrac{3}{2x}$. $\qquad$ (b) $x =$ .... [2]

2 Given that $\qquad S = \dfrac{RV}{3V-1}$,

(a) calculate the value of S when $R = 100$ and $V = -13,$ $\qquad$ *Answer* (a) $S =$ .... [1]

(b) express V in terms of R and S. $\qquad$ (b) $V =$ .... [3]

3 (a) Factorise $\quad x^2 - 7x + 12.$ $\qquad$ *Answer* (a).. [1]

(b) Solve $\quad (x+1)(3x-2) = 0.$ $\qquad$ (b) $x =$ .... or .... [1]

Answers: (a) $(x-3)(x-4)$; (b) –1 or 2/3.

4 (a) Expand and simplify $\quad (x-1)(x^2+x+1).$ $\qquad$ *Answer* (a)................... [2]

(b) Factorise $\quad ax - bx - 3ay + 3by.$ $\qquad$ (b).. [2]

Answers: (a) $x^3 - 1$; (b) $(a-b)(x-3y)$.

5
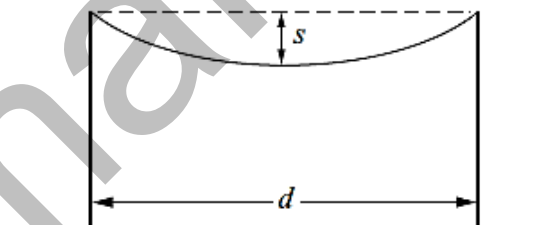
Two vertical posts of the same height stand on horizontal ground.
The distance between the posts is d centimetres.
When a wire of length w centimetres is suspended between the posts, the sag in the middle is s centimetres.
The sag is given by the formula $s = \sqrt{\dfrac{3d(w-d)}{8}}$.

(a) Find s when $d = 800$ and $w = 803.$ $\qquad$ *Answer* (a) $s =$ .... [1]

(b) Express w in terms of d and s. $\qquad$ (b) $w =$ .... [3]

Answers: (a) 30; (b) $V = \dfrac{8s^2 + 3d^2}{3d}$.

6 Express as a single fraction in its simplest form $\dfrac{2}{x-3} - \dfrac{1}{x+2}$.   *Answer* .... [2]

Answer: $\dfrac{x+7}{(x-3)(x+2)}$.

7 Solve the equation $\dfrac{4}{x+3} = \dfrac{x-1}{3}$.   *Answer* .... [3]

Answers: 3 and – 5.

8

(a) Factorise completely $5a^2 - 20$.   *Answer (a)* .... [2]

A formula connecting x and y is $y = \dfrac{k}{x^3}$, where k is a constant.

Given that $y = -1$ when $x = 2$, calculate the value of

(i) k,   *Answer (b)*(i) $k = $ .... [1]

(ii) x when $y = 64$.   (ii) $x = $ .... [1]

Answers: (a) $5(a + 2)(a - 2)$; (b)(i) $- 8$, (ii) $-\dfrac{1}{2}$.

9

(a) Factorise $3tx - 2sx + 15ty - 10sy$.   *Answer (a)* .... [2]

(b) Solve the equation $\dfrac{x-2}{4} + \dfrac{x+1}{3} = 1$.   (b) $x = $.................................... [2]

(c) Factorise $2y^2 - 3y - 2$.   (c) .... [2]

Answers: (a) $(3t - 2s)(x + 5y)$; (b) 2; (c) $(2y + 1)(y - 2)$.

10 (a) Given that $x = 6$ is a solution of $\dfrac{x^2}{3} + k = 0$, find the value of k.

(b) Solve $2y^2 - 3y - 2 = 0$.   *Answer* (a) $k = $ ....[1]

(b) $y = $ .... or ....[2]

Answer: (a) −12   (b) 2 or −½

23

11 $C = \dfrac{5}{9}(F - 32)$

   (a) Calculate C when $F = -4$.

   (b) Express F in terms of C.

Answer (a) $C =$ ....[1]

(b) $F =$ ....[2]

Answer: (a) -20 (b) $\dfrac{9C + 160}{5}$ **J06/1/Q19**

12 (a) Simplify

     (i) $x(3x + 2) - (2x + 4)$,

     (ii) $\dfrac{ax^2 - x^2}{ax - x}$.

   (b) Factorise completely $7x^2 - 63$.

Answer (a)(i) ....[1]

(ii) ....[2]

(b) ....[2]

Answer: (a)(i) $3x^2 - 4$ (ii) x (b) $7(x - 3)(x + 3)$ **J06/1/Q23**

13 Factorise

   (a) $2x^2 - 7x - 15$,

   (b) $2yt - 8ys - zt + 4zs$.

Answer (a) ....[2]

(b) ....[2]

Answer: (a) $(2x + 3)(x - 5)$ (b) $(2y - z)(t - 4s)$ **J07/1/Q21**

14 (a) Solve

     (i) $9 - k < 7$,

     (ii) $\dfrac{5}{2t} = \dfrac{1}{12}$.

Answer (a) (i) ....[1]

(ii) $t =$ ....[1]

Answer: (a)(i) $k > 2$ (ii) 30 **J07/1/Q22**

15 Express as a single fraction in its simplest form

$$\dfrac{3}{2t - 1} - \dfrac{2}{t + 2}.$$

Answer ....[3]

    J08/1/Q15

Answer: $\dfrac{8 - t}{(2t - 1)(t + 2)}$

16 (a) Factorise completely

    (i) $15x^2 + 10x$,

    Answer (a)(i) ....[1]

    (ii) $t^2 - 2t - 15$.

    (ii) ....[1]

(b) Solve $4(x - 0.3) = 3(x - 0.2)$.

    (b) $x =$ ....[2]

Answers: **(a)(i)** $5x(3x + 2)$ **(ii)** $(t + 3)(t - 5)$ **(b)** 0.6 **J08/1/Q19**

17   (a) Factorise $x^2 - y^2$.

    Answer (a) ....[1]

(b) Evaluate $102^2 - 98^2$.

    Answer (b) .... [1]

Answer: **(a)** $(x - y)(x + y)$ **(b)** 800 **J09/1/Q4**

18   (a) Simplify $4a^3 \times a^2$.

    Answer (a) ....[1]

) Simplify fully $3x(x + 5) - 2(x - 3)$.

    Answer (b) .... [2]

Answer: **(a)** $4a^5$ **(b)** $3x^2 + 13x + 6$ **J09/1/Q7**

19 Given that n is an integer and $n > 1$, decide whether each statement in the table is **true** or **false**.

For each statement write **true** or **false** in the table.
If you write false, give an example to justify your decision.

Statement	True or False	Example (if false)
$n^3 > 1$		
$\dfrac{1}{n} > \dfrac{1}{n^2}$		
$(n - 1)(n + 3)$ is always odd		

    [2]

Answer: TRUE, TRUE, FALSE with valid example   **J10/11/Q7**

20   (b) Factorise completely

    (i) $4x^2 - 25y^2$,

    Answer (b)(i) .... [1]

    (ii) $5ax - 5a^2 - 2x + 2a$.

    Answer (b)(ii) .... [2]

(b)(i) $(2x - 5y)(2x + 5y)$ **(b)(ii)** $(5a - 2)(x - a)$ **J10/11/Q21**

21   **(a)** Solve

      (i) $5 - 2(3x - 1) = 2x + 1$,   *Answer* $(a)(i)$ $x =$ .... [2]

      (ii) $\dfrac{2}{5t} = \dfrac{3}{4}$.   *Answer* $(a)(ii)$ $t =$ .... [2]

Answer: **(a)(i)** 0.75   **(a)(ii)** $\dfrac{8}{15}$ **J10/11/Q24**

22   **(a)** Solve $\dfrac{3}{x-1} = 2$.   *Answer* (a) $x =$ .... [1]

) Given that $p = 2t - r$, express t in terms of p and r.   *Answer* (b) $t =$ .... [1]

Answer: **(a)** 2.5   **(b)** $\dfrac{p+r}{2}$ **J10/12/Q6**

23   **(a)** Factorise completely

      (i) $3x^2 - 12x$,   *Answer* $(a)(i)$ .... [1]

      (ii) $x^2 - xy - 2y^2$.   *Answer* $(a)(ii)$ .... [1]

    (b) Simplify $\dfrac{x^2 + 4x}{x^2 - 16}$.   *Answer* (b) .... [2]

Answer: **(a)(i)** $3x(x-4)$ **(ii)** $(x+y)(x-2y)$ **(b)** $\dfrac{x}{x-4}$ **J10/12/Q21**

24   Factorise completely

    (a) $12ab^2 - 8a^2b$,   *Answer* .... [1]

    (b) $2x^2 + 3x - 20$.   *Answer* .... [2]

Answers: **(a)** $4ab(3b-2a)$ **(b)** $(2x-5)(x+4)$ **J11/11/Q9**

25   **(a)** Factorise completely $9pq - 12q^2$.   *Answer* .... [1]

    (b) Factorise completely $8px + 4py - 6x - 3y$.   *Answer* .... [2]

Answer: **(a)** $3q(3p-4q)$ **(b)** $(4p-3)(2x+y)$ **J11/12/Q15**

26   Factorise completely

    (a) $12x^2 - 15x^3$,   *Answer* .... [1]

    (b) $x^2 - x - 6$.   *Answer* .... [1]

Answer: **(a)** $3x^2(4-5x)$ **(b)** $(x-3)(x+2)$ **J12/11/Q4**

27 Solve

(a) $5x - 2 = 1$, *Answer* $x = $ ....[1]

(b) $3 - y \leqslant 1$, *Answer* ....[1]

(c) $\dfrac{2t-1}{4} = \dfrac{1-t}{3}$. *Answer* $t = $ ....[2]

Answer: (a) $\frac{3}{5}$ (b) $(y) \geq 2$ (c) $\frac{7}{10}$ J12/11/Q18

28 $\dfrac{1}{b} = \dfrac{1}{c} + \dfrac{1}{d}$

(a) Evaluate b when $c = 3$ and $d = 8$. *Answer* $b = $ ....[2]

(b) Rearrange the formula to make d the subject. *Answer* $d = $ ....[3]

Answer: (a) $\frac{24}{11}$ (b) $\frac{bc}{c-b}$ J12/11/Q22

29 $b = m(a - c)$

(a) Evaluate b when $m = 5$, $a = 8$ and $c = -3$. *Answer* $b = $ ....[1]

(b) Rearrange the formula to make c the subject. *Answer* $c = $ ....[2]

  J13/11/Q10

Answer: (a) 55 (b) $\dfrac{ma-b}{m}$

30 (a) Factorise fully $10x^2y + 15xy^2$. *Answer* ....[1]

(b) Factorise $25a^2 - b^2$. *Answer* ....[1]

(c) Simplify $\dfrac{3}{(x+1)^2} - \dfrac{2}{x+1}$. *Answer* ....[2]

(d) Simplify $\dfrac{3a^2}{10bc} \div \dfrac{9a}{5b^2c}$. *Answer* ....[2]

Answer: (a) $5xy(2x + 3y)$ (b) $(5a - b)(5a + b)$ (c) $\dfrac{1-2x}{(x+1)^2}$ (d) $\dfrac{ab}{6}$ J13/11/Q25

31 Make a the subject of the formula $y = \dfrac{a-4}{3-a}$. *Answer* $a = $ ....[3]

Answer: $\dfrac{3y+4}{y+1}$ J14/11/Q9

32   **(a)** Expand and simplify

        (i) $4(2t+3)+5,$ *Answer* .... [1]

        (ii) $6p+3q-2(2p-5q).$ *Answer* .... [1]

     (b) Factorise completely   *Answer* .... [1]

       $25x^3y^2 - 15x^2y.$

Answers: **(a)(i)** $8t+17$ **(ii)** $2p+13q$ **(b)** $5x^2y(5xy-3)$ **J14/11/Q17**

33   **(a)** Given that $x^2 - 14x + 40 = (x-a)^2 + b,$ find the values of a and b.

                 Answer $a =$ ....

                       $b =$ .... [2]

     (b) Solve the equation $3x^2 + 7x - 6 = 0$ by factorisation.

                 Answer $x =$ .... or .... [2]

Answers: **(a)** $a = 7$ $b = -9$ **(b)** $\frac{2}{3}$ -3 **J14/11/Q20**

34   Express $\frac{1}{x} + 2 - \frac{3}{x+1}$ as a single fraction in its simplest form.   *Answer* .... [3]

Answer: $\dfrac{2x^2 + 1}{x(x+1)}$ **J15/11/Q11**

35   **(a)** Factorise completely $3 - 12a^2.$ *Answer* .... [2]

     (b) Factorise $x^2 - 6y + 2xy - 3x.$ *Answer* .... [2]

                                          J15/11/Q17

Answers: **(a)** $3(1-2a)(1+2a)$ **(b)** $(x-3)(x+2y)$

36   Factorise completely $3xy - 20 + 5x - 12y.$ *Answer* .... [2]

Answer: $(x-4)(3y+5)$ **J16/11/Q10**

37   **(a)** Given that $a = 3$ and $b = -7$, evaluate

        (i) $2a - b,$ *Answer* .... [1]

        (ii) $a^2 + b^2.$ *Answer* .... [1]

     (b) $A = 2r^2 + 5$

       Rearrange the formula to make r the subject.   *Answer* $r =$ .... [2]

                                            J17/11/Q16

Answers: **(a)(i)** 13 **(ii)** 58 **(b)** $(\pm)\sqrt{\dfrac{A-5}{2}}$

38

(a) Solve $\dfrac{6}{x+1} = \dfrac{5}{x-3}$.

Answer $x =$ .... [2]

(b) $\qquad$ $f(x) = x - 3$ $\qquad$ $g(x) = x^2 + 1$ $\qquad$ *Answer* .... [1]

(i) Find $f(-5)$.

(ii) Find m given that $g(m-3) = 17$. $\qquad$ *Answer* $m =$ .... or .... [3]

Answers: **(a)** 23 **(b)(i)** −8 **(ii)** −1 or 7 $\qquad$

39

Factorise completely

(a) $9a - 12a^2$, $\qquad$ *Answer (a)* .... [1]

(b) $4y^2 - 1$, $\qquad$ *(b)* .... [1]

(c) $x^2 - 7x + 12$. $\qquad$ *(c)* .... [1]

Answers: **(a)** $3a(3 - 4a)$; **(b)** $(2y - 1)(2y + 1)$; **(c)** $(x - 3)(x - 4)$. $\qquad$

40

(a) Factorise $x^2 - 5x + 6$.

(b) Given that $x = 4$ is a solution of the equation $x^2 + 3x + c = 0$, find the value of c.

Answers: **(a)** $(x - 2)(x - 3)$; **(b)** -28.

41

(a) (i) Factorise $ax - bx$.

(ii) Hence evaluate $1426 \times 0.6789 - 426 \times 0.6789$.

(b) Solve the equation

$$3(x - 5) - 2 = 7 - (1 - x).$$

Answers: **(a)(i)** $x(a - b)$, **(ii)** 678.9; **(b)** $x = 11\frac{1}{2}$. $\qquad$

42

(a) Factorise $(a - 2b) - 3c(a - 2b)$. $\qquad$ *Answer (a)* ....[1]

(b) Simplify $5t(t + 3) - 3(5t - 2)$. $\qquad$ *(b)* .... [1]

(c) Solve the simultaneous equations

$\qquad$ *(c) x =* ....

$$2x - 3y = 15,$$
$$3x - 7y = 27\tfrac{1}{2}.$$

$\qquad$ *y =* .... [3]

Answers: (a) $(a - 2b)(1 - 3c)$, $5t^2 + 6$; (c) $x = 4\frac{1}{2}$, $y = -2$.

43 (a) Factorise fully $5x^2 - 10x$.   *Answer (a)* .... [1]

(b) Solve $3y + 6 = 7y - 10$.   *(b) y =* .... [1]

(c) Solve $3p(p + 2) = 0$.   *(c) p =* .... or .... [1]

Answers: (a) $5x(x - 2)$; (b) $y = 4$; (c) $p = 0$ or -2.

44 (a) Simplify $25x^2 \div 5x^{-4}$.   *Answer (a)* ....[1]

(b) Solve $(2x - 3)(x + 2) = 0$.   *(b) x =* .... or ....[1]

Answer: (a) $5x^6$;   (b) $1\frac{1}{2}$ or -2.

45 Solve the equations

(a) $\dfrac{24}{x - 4} = 1$,   *Answer (a) x =*...[1]

(b) $12 - 2(5 - y) = 5y$.   *(b) y =*...[2]

Answer: (a) $x = 28$ (b) $y = \frac{2}{3}$

46 Solve the simultaneous equations $2x - y = 16$,   *Answer x =* ....
$3x + 2y = 17$.   *y =*...[3]

Answer: $x = 7$; $y = -2$

47 Factorise completely

(a) $15a^2 + 12a^3$,   *Answer (a)* ....[1]

(b) $1 - 16b^2$,   *(b)* ....[1]

(c) $6cx - 3cy - 2dx + dy$.   *(c)* ....[2]

Answer: (a) $3a^2(5 + 4a)$ (b) $(1 - 4b)(1 + 4b)$ (c) $(3c - d)(2x - y)$

48 (a) Expand and simplify $(p - 5)(p + 4)$.

        Answer (a) ....[1]

(b) Factorise completely

(i) $4x^2 + 12xy + 9y^2$,   *(b)(i)* ....[2]

(ii) $3m^2 - 48$.   *(ii)* ....[2]

Answers: (a) $p^2 - p - 20$,   (b)(i) $(2x + 3y)^2$ or $(2x + 3y)(3x + 3y)$,   (ii) $3(m - 4)(m + 4)$.   **N08/1/Q21**

49 The force acting on an object during a collision is given by the formula

$$F = \frac{mv - mu}{t}.$$

(a) Given that $m = 4$, $v = 5$, $u = 3$ and $t = 0.01$, find the value of F.

Answer (a) $F = $ ....[1]

(b) Rearrange the formula to make m the subject.   *Answer (b)* $m = $ ....[2]

N09/1/Q9

Answers: (a) 800   (b) $\dfrac{Ft}{v - u}$

50 **(a)** Factorise completely

    (i) $21a^2 - 14a$,   *Answer (a)(i)* ....[1]

    (ii) $x^2 - 3x - 40$.   *Answer (a)(ii)* ....[1]

(b) Given that $y = 3$ is a solution of the equation $2y^2 + ky - 27 = 0$, find the other solution.

Answer (b) $y = $ ....[2]

Answers: (a)(i) $7a(3a - 2)$ (ii) $(x - 8)(x + 5)$ (b) $-4\frac{1}{2}$ **N09/1/Q19**

51 **(a)** Remove the brackets and simplify $4(7x - 3) - 3(5x - 4)$.

Answer (a) ....[1]

(b) Express as a single fraction in its simplest form $\dfrac{4}{3y} - \dfrac{5}{4y}$.

Answer (b) ....[1]

(c) Simplify $(4a^2b) \times (3ab^3)$.   *Answer (c)* ....[1]

Answers: (a) $13x$ (b) $1/(12y)$ (c) $12a^3b^4$ **N10/11/Q12**

52 **(a)** Factorise completely $16a^2 - 6a$.   *Answer (a)* ....[1]

(b) Factorise completely $6x + 3xy - 4y - 8$.   *Answer (b)* ....[2]

Answers: (a) $2a(8a-3)$ (b) $(3x-4)(y+2)$ **N10/11/Q13**

53 Factorise

    (a) $4t^2 - 9$,   *Answer (a)* ....[1]

    (b) $3x^2 + 5x - 2$.   *Answer (b)* ....[1]

Answer. (a) $(2t-3)(2t+3)$ (b) $(3x-1)(x+2)$

54 Make x the subject of the formula $y = 2x^2 + 3$.   *Answer* $x =$ ....[2]

Answer. $\sqrt{\dfrac{y-3}{2}}$

55 Factorise completely $2xy - 3x - 10y + 15$.   *Answer* ....[2]

Answer. $(x-5)(2y-3)$

56 (a) Factorise $9x^2 - 1$.   *Answer* ....[1]

(b) Solve the equation $2y^2 + 29y - 15 = 0$.   *Answer* $y =$ .... or ....[3]

Answer. (a) $(3x-1)(3x+1)$ (b) -15 and $\frac{1}{2}$

57 Factorise completely

(a) $20p + 25p^2$,   *Answer* ....[1]

(b) $9 - 4t^2$,   *Answer* ....[1]

(c) $9 + 35x - 4x^2$.   *Answer* ....[1]

Answers: (a) $5p(4 + 5p)$ (b) $(3 - 2t)(3 + 2t)$ (c) $(9 - x)(1 + 4x)$

58 Factorise $2ac - 3bc - 6bd + 4ad$.   *Answer* ....[2]

Answer. $(2a - 3b)(c + 2d)$.

59
$$s = \frac{n}{2}(a + b)$$

(a) Evaluate s when $n = 200$, $a = 3.6$ and $b = 5.7$.   *Answer* $s =$ ....[1]

(b) Rearrange the formula to make b the subject.   *Answer* $b =$ ....[2]

Answers: (a) 930; (b) $\dfrac{2s - an}{n}$.

60
$$4 = \sqrt{\frac{cx + 1}{dx - 1}}$$

Find x in terms of c and d.   *Answer* $x =$ ....[3]

Answer. $\dfrac{17}{16d - c}$

61 (a) Expand and simplify $10 - 3(3x - 2)$.   *Answer* ....[1]

(b) Simplify fully $\dfrac{3x^2 + 16x + 5}{9x^2 - 1}$.   *Answer* ....[3]

Answers: (a) $16 - 9x$ (b) $\dfrac{x+5}{3x-1}$

62 Factorise completely

(a) $5 - 20t^2$,

Answer .... [2]

(b) $3y^2 - 2xy - 6x + 9y$.

Answer .... [2]

Answers: (a) $5(1-2t)(1+2t)$ (b) $(3y-2x)(y+3)$

63 (a) Simplify $8 - 3(2t+1)$.

Answer .... [1]

(b) Simplify $\dfrac{(2x^2y)^3}{6x^4y^4}$.

Answer .... [2]

Answers: (a) $5 - 6t$ (b) $\dfrac{4x^2}{3y}$

64 (a) Factorise $25a^2 - 5a$.

Answer .... [1]

(b) Factorise $9b^2 - 16$.

Answer .... [1]

(c) Factorise $4xy + 3t + 6y + 2tx$.

Answer .... [2]

Answers: (a) $5a(5a-1)$ (b) $(3b-4)(3b+4)$ (c) $(2x+3)(2y+t)$

Algebra Paper 2

1

   (a) Remove the brackets and simplify $(q+3r)\,(2q-r)$.   [2]

   (b) Given that $m = -2$ and $n = 4$, evaluate

        (i) $5m^3$,   [1]

        (ii) $\dfrac{m}{n} + \dfrac{n}{m}$.   [1]

   (c) Factorise completely $3y^2 - 3$.   [2]

   (d) Peter has Rs200 and Paul has Rs2120.
       When they are each given Rsx, Paul has 5 times as much as Peter.
       Write down an equation in x and solve it.   [3]

<div align="right">J02/2/Q2</div>

2

   (a) Factorise completely $2tv + t - 10v - 5$.   [2]

   (b) Make k the subject of the formula

$$\sqrt{\frac{h}{k}} = 3.$$

                       [2]

   (c) Solve the equation $x^2 - 23x + 81 = 0$, giving both answers correct to two decimal places.   [4]

<div align="right">J04/2/Q2</div>

Answers: **(a)** $(t-5)(2v+1)$; **(b)** $k = \dfrac{h}{9}$; **(c)** 18.66 or 4.34;

3

   (a) Remove the brackets and simplify

        (i) $4(3 - 2p) - 3(1 - p)$,   [1]

        (ii) $(3q - r)(q + 2r)$.   [2]

   (b) Factorise completely $18t^2 - 2$.   [2]

   (c) Given that $y = 18 + 3x^2$,

        (i)   find the value of y when $x = -2$,   [1]

        (ii)   find the values of x when $y = 93$,   [2]

        (iii)   express x in terms of y.   [2]

<div align="right">J05/2/Q1</div>

Answers: **(a)(i)** $9 - 5p$, **(ii)** $3q^2 + 5qr - 2r^2$; **(b)** $2(3t + 1)(3t - 1)$; **(c)(i)** 30, **(ii)** ± 5, **(iii)** $\sqrt{\dfrac{y - 18}{3}}$.

4

   (a) Solve the equation $3x^2 - 4x - 5 = 0$, giving your answers correct to two decimal places.   [4]

   (b) Remove the brackets and simplify $(3a - 4b)^2$.   [2]

(c) Factorise completely $12 + 8t - 3y - 2ty$. [2]

5　It is given that $y = \dfrac{3x^2 - 12}{5}$.

(a) Find y when $x = -3$. [1]

(b) Find the values of x when $y = 0$. [2]

(c) For values of x in the range $-3 \leqslant x \leqslant 2$, write down

　　(i) the largest value of y, [1]

　　(ii) the smallest value of y. [1]

(d) Express x in terms of y. [2]

(e) It is also given that $y = \dfrac{t - 3}{2}$ when $x = t$.

　　(i) Show that t satisfies the equation $6t^2 - 5t - 9 = 0$. [1]

　　(ii) Solve the equation $6t^2 - 5t - 9 = 0$, giving each answer correct to two significant figures. [4]

6　**(c)** The recommended maximum heart rate, H, for a man during exercise, is given by the formula

$$H = \frac{4}{5}(220 - n) ,$$

　　where n years is the age of the man.

　　(i) Calculate H when $n = 25$. [1]

　　(ii) Calculate n when $H = 144$. [1]

　　(iii) Make n the subject of this formula. [2]

7　**(a)** Express as a single fraction in its simplest form $\dfrac{4}{x + 3} - \dfrac{3}{2x - 1}$. [3]

(b) It is given that $k = \sqrt{2lm + 3n}$.

　　Express m in terms of k, l and n. [2]

(c) Solve the equation $3x^2 - 4x - 16 = 0$.
　　Give your answers correct to 2 decimal places. [4]

8 (a) Express as a single fraction in its simplest form

   (i) $\dfrac{1}{2x} - \dfrac{2}{5x}$, *Answer* ....[1]

   (ii) $\dfrac{4}{x} + \dfrac{7}{x-3}$. *Answer* ....[2]

Answers: 1(a)(i) $\dfrac{1}{10x}$ (ii) $\dfrac{11x-12}{x(x-3)}$ **J11/22/Q1**

---

9 (a) Solve $4(x-2) = 7 - x$. *Answer* $x =$ .... [2]

   (b) Solve the simultaneous equations.

   $2x + y = 7$ *Answer* $x =$ ....
   $4x - 3y = 19$ $y =$ .... [3]

   (c) (i) Write down the integer values that satisfy $-1 \leqslant n < 2$.

   Answer .... [1]

   (ii) Solve $2 - 3y < 8$. *Answer* .... [2]

Answer: (a) $x = 3$ (b) $x = 4, y = -1$ (c)(i) $-1, 0, 1$ (ii) $y > -2$ **J13/21/Q1**

---

10 (a) Express as a single fraction in its simplest form $\dfrac{x}{(x-4)^2} - \dfrac{2}{x-4}$. *Answer* .... [2]

   (b) Solve the simultaneous equations.

   $2x - 3y = 14$ *Answer* $x =$ ....
   $6x + 4y = 3$ $y =$ .... [3]

   (c) Solve $x(x-4) = 6 + x$. *Answer* $x =$ .... or .... [3]

   (d) Simplify $\dfrac{y^2 - 9}{2y^2 - y - 15}$. *Answer* .... [3]

Answers: (a) $\dfrac{8-x}{(x-4)^2}$ (b) $x = 2.5$ $y = -3$ (c) $x = 6$ or -1 (d) $\dfrac{y-3}{2y+5}$ **J14/21/Q1**

---

11 (a) Expand the brackets and simplify $(x-1)(x^2 + x + 1)$. *Answer* .... [2]

   (b) Solve the equation $\dfrac{3x}{x+2} - \dfrac{4}{x-2} = 3$. *Answer* .... [3]

   (c) Solve these simultaneous equations. *Answer* $x =$ ....
   $4x - 3y = 4$
   $4y - 3x = -6.5$ $y =$ .... [4]

Answers: (a) $x^3 - 1$ (b) 0.4 (c) $x = -0.5, y = -2$ **J15/21/Q6**

36

12

(a) Solve the equation $(x-5)^2 = 81$. [2]

(b) Express as a single fraction in its simplest form

$$\frac{a}{2a-1} - \frac{2}{a+1}.$$ [3]

(c) Given that $A = h(4m + h)$,

express m in terms of h and A. [3]

Answers: (a) 14 and -4; (b) $\dfrac{(a-1)(a-2)}{(2a-1)(a+1)}$; (c) $m = \dfrac{A - h^2}{4h}$ N02/2/Q2

13

(a) Solve the equation $(2x-3)(x-4) = 18$. [3]

(b) A formula used in connection with a mirror is $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$.

   (i) Given that $v = 9$ and $f = 5$, find u. [1]

   (ii) Express v in terms of u and f. [2]

(c) A man bought a eggs at r cents per dozen.
He sold them for s cents each.

Find an expression, in terms of a, r and s, for the profit, in cents, that he made. [2]

Answers: (a) $-\dfrac{1}{2}$ and 6; (b)(i) $11\dfrac{1}{4}$, (ii) $v = \dfrac{fu}{u-f}$; (c) $\dfrac{a(12s-r)}{12}$. N03/2/Q3

14

(a) It is given that $S = \dfrac{n(a+l)}{2}$.

   (i) Find the value of S when $n = 20$, $a = -5$ and $l = 17$. [1]

   (ii) Express l in terms of S, n and a. [2]

(b) Solve the equations

   (i) $5t^2 = 12t$, [2]

   (ii) $\dfrac{y-1}{8} = \dfrac{2}{y-1}$. [3]

(c) Solve the equation $3x^2 + 9x + 5 = 0$, giving both answers correct to two decimal places. [4]

Answers: (a)(i) 120, (ii) $\dfrac{2s-an}{n}$; (b)(i) 0, 2.4, (ii) 5, –3, (iii) –0.74, –2.26. N04/2/Q5

15

   (a) Solve the equation $\dfrac{3t+1}{2} = 4$.   [2]

   (b) Solve the simultaneous equations

$$2x + y = 12,$$
$$3y - 2x = 56.$$
    [2]

   (c) Simplify $\dfrac{3y^2 + 8y + 4}{y^2 - 4}$.   [3]

   (d) Given that $3h + 2x = 2f - gx$, express x in terms of f, g and h.   [3]

Answers: **(a)** $2\dfrac{1}{3}$; **(b)** $x = -2\dfrac{1}{2}$, $y = 17$; **(c)** $\dfrac{3y+2}{y-2}$; **(d)** $\dfrac{2f-3h}{g+2}$.   N05/2/Q2

16

   (a) (i) Factorise completely $5x^2 - 20$.   [2]

       (ii) Simplify $\dfrac{5x^2 - 20}{10x^2 + 10x - 20}$.   [2]

   (b) Express as a single fraction in its simplest form

$$\dfrac{4}{y-3} - \dfrac{3}{y+5}.$$
    [3]

   (c) Given that $T = 2\pi\sqrt{\dfrac{L}{g}}$,

      express g in terms of π, T and L.   [3]

Answer: **(a)(i)** $5(x+2)(x-2)$ **(ii)** $\dfrac{x-2}{2(x-1)}$ **(b)** $\dfrac{y+29}{(y-3)(y+5)}$ **(c)** $\dfrac{4\pi^2 L}{T^2}$ N06/2/Q1

17

   (a) It is given that $x = a + \sqrt{a^2 + b^2}$.

      (i) Calculate x when $a = 0.73$ and $b = 1.84$.
          Give your answer correct to 2 decimal places.   [2]

      (ii) Express b in terms of x and a.   [3]

   (b) A shopkeeper sells pens and pencils.
      Each pen costs \$5 and each pencil costs \$3.
      One day he sold x pens.
      On the same day he sold 9 more pens than pencils.

      (i) Write down an expression, in terms of x, for his total income from the sale of these pens and
          pencils.   [2]

      (ii) This total income was less than \$300.

          Form an inequality in x and solve it.   [2]

      (iii) Hence write down the maximum number of pens that he sold.   [1]

Answers: **(a)(i)** 2.71, **(ii)** $\pm\sqrt{x^2 - 2ax}$, **(b)(i)** $8x - 27$, **(ii)** $8x - 27 < 300$, $x < 40.875$, **(iii)** 40.   N07/2/Q2

18

(a) Solve the equation

$$\frac{2p+1}{3} = 1 + \frac{p-3}{2}.$$ [3]

(b) Simplify

$$\frac{2v-6}{v^2-2v-3}.$$ [3]

(c) The tens digit of a number is x and the units digit is y.
Hence the value of the number is $10x + y$.
For example, if $x = 5$ and $y = 6$, the number would be $10 \times 5 + 6 = 56$.

   (i) When the digits x and y are reversed, the value of the number is increased by 63.
   Show that $y - x = 7$. [2]

   (ii) The sum of the original number and the number with reversed digits is 99.

   (a) Show that $x + y = 9$. [1]

   (b) Hence find the value of x and the value of y. [2]

Answers: (a) $p = -5$, (b) $\dfrac{2}{v+1}$, (c)(i) $(10y + x) - (10x + y) = 63$ seen, leading to $y - x = 7$, **N08/2/Q3**
(ii)(a) $(10x + y) + (10y + x) = 99$ seen, leading to $x + y = 9$, (b) $x = 1, y = 8$.

19 Solve the equations

(a) $2^y = 8$, [1]

(b) $3p + 4 = 8 - 2(p-3)$, [2]

(c) $\dfrac{18}{q} - \dfrac{16}{q+2} = 1$, [3]

(d) $5x^2 + x - 7 = 0$, giving each solution correct to 2 decimal places. [4]

N09/2/Q1

20

(a) $P = \dfrac{7Q}{4} + 15$

   (i) Find P when $Q = -40$. [1]

   (ii) Express Q in terms of P. [2]

(b) Factorise completely

   (i) $7c^2 - 28d^2$, [2]

   (ii) $3x^2 - 7x - 6$. [2]

(c) Solve the equation $\dfrac{4}{7-y} = 5$. [2]

Answers: (a)(i) -55 (ii) $4(P-15)/7$ (b)(i) $7(c-2d)(c+2d)$ (ii) $(3x+2)(x-3)$ (c) 6.2 **N10/21/Q1**

21   (a)   Simplify

       (i) $\dfrac{x+y}{8x+8y}$,   [1]

       (ii) $x(3x-2)-(3x^2-5)$.   [2]

   (b)   Solve the equation $3t-4=7+2(t+3)$.   [2]

   (c)   Factorise $5px-7qx+10py-14qy$.   [2]

   (d)   (i)   When $x=-2$, which of the two expressions, $3x+4$ and $2-x$, has the greater value?
                You must show your working.   [2]

          (ii)   Solve the inequality $3x+4<2-x$.   [2]

22

   (a)   Solve $5t(3t+7)=0$.   *Answer* $t=$ .... or .... [2]

   (b)   Solve the simultaneous equations.   *Answer* $x=$ ....
$$3x+4y=1$$
$$5x-8y=9$$
                                                          $y=$ .... [3]

   (c)   Express as a single fraction $\dfrac{5}{p-2}-\dfrac{4}{2p+3}$ *Answer*   .... [3]

   (d)   Simplify $\dfrac{q^2-1}{2q^2-3q+1}$.   *Answer*   .... [3]

Answers: (a) 0 $\dfrac{-7}{3}$ (b) $x=1$ $y=\dfrac{-1}{2}$ (c) $\dfrac{6p+23}{(p-2)(2p+3)}$ (d) $\dfrac{q+1}{2q-1}$

---

23   (a) $A=h(4m+h)$

       Express m in terms of A and h.   *Answer*   .... [3]

   (b)   Factorise completely $3ax+5bx-6ay-10by$.   *Answer*   .... [2]

   (c)   Solve the equation $\dfrac{5x-1}{9}=\dfrac{9}{5x-1}$.   *Answer* $x=$ .... or .... [3]

Answers: (a) $m=\dfrac{A-h^2}{4h}$ (b) $(3a+5b)(x-2y)$ (c) $x=2$ or -1.6

24 **(a)** **(i)** Simplify $5p - (1 - 5p) + 2$.

Answer .... [2

(ii) Solve the inequality $3 - 2x > 5$.

Answer .... [2

(b) $y = \dfrac{A + 2x}{x}$.

(i) Find y when $x = A$.

Answer $y =$ .... [1]

(ii) Rearrange the formula to make x the subject.

Answer $x =$ .... [3]

(c) Ada is x years old and Bill is y years old.
Last year, Bill was 6 times as old as Ada.

(i) Form an equation in x and y and show that it simplifies to $y = 6x - 5$. [1]

(ii) In 19 years time, Bill will be twice as old as Ada.

Form another equation in x and y and show that it simplifies to $y = 2x + 19$. [1]

(iii) Hence find the present ages of Ada and Bill.

Answer Ada's age .... years

Bill's age .... years [2]

Answers: (a)(i) $10p + 1$ (ii) $x < -1$ (b)(i) 3 (ii) $\dfrac{A}{y - 2}$ (c)(iii) $x = 6$ $y = 31$ **N12/21/Q2**

41

25

(a) Solve $\dfrac{2}{3-x} = 1$.

Answer ....[1]

(b) Factorise

    (i) $5x + 5y$,

Answer ....[1]

    (ii) $9x^2 - 16$.

Answer ....[1]

(c) (i) Factorise $2x^2 + 5x - 12$.

Answer ....[1]

    (ii) Use your answer to **part (c)(i)** to solve the equation $2x^2 + 5x - 12 = 0$.

Answer $x =$ .... or .... [1]

9

(d) A source of light is observed from a distance of d metres.
The amount of light received, L units, is inversely proportional to the square of the distance.

Given that $L = 9$ when $d = 2$, find the value of L when $d = 3$.

Answer ....[2]

26

(a) Evaluate $\sqrt[3]{\dfrac{543}{28.6 - 1.35}}$.

Answer .... [1]

(b) Factorise completely $9p^2 - 6pq$.

Answer .... [1]

(c) Expand the brackets and simplify $(3a + b)^2$.

Answer .... [1]

(d) Express as a single fraction in its simplest form $\dfrac{4}{2t+1} - \dfrac{3}{3t+1}$.

Answer .... [3]

(e) Find the integer values of n such that

$$4(2-n) > 17 \text{ and}$$

$$n > -6.$$ *Answer* .... [2]

(f) Abebi, Bella and Chuku share \$112.

Abebi receives \$$x$.
Bella receives \$12 less than Abebi.
Chuku receives twice as much as Bella.

Answer \$ .... [3]

Form an equation in x and solve it to find how much Chuku receives.

Answers: (a) 2.71 (b) $3p(3p - 2q)$ (c) $9a^2 + 6ab + b^2$ (d) $\dfrac{6t+1}{(2t+1)(3t+1)}$ (e) $-5, -4, -3$ (f) 50 **N16/21/Q2**

27

(a) Solve $\dfrac{y}{2y+3} = \dfrac{2}{y+5}$.

Answer $y =$ .... or .... [3]

(b) Make t the subject of the formula $p = \dfrac{4t+1}{2-t}$.

Answer .... [3]

(c) Simplify fully $\dfrac{3x^2 - 14x + 8}{x^2 - 16}$.

Answer .... [3]

Answers: (a) $y = -3, y = 2$ (b) $t = \dfrac{2p-1}{4+p}$ (c) $\dfrac{3x-2}{x+4}$ **N17/21/Q5**

Indices : Powers and Roots Paper 1

1 (a) Calculate the value of $9^{\frac{1}{2}} + 9^0$.

Answer (a) .... [1]

(b) The reciprocal of 2^{-3} is 2^n.
Write down the value of n.

(b) $n = $ .... [1]

---

2 (a) It is given that $5^{-2} \times 5^k = 1$.
Write down the value of k.

Answer (a) $k = $ .... [1]

(b) It is given that $\sqrt[3]{7} = 7^m$.
Write down the value of m.

(b) $m = $ .... [1]

Answers: (a) 2; (b) 1/3.

---

3 (a) Simplify $(3x^3)^2$.

Answer (a) .... [1]

(b) Given that $(16)^{-\frac{1}{2}} \times k = 1$, evaluate k.

(b) $k = $ .... [1]

Answers: (a) $9\,x^6$; (b) 4.

---

4 (a) The number 222.222 is written in the answer space.
Circle the digit which represents the value 2×10^0.

Answer (a) 222.222 [1]

(b) Write 5×10^{-2} as a fraction in its simplest form.

(b) ....[1]

(c) Evaluate $8^{\frac{2}{3}}$.

(c) .... [1]

Answers: (a) Units digit ringed; (b) $\dfrac{1}{20}$; (c) 4.

---

5 (a) Evaluate $5^2 + 5^0$.

Answer (a) ....[1]

(b) Simplify

(b)(i) .... [1]

(i) $\left(\dfrac{1}{x}\right)^{-2}$,

(ii) .... [1]

(ii) $\left(x^6\right)^{\frac{1}{2}}$.

Answer. (a) 26   (b)(i) x^2 (ii) x^3

---

6 Evaluate

(a) 17^0,

(b) $4^{\frac{5}{2}}$,

(c) $(0.2)^{-2}$.

Answer (a) ....[1]

(b) ....[1]

(c) ....[1]

Answer: (a) 1   (b) 32   (c) 25

J07/1/Q12

7 Evaluate

(a) 9^0,

(b) 9^{-2},

(c) $9^{\frac{3}{2}}$.

Answer (a) ....[1]

(b) ....[1]

(c) ....[1]

Answers: (a) 1   (b) $\dfrac{1}{81}$ (c) 27

J08/1/Q8

8 Evaluate

(a) 0.2×0.06,

(b) $3 \div 0.01$,

(c) $27^{\frac{1}{3}}$.

Answer (a) ....[1]

Answer (b) ....[1]

Answer (c) ....[1]

Answer: (a) 0.012   (b) 300   (c) 3

J10/11/Q10

9 Evaluate

(a) $9^1 + 9^0$,

(b) $\left(\dfrac{1}{9}\right)^{\frac{1}{2}}$.

Answer (a) ....[1]

Answer (b) ....[1]

Answer: (a) 10   (b) $\dfrac{1}{3}$

J10/12/Q4

10 Evaluate

(a) $8^{\frac{2}{3}}$,

(b) $\left(\dfrac{1}{6}\right)^{-2}$.

Answer ....[1]

Answer ....[1]

Answers: (a) 4   (b) 36

J11/11/Q6

11

(a) Evaluate $\left(\frac{1}{4}\right)^{-2}$.

Answer .... [1]

(b) Evaluate $64^{\frac{2}{3}}$.

Answer .... [1]

(c) Simplify $\left(\frac{4x^2 y^9}{x^4 y}\right)^{\frac{1}{2}}$.

Answer .... [2]

Answer. (a) 16 (b) 16 (c) $\dfrac{2y^4}{x}$

J11/12/Q21

12

15 (a) Write 8^3 in the form 2^k.

Answer .... [1]

(b) Evaluate $\dfrac{9 \times 2^{12} - 3 \times 2^{10}}{3 \times 2^8}$.

Answer .... [2]

Answer. (a) 2^9 (b) 44

J12/11/Q15

13

(a) Evaluate

(i) $5^0 + 5^2$,

Answer .... [1]

(ii) $36^{\frac{1}{2}}$,

Answer .... [1]

(iii) $\left(2^{\frac{2}{3}}\right)^6$.

Answer .... [1]

(b) $\left(\frac{1}{3}\right)^k = 9$

Answer $k =$ .... [1]

Find the value of k.

Answer. (a)(i) 26 (ii) 6 (iii) 16 (b) −2

J13/11/Q20

14

(a) Solve $2(5^p) = 250$.

Answer $p =$ .... [1]

(b) Simplify

(i) $1 \div x^{-5}$,

Answer .... [1]

(ii) $\dfrac{3a}{4} \div \dfrac{9a^2}{8}$.

Answer .... [1]

Answers: (a) 3 (b)(i) x^5 (ii) $\dfrac{2}{3a}$

J14/11/Q13

15 Given that $6^x = 9$, write down the value of

(a) 6^{-x},

Answer .... [1]

(b) $6^{\frac{x}{2}}$,

Answer .... [1]

(c) $6^0 + 6^x$.

Answer .... [1]

Answers: (a) $\dfrac{1}{9}$ (b) 3 (c) 10

---

16 **3 (a)** Evaluate

(i) 3^{-2},

Answer .... [1]

(ii) $125^{\frac{2}{3}}$.

Answer .... [1]

(b) Simplify $\left(\dfrac{2a^2 b^5}{18a^4 b}\right)^{\frac{1}{2}}$.

Answer .... [2]

Answers: (a)(i) $\dfrac{1}{9}$ (ii) 25 (b) $\dfrac{b^2}{3a}$

---

17 Find the value of

(a) $\sqrt{0.0081}$,

Answer (a) .... [1]

(b) $7^3 \times 7^{-3}$.

(b) .... [1]

Answers: (a) 0.09; (b) 1.

---

18 Evaluate

(a) $7\frac{1}{4} - 6\frac{2}{3}$,

(b) $3\sqrt{2} \times 5\sqrt{2}$.

Answers: (a) 42.5%; (b) 4.85

19

(a) On a journey, a cyclist travelled 1 kilometre in x minutes.
On a second journey, the cyclist travelled for y hours at the same average speed as on the first journey.
Find an expression, in terms of x and y, for the number of kilometres he travelled on the second journey.

(b) $\dfrac{t \times t^3}{\sqrt{t}} = t^n$.

Find the value of n.

Answers: (a) $\dfrac{60y}{x}$; (b) 3 ½

20

Find a, b and c when

(a) $3^a \div 3^5 = 27$,

(b) $125^b = 5$,

(c) $10^c = 0.001$.

Answer (a) $a =$ .... [1]

(b) $b =$ .... [1]

(c) $c =$ .... [1]

Answers: (a) 8; (b) $\dfrac{1}{3}$; (c) –3.

21

(a) Express $\dfrac{17}{40}$ as a percentage.

(b) Evaluate $\left(\dfrac{1}{3}\right)^{-2}$.

Answer (a) ....% [1]

(b) .... [1]

Answers: (a) 42.5%; (b) 9.

22

(a) Find the fraction which is exactly halfway between $\dfrac{5}{9}$ and $\dfrac{8}{9}$.

(b) **Estimate** the value of $\sqrt{5000}$, giving your answer correct to **one** significant figure.

(c) Evaluate $3^0 \times 4^{\frac{3}{2}}$.

Answer (a) ....[1]

(b) ....[1]

(c) ....[1]

Answer: (a) $\dfrac{13}{18}$; (b) 70; (c) 8.

23

(a) Add 620 grams to 3.7 kilograms.
Give your answer in kilograms.

Answer (a) ....kg [1]

(b) Write the following numbers in order of size, starting with the smallest.

$$3^1 \qquad 3^{-1} \qquad (-1)^3 \qquad 3^0$$

Answer (b) .... , ... , ... , ... [1]
smallest

Answer: (a) 4.32 kg   (b) $(-1)^3, 3^{-1}, 3^0, 3^1$ **N07/1/Q3**

24

Evaluate

(a) $3\frac{1}{5} - 2\frac{2}{3}$,

Answer (a) ....[1]

(b) $4^{\frac{3}{2}}$.

(b) ....[1]

Answers: (a) $\frac{8}{15}$,   (b) 8.   **N08/1/Q3**

25

(a) Evaluate $4^0 + 4^1$.

Answer (a) ....[1]

(b) Evaluate $\left(\frac{1}{4}\right)^{-2}$.

Answer (b) ....[1]

Answers: (a) 5 (b) 16   **N10/11/Q4**

26

(a) Evaluate $5^0 - 5^{-1}$.

Answer (a) ....[1]

(b) Simplify $(5x^3)^2$.

Answer (b) ....[1]

(c) Simplify $\left(\frac{16}{n^{16}}\right)^{\frac{1}{2}}$.

Answer (c) ....[1]

Answer: (a) $\frac{4}{5}$ (b) $25x^6$ (c) $\frac{4}{n^8}$ **N10/12/Q10**

27

(a) Simplify $(3a^4)^2$.

Answer .... [1]

(b) Evaluate $\left(\frac{1}{4}\right)^{-2}$.

Answer .... [1]

(c) Given that $x^3 = 27^0$, find x .

Answer $x =$ .... [1]

(d) Evaluate $\dfrac{12^{\frac{1}{2}}}{3^{\frac{3}{2}}}$.

Answer .... [1]

Answers: (a) $9a^8$ (b) 16 (c) 1 (d) $\frac{2}{3}$ **N11/11/Q18**

28 (a) Evaluate $4^0 - 4^{-2}$.

Answer .... [1]

(b) Simplify $(2x^2)^3$.

Answer .... [1]

Answer. (a) $\dfrac{15}{16}$ (b) $8x^6$

---

29 (a) Evaluate

(i) $5^1 + 5^0$,

Answer ....[1]

(ii) $\left(\dfrac{4}{3}\right)^{-2}$.

Answer ....[1]

(b) Simplify $\left(2x^2\right)^3$.

Answer ....[1]

Answers: (a)(i) 1 (ii) $\dfrac{9}{16}$ (b) $8x^6$

---

30 Giving each answer as a fraction in its lowest terms, evaluate

(a) $\dfrac{3 \times (2)^3}{6 \times 9}$,

Answer ....[1]

(b) $\left(\dfrac{3^2}{2}\right)^{-2}$.

Answer ....[1]

Answers: (a) $\dfrac{4}{9}$ (b) $\dfrac{4}{81}$

---

31 (a) Find n when $3^3 \times 3 \times 3^5 = 3^n$.

Answer $n = $ ....[1]

(b) Find the value of $32^{\frac{3}{5}}$.

Answer ....[1]

(c) Find the value of $\left(\dfrac{1}{5}\right)^{-2}$.

Answer ....[1]

Answers: **(a)** 9; **(b)** 8; **(c)** 25.

---

32 (a) Evaluate $9^{-\frac{1}{2}}$.

Answer .... [1]

(b) Evaluate $10^3 - 10^0$.

Answer .... [1]

(c) Solve $x^{\frac{3}{2}} = 8$.

Answer $x = $ .... [1]

Answers: (a) $\dfrac{1}{3}$ (b) 999 (c) 4

33 (a) Evaluate $2^3 - 2^0$.

Answer .... [1]

(b) Simplify $\dfrac{12xy}{9x^2}$.

Answer .... [1]

Answers: (a) 7 (b) $\dfrac{4y}{3x}$

34 (a) Evaluate $9^2 - 9^0$.

Answer .... [1]

(b) Evaluate $9^{-\frac{1}{2}}$.

Answer .... [1]

Answers: (a) 80 (b) $\dfrac{1}{3}$

Variation Paper 1

1 V varies inversely as P.
When $P = 3$, $V = 1$.

(a) Express V in terms of P.

(b) Complete the table in the answer space.

Answer (a) .... [2]

(b)

P	3	5	
V	1		9

[2]

---

2 It is given that $p = \dfrac{12}{\sqrt{q}}$.

(a) Describe the relationship between p and q in words by completing the sentence in the answer space.

(b) Calculate q when $p = 4$.

Answer (a) p is .... proportional to the square root of q. [1]

(b) $q = $ ....[1]

Answer: (a) Inversely   (b) 9

---

3 It is given that y is directly proportional to the square of x and that $y = 1$ when $x = \frac{1}{2}$.

Find

(a) the formula for y in terms of x,

(b) the values of x when $y = 9$.

Answer (a) $y = $ ....[2]

(b) $x = $ ....[1]

Answers: (a) $y = 4x^2$ (b) 1.5 and -1.5

---

4 y is directly proportional to the square root of x.
Given that $y = 12$ when $x = 36$,

find

(a) the formula for y in terms of x,

(b) the value of x when $y = 10$.

Answer (a) $y = $ .... [2]

Answer (b) $x = $ .... [1]

Answer: (a) $2\sqrt{x}$ (b) 25

5 y is inversely proportional to x^2.
Some values of y and x are given in the table below.

x	3	2	q
y	4	p	1

Find

(a) the formula for y in terms of x,   Answer   (a) $y=$ .... [2]

(b) the value of p,   Answer   (b) $p=$ .... [1]

(c) the two values of q.   Answer   (c) $q=$............... , ... [1]

Answer: (a) $y = 36 / x^2$ (b)   9   (c) ± 6 J10/11/Q17

---

6   It is given that y is inversely proportional to the square of x and that $y = 48$ when $x = \frac{1}{2}$.

Find

(a) the formula for y in terms of x,   Answer   (a) $y =$ .... [2]

(b) the values of x when $y = 3$.   Answer   (b) $x =$ .... , ... [1]

Answer: (a) $\dfrac{12}{x^2}$ (b)   2   - 2   J10/12/Q12

---

7 y is directly proportional to the square of x.

Given that $y = 2$ when $x = 4$, find y when $x = 10$.   Answer $y =$ .... [2]

Answer: 12.5   J11/12/Q8

53

8 R is directly proportional to the **cube** of p.
When $p = 2$, $R = 24$.

(a) Find the formula for R in terms of p.   *Answer* $R = $ ....[1]

(b) Find the value of p when $R = 192$.   *Answer* $p = $......................................[2]

(c) Which of the diagrams below represents the graph of R against p?

| Diagram 1 | Diagram 2 | Diagram 3 | Diagram 4 | Diagram 5 |

Answer   Diagram ....[1]

Answer: (a) $3p^3$ (b) 4 (c) 2   **J13/11/Q21**

---

9 (a) A spherical tennis ball and a spherical beach ball have diameters in the ratio $1 : 3$.
The surface area of the beach ball is $153\,\text{cm}^2$.

Calculate the surface area of the tennis ball.   *Answer*   .... cm^2 [2]

(b) y is inversely proportional to the cube of x.
When $x = 2$, $y = 9$.

Find y when $x = 5$.   *Answer* $y = $ .... [3]

Answers: (a) 17 (b) $\dfrac{72}{125}$ **J14/11/Q23**

---

10 y is inversely proportional to the square of x.

Given that $y = 3$ when $x = 2$, find y when $x = 5$.   *Answer* $y = $ .... [2]

        J15/11/Q9

Answer: $\dfrac{12}{25}$

---

11 5 (a) y is directly proportional to the square of x.

Given that $y = 8$ when $x = 4$, find y when $x = 3$.   *Answer* $y = $ .... [2]

(b) p is inversely proportional to q.
It is known that $p = 15$ for a particular value of q.

Write down the value of p when this value of q is doubled.

Answer $p = $ .... [1]

12 y is inversely proportional to $(x + 2)$.

     (a) Write down an expression for y in terms of x and a constant k.

     (b) It is given that $y = 4$ when $x = 3$.
        Find y when $x = 8$.

Answer (a) $y =$ .... [1]

(b) $y =$ .... [2]

Answers: **(a)** $k \div (x + 2)$; **(b)** 2.   **N01/Q15**

13

The force of attraction between two magnets is F Newtons.
This force is inversely proportional to the square of the distance, d centimetres, between the magnets.

     (a) When the magnets are a certain distance apart, the force is 10 Newtons.
        What is the force when this distance is doubled?

     (b) (i) Write down a formula connecting F, d and a constant k.

        (ii) When the magnets are 3 cm apart, the force is 2 Newtons.
            Find the force when they are 5 cm apart.

Answers: **(a)** 2 ½ N; **(b)(i)** $F = \dfrac{k}{d^2}$, **(ii)** 0.72 N.   **N02/1/Q19**

14   **(a)** y is directly proportional to x^2.
        It is known that $y = 10$ for a particular value of x.
        Find the value of y when this value of x is halved.

     (b) Seven men can paint a bridge in 15 days.

        (i) How long would it take 3 men?

        (ii) The bridge was painted in t days.
            Write down an expression, in terms of t, for the number of men needed to paint the bridge.

Answers: $2\dfrac{1}{2}$; **(b)(i)** 35, **(ii)** $\dfrac{105}{t}$.   **N03/Q10**

15

(a) When an object is falling, the air resistance varies as the square of the speed.
At a certain speed, the resistance is 30 newtons.
What is the resistance at twice this speed?

Answer (a) .... newtons [1]

(b) y is inversely proportional to x.
Given that $y = 6$ when $x = 4$, find the value of y when $x = 3$.

(b) y = ...[2]

Answer: (a) 120 newtons (b) 8   **N07/1/Q12**

16

T is inversely proportional to the square of L.
Given that $T = 9$ when $L = 2$, find

(a) the formula for T in terms of L,

Answer (a) T = ...[2]

(b) the values of L when $T = 25$.

(b) L = ...[1]

Answers: (a) $T = \dfrac{36}{L^2}$,   (b) $\pm\dfrac{6}{5}$.   **N08/1/Q10**

17

y is inversely proportional to x.
Given that $y = 250$ when $x = 4$, find y when $x = 80$.   *Answer y =* ....[2]

Answer: $12\frac{1}{2}$ **N09/1/Q6**

18

Answer y = ...[2]

y varies inversely as the square of x.
Given that $y = 4$ when $x = 3$, find the value of y when $x = 2$.

Answer: 9   **N10/11/Q7**

19

y is directly proportional to the square of x.

Given that $y = 50$ when $x = 5$, find the value of y when $x = 3$.

Answer   y = ...[2]

Answer: 18   **N10/12/Q7**

20 The time taken to fill a tank with water varies inversely as the area of cross-section of the inlet pipe. The time taken is 40 minutes when the area is $3\,cm^2$.

(a) Find the number of minutes taken to fill the tank when the area is $5\,cm^2$.

Answer .... [2]

(b) It is given that the area is A square centimetres.

Find the expression, in terms of A, for the number of minutes taken to fill the tank.

Answer .... [1]

(c) Water flowed into the empty tank through a pipe of area $4\,cm^2$.
It flowed for 9 minutes.

Find, in its simplest form, the fraction of the tank that now contained water.

Answers: (a) 24 (b) $\dfrac{120}{A}$ (c) $\dfrac{3}{10}$

---

21 y is inversely proportional to x.
The table shows some values of x and y.

x	3	4	q	n
y	20	p	5	m

(a) Find p. Answer $p =$ .... [1]

(b) Find q. Answer $q =$ .... [1]

(c) Express m in terms of n. Answer $m =$ .... [1]

Answer: (a) 15 (b) 12 (c) $\dfrac{60}{n}$

---

22 y is inversely proportional to x.

Given that $y = \dfrac{1}{5}$ when $x = 20$, find y when $x = \dfrac{1}{7}$. Answer $y =$ .... [2]

Answer: 28

---

23 When the speed of a car is $v\,m/s$, its braking distance is $d\,m$.
d is directly proportional to the square of v.
When the speed of the car is $8\,m/s$ the braking distance is $5\,m$.

Find the formula for d in terms of v and hence find the braking distance when the speed of the car is $40\,m/s$.

Answer Formula $d =$ ....

Braking distance = ... m [3]

Answer: $d = \dfrac{5v^2}{64}$ and 125.

24 **(a)**

p	27	33
q	9	r

Given that p is directly proportional to q, find the value of r.

Answer $r =$ .... [1]

(b)

x	2	10
y	25	1

Complete the sentence below describing the relationship between x and y.

y is inversely proportional to .... [1]

(c) M is directly proportional to L^3.

How many times larger is M when L is multiplied by 2?

Answer .... [1]

Answers: **(a)** 11 **(b)** x^2 **(c)** 8

25 y is inversely proportional to x.

Given that $y = -50$ when $x = 3$, find y when $x = -10$.   *Answer* $y =$ .... [2]

Answer: 15

26 y varies directly as the square of x.

Given that $y = \dfrac{1}{5}$ when $x = \dfrac{1}{2}$, find y when $x = 10$.   *Answer* $y =$ .... [2]

Answer: 80

Standard Form Paper 1

1 (a) Calculate the value of $16 - 8 \div 2$. *Answer* (a) .... [1]

(b) Express 0.0032 in standard form. (b) .... [1]

---

2 $p = 3.2 \times 10^{11}$ and $q = 8 \times 10^{-4}$.
Expressing your answers in standard form, evaluate

(a) q^2, *Answer* (a).................................. [1]

(b) $p \div q$. (b)............................. [1]

Answers: **(a)** 6.4×10^{-7}; **(b)** 4×10^{14}.

---

3 (a) Express 217.3×10^2 in standard form. *Answer* (a) .... [1]

(b) Arrange the following numbers in order starting with the smallest.

$$217.3 \times 10^2 , \quad 22.6 \times 10^3 , \quad 0.031 \times 10^5 , \quad 2.5 \times 10^4 .$$

swer (b) .... , ... , ... , ... [2]

Answers: **(a)** 2.173×10^4; **(b)** 0.031×10^5, 217.3×10^2, 22.6×10^3, 2.5×10^4.

---

4 (b) The distance of Saturn from the Sun is 1507 million kilometres.
Express 1507 million in standard form.

Answer (b) ....[1]

(b) 1.507×10^9

---

5 The Earth is 1.5×10^8 kilometres from the Sun.

(a) Mercury is 5.81×10^7 kilometres from the Sun.

How much nearer is the Sun to Mercury than to the Earth?
Give your answer in standard form.

(b) A terametre is 10^{12} metres.

Find the distance of the Earth from the Sun in terametres.

Answer (a) .... km [2]

(b) .... terametres [2]

6   (a) Convert 0.8 kilometres into millimetres.   *Answer* (a) .... mm [1]

b) Evaluate $(6.3 \times 10^6) \div (9 \times 10^2)$,   giving your answer in standard form.

Answer (b) .... [2]

7   Some data about two planets, Earth and Mars, is shown in the table.

Planet	Average temperature (°C)	Mass (tonnes)	Volume (km³)
Earth	15	5.98×10^{21}	1.08×10^{12}
Mars	−63	6.58×10^{20}	162 000 million

(a) How much greater is the average temperature on Earth than that on Mars?

Answer (a) ....°C [1]

(b) Write down the volume of Mars in standard form.   *Answer* (b) .... km³ [1]

(c) Calculate the difference in mass between Earth and Mars.
Give your answer in standard form.

Answer (c) ....tonnes [2]

8   (a) Vicky's fingernail grows one nanometre in one second.
One nanometre is 1×10^{-9} metres.
Vicky calculates how much her fingernail grows in one hour.

Find this length, in standard form, giving your answer

(i)   in metres,   *Answer* .... m [1]

(ii) in millimetres.   *Answer* .... mm [1]

(b) It is given that $2 \times 10^3 (d + 3 \times 10^2) = 8 \times 10^6$.   *Answer* $d =$ .... [2]

Find d.

9 **(a)** The mass of one grain of rice is 0.000 02 kg.

Write 0.000 02 in standard form. *Answer* .... [1]

(b) The table shows the amount of rice grown in some countries in 2002.

	China	Brazil	India	Vietnam
Amount (tonnes)	1.2×10^8	7.6×10^6	8.0×10^7	2.1×10^7

(i) Write these amounts in order, smallest first.

Answer .... , ... , ... , ... [1]
           smallest

(ii) Calculate the difference in the amount of rice grown in Brazil and Vietnam.
Give your answer in standard form.

Answer .... tonnes [1]

Answer. **(a)** 2×10^{-5} **(b)(i)** 7.6×10^6, 2.1×10^7, 8.0×10^7, 1.2×10^8 **(ii)** 1.34×10^7 **J11/12/Q13**

10

A swarm of locusts contains 40 billion locusts.
A billion is a thousand million.

(a) Write down, in standard form, the number of locusts in this swarm.

Answer .... [1]

(b) Each locust eats 2 grams of food every day.

Find the amount of food eaten by this swarm in one week.
Give your answer in **kilograms** using standard form.

Answer .... kg [2]

Answer. **(a)** 4×10^{10} **(b)** 5.6×10^8 **J12/11/Q17**

---

11 $p = 2.4 \times 10^2$ $q = 6 \times 10^3$

Giving your answers in standard form, find

(a) $p + q$, *Answer* .... [1]

(b) $2p \div q$. *Answer* .... [2]

Answers: **(a)** 6.24×10^3 **(b)** 8×10^{-2} **J13/11/Q17**

12 (a) Light travels at a speed of 3×10^8 m/s.

Calculate the time it takes for light to travel 6 km. *Answer* .... s [2]
Give your answer in standard form.

(b) One molecule of water is made up of two atoms of hydrogen and one atom of oxygen.
The mass of one atom of hydrogen is 1.67×10^{-24} g.
The mass of one atom of oxygen is 2.66×10^{-23} g.

Calculate the mass of one molecule of water.
Give your answer in standard form. *Answer* .... g [2]

Answers: **(a)** 2×10^{-5} **(b)** 2.99×10^{-23} **J14/11/Q19**

13
$$p = 4 \times 10^5 \qquad q = 7 \times 10^6$$

Expressing each answer in standard form, find

(a) p^2, *Answer* .... [1]

(b) $p + q$. *Answer* .... [1]

Answers: **(a)** 1.6×10^{11} **(b)** 7.4×10^6 **J15/11/Q5**

---

14 (a) Write the number 0.034×10^{-3} in standard form. *Answer* .... [1]

(b) Arrange the following numbers in order, starting with the smallest.

$$0.034 \times 10^{-3} \qquad 33.7 \times 10^{-6} \qquad 0.42 \times 10^{-5}$$

Answer .... , ... , ... [1]
    smallest

Answers: **(a)** 3.4×10^{-5} **(b)** 0.42×10^{-5} 33.7×10^{-6} 0.034×10^{-3} **J16/11/Q6**

---

15 (a) Write 0.000 186 in standard form. *Answer* .... [1]

(b) $$s = 1.3 \times 10^7 \qquad t = 8 \times 10^8$$

Giving each answer in standard form, find

(i) t^2, *Answer* .... [1]

(ii) $t - s$. *Answer* .... [2]

Answer: **(a)** 1.86×10^{-4} **(b)(i)** 6.4×10^{17} **(ii)** 7.87×10^8 **J17/11Q14**

16

The population of Europe is approximately 7.0×10^8.
The population of Asia is approximately 3.5×10^9.
The mean mass of the population of Europe is approximately 62 kg.

(a) Giving your answers in standard form, estimate

(i) the total mass of the population of Europe,

(ii) how many more people live in Asia than in Europe.

(b) Express the population of Europe as a percentage of the population of Asia.

Answers: **(a)(i)** 4.34 x 10^{10}kg; **(ii)** 2.8 x 10^9; **(b)** 20%.   N02/1/Q12

17

(a) Write down the following numbers in order of size, starting with the smallest

$$-0.29, \quad -1.5, \quad 0, \quad -0.3, \quad -4.$$

Answer (a) ...., ..., ..., ..., ... [1]

(b) The thickness of a sheet of paper is 8×10^{-4} cm.
Find the thickness of two sheets of paper, giving your answer in standard form.

Answer (b) .... cm [1]

Answers: **(a)** −4, −1.5, −0.3, −0.29, 0; **(b)** 1.6 x 10^{-3}.   N03/Q8

18

An atom of helium has a mass of 6.8×10^{-27} kilograms.

(a) Express this mass in grams.   *Answer (a)* ....g [1]
Give your answer in standard form.

(b) A room contains 9×10^{22} atoms of helium.   *(b)* ....g [2]

Find the mass of helium in the room.
Give your answer in grams as a normal decimal number.

Answers: **(a)** 6.8 × 10^{-24} g; **(b)** 0.612 g.   N04/Q12

19

The population of a country is 3.2×10^6.
There are 8×10^5 children.

(a) What fraction of the whole population are children?
Give your answer in its simplest form.   *Answer (a)* ....[1]

(b) Find the number of adults.   *(b)* .... [1]
Give your answer in standard form.

Answers: **(a)** $\frac{1}{4}$; **(b)** 2.4 x 10^6.   N05/1/Q5

20 .0 The distance from the Earth to the Sun is e kilometres, where $e = 1.5 \times 10^8$.
The distance from the Sun to Mercury is m kilometres, where $m = 6 \times 10^7$.

(a) Express $e : m$ as the ratio of two integers in its simplest form.

(b)

Earth   Sun   Mercury
●   ●   •
$\longleftarrow$ —— 1.5×10^8 km —— $\longrightarrow$$\longleftarrow$ 6×10^7 km $\longrightarrow$

The diagram shows when the Earth, the Sun and Mercury are in a straight line, with
the Sun between the Earth and Mercury.
Find the distance from the Earth to Mercury.
Give your answer in standard form.

Answer (a) .... : ....[1]

(b) ....km [2]

Answer: (a) 5:2 ;   (b) 2.1 × 10⁸.   **N06/1/Q10**

---

21 It is given that $m = 2.1 \times 10^7$ and $n = 3 \times 10^4$.
Expressing your answers in standard form, find

(a) $m \div n$,   *Answer (a)* ....[1]

(b) $n^2 + m$.   *(b)* ....[2]

Answers: **(a)** 7 × 10²,   **(b)** 9.21 × 10⁸.   **N08/1/Q7**

22

Tom estimated the population of five countries in 2020.
The table below shows these estimates.

Country	Population
Australia	2.35×10^7
Brazil	1.95×10^9
China	1.4×10^9
Japan	1.36×10^8
United Kingdom	6.9×10^7

(a) Which country did he estimate would have a population about 20 times that of the United Kingdom?

Answer (a) ....[1]

(b) How many more people did he estimate would be in Japan than in Australia? Give your answer in standard form.

Answer (b) ....[2]

Answers: **(a)** China   **(b)** 1.125×10^8

---

23 The speed of light is given as 3×10^5 km/s.
Writing each answer in standard form, calculate

(a) the distance, in kilometres, that light travels in one minute,

Answer (a) .... km [1]

(b) the time, in seconds, that light takes to travel 150 km.

Answer (b) .... seconds [2]

Answers: **(a)** 1.8×10^7 **(b)** 5×10^{-4}

24

(a) Express 0.000 070 6 in standard form.   *Answer (a)* ....[1]

(b) A house was bought for \$20 000 and sold for \$50 000.

Calculate the percentage profit.   *Answer (b)* ....% [1]

Answer. **(a)** 7.06×10^{-5} **(b)** 150

---

25 A large tank contained 2.3×10^6 litres of oil.
During a 4 week period, 1.2×10^5 litres were used.

(a) Calculate how many litres of oil remain in the tank after the 4 weeks. Give your answer in standard form.

Answer .... [1]

(b) Giving your answer in standard form, calculate the average number of litres used each week.

Answer .... [1]

Answers: **(a)** 2.18×10^6 **(b)** 3×10^4

26

(a) Express the number 0.000 042 in standard form.   *Answer* .... [1]

(b) Calculate $(7 \times 10^{-3}) \times (3 \times 10^9)$, giving your answer in standard form.

Answer .... [1]

Answer. **(a)** 4.2×10^{-5} **(b)** 2.1×10^7

27 **(a)** Write the number 0.000 034 in standard form.   *Answer* ....[1]

(b) Expressing your answer in standard form, find $(5 \times 10^8) \times (4 \times 10^7)$.

    Answer ....[1]

Answers: **(a)** 3.4×10^{-5} **(b)** 2×10^{16} **N12/11/Q6**

28 **(a)** Evaluate $5 \times 10^0 + 3 \times 10^1 + 1 \times 10^2$.   *Answer* ....[1]

(b) Find $(5 \times 10^8) \times (2.4 \times 10^{-3})$.
Give your answer in standard form.   *Answer* ....[1]

Answers: **(a)** 135 **(b)** 1.2×10^6 **N13/11/Q6**

29 **(a)** In 2013 the population of China was approximately 1 360 000 000.

Write this number in standard form.

    Answer .... [1]

(b) $p = 8 \times 10^5$ $q = 7 \times 10^3$

Giving your answers in standard form, find

(i) pq,   *Answer* .... [1]

(ii) $p - q$.   *Answer* .... [2]

Answers: **(a)** 1.36×10^9; **(b)** 5.6×10^9; **(c)** 7.93×10^5.   **N14/11/Q19**

30 **(a)** The mass of a dust particle is approximately 0.000 075 3 g.

Write this mass in standard form.   *Answer* .... g [1]

(b) The mass of the Earth is 5.972×10^{24} kg.
The mass of the Moon is 7.3×10^{22} kg.

Find the total mass, in kg, of the Earth and the Moon.
Give your answer in standard form.   *Answer* .... kg [2]

Answers: **(a)** 7.53×10^{-5} **(b)** 6.045×10^{24} **N15/11/Q16**

31 **(a)** Write the number 450 000 000 in standard form.   *Answer* .... [1]

(b) Giving your answer in standard form, evaluate $\dfrac{1.5 \times 10^5}{5 \times 10^{-3}}$.

    Answer .... [2]

32   **(a)** Write the number 0.000 12 in standard form.

Answer .... [1]

(b) Giving your answer in standard form, evaluate $5.5 \times 10^7 - 2.1 \times 10^6$.

Answer .... [2]

Standard Form Paper 2

1 The mass and diameter of the planets in the inner solar system are shown in the table.

Planet	Mass (kg)	Diameter (km)
Mercury	3.30×10^{23}	4880
Venus	4.87×10^{24}	12 100
Earth	5.97×10^{24}	12 800
Mars	6.42×10^{23}	6790

(a) List the planets in order of mass, starting with the lowest. [1]

(b) Find the radius, in kilometres, of Mars, giving your answer correct to 1 significant figure. [1]

(c) Giving your answer in standard form, find the total mass, in kilograms, of Venus and Mars. [1]

(d) [Volume of a sphere $= \frac{4}{3}\pi r^3$]

Giving your answer in standard form, find the volume, in cubic kilometres, of the Earth. [2]

Answers: (a) Mercury, Mars, Venus, Earth; (b) 3000; (c) 5.51×10^{24}; (d) 1.10×10^{12}. **J10/22/Q3**

Conversion and Limits of Accuracy Paper 1

1   (a) Add together 37 kilograms and 40 grams.
         Give your answer in kilograms.   *Answer (a)* .... kg [1]

(b) The length of a piece of string is 0.026 metres, correct to the nearest millimetre.
Write down, in millimetres, the lower bound of this length.

(b)..................................... mm [1]

Answers: **(a)** 37.04 kg; **(b)** 25.5 mm.   J03/1/Q8

---

2   (a) Maryam's height is 1.52 m correct to the nearest centimetre.
         State the lower bound of her height.

(b) The length of each of Maryam's paces is 0.55 m.
She walks at a constant speed of 2 paces per second.
Calculate the distance, in kilometres, that she walks in one hour.

Answer (a) .... [1]

(b) .... km [2]

Answers: **(a)** 1.515 m; **(b)** 3.96 km.   J04/1/Q16

---

3   (a) The population of a city is given as 280 000, correct to the nearest ten thousand.
         State the greatest possible error in the given value.

(b) The dimensions of a rectangular card are 7 cm by 4 cm, correct to the nearest centimetre.
Calculate the smallest possible perimeter of the card.

Answer (a) .... [1]

(b) ....cm [1]

Answers: **(a)** 5 000; **(b)** 20 cm.   J05/1/Q6

---

4   Complete the statements in the answer spaces.

Answer (a) 4872 correct to 1 significant figure is .... [1]

(b) 4872 correct to .... significant figures is 4870. [1]

Answer: **(a)** 5000   **(b)**   3   J06/1/Q4

5 The thickness of an oil film is 0.000004 cm.

(a) Express 0.000004 in standard form.   Answer (a) ....[1]

(b) The oil covers an area of 20 m².
Calculate the volume of the oil in cubic centimetres.   (b) .... cm³ [2]

Answer: (a) 4×10^{-6} (b)   0.8 cm³   **J06/1/Q9**

---

6 (b) The length and width of a rectangle are 50 cm and 15 cm respectively.
Each measurement is correct to the nearest centimetre.

(i) Write down the upper bound of the length.

(ii) Find the least possible perimeter of the rectangle.

Answer (b)(i) ....cm [1]

(ii) ....cm [1]
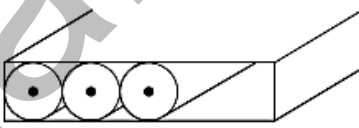
(b)(i) 50.5 cm   (ii) 128 cm   **J06/1/Q16b**

---

7 By writing each number correct to 1 significant figure, estimate the value of

$$\frac{8.62 \times 2.04^2}{0.285}.$$

Answer ....[2]

Answer: 100 or 120   **J08/1/Q4**

8

Pencils are packed in a box.
Each pencil has a diameter of 7 mm, correct to the nearest millimetre.

(a) Write down the lower bound of the diameter of a pencil.   (a) ......mm [1]

(b) Find the smallest width of a box that can **always** hold 8 pencils side by side.
Give your answer in centimetres.   Answer   (b) .... cm [2]

Answer: (a)   6.5   (b)   6   **J10/11/Q9**

---

9 By writing each number correct to 1 significant figure, estimate the value of

$$\frac{48.9 \times 0.207^2}{3.94}.$$

Answer   .... [2]

Answer: 0.5   **J10/12/Q5**

10 (a) A box has volume $2.5\,m^3$.

Express this volume in cm^3. *Answer* (a) .... cm^3 [1]

(b) John has a length of string.
The string is $4\,m$ long, correct to the nearest $10\,cm$.

   (i) Write down the lower bound of the length of the string.
   Give your answer in centimetres. *Answer* (b)(i) .... cm [1]

   (ii) John cuts off ten pieces of string.
   Each piece is $5\,cm$ long, correct to the nearest centimetre.

   Find the minimum possible length of string remaining.
   Give your answer in centimetres.

   Answer (b)(ii) .... cm [2]

Answer: (a) 2 500 000 (b)(i) 395 (ii) 340 **J10/12/Q22**

---

11 (a) By writing each number correct to one significant figure, estimate the value of
$$\frac{7.84 \times 326}{0.18}.$$
Answer .... [2]

(b) An athlete ran a race in 9.58 seconds, correct to the nearest hundredth of a second.

What is the shortest possible time that the athlete could have run the race?

Answer ....seconds [1]

Answers: (a) 12000 (b) 9.575 **J11/11/Q14**

---

12 The table shows the height, in metres, above sea level of the highest and lowest points in some continents.
A negative value indicates a point below sea level.

	Asia	Africa	Europe	South America
Highest point (m)	8850	5963	5633	6959
Lowest point (m)	−409	−156	−28	−40

(a) What is the height above sea level of the highest point in Africa?
Give your answer in **kilometres**. *Answer* .... km [1]

(b) In South America, how much higher is the highest point than the lowest point?
Give your answer in metres. *Answer* .... m [1]

(c) How much higher is the lowest point in Europe than the lowest point in Asia?
Give your answer in metres. *Answer* .... m [1]

Answer: (a) 5.963 (b) 6999 (c) 381 **J11/12/Q17**

13

(a) Express 0.047 852 correct to two decimal places. *Answer* .... [1]

(b) Estimate the value of $\sqrt{200}$, giving your answer correct to two significant figures.

Answer .... [1]

(c) By writing each number correct to one significant figure, estimate the value of

$$\frac{212 \times 1.97^2}{0.763}.$$

Answer .... [2]

Answer: (a) 0.05 (b) 14 (c) 1000 | **J11/12/Q19**

14

(a) A bag containing fruit has mass 3.813 kilograms.
When the bag is empty its mass is 257 grams.

Find, in kilograms, the mass of the fruit. *Answer* .... kg [1]

(b) The area of a shape is $1.2\,\text{m}^2$.

Convert this area to cm^2. *Answer* .... cm^2 [1]

Answer: (a) 3.556 (b) 12000 | **J13/11/Q3**

15

(a) By writing each number correct to one significant figure, estimate the value of

$$\frac{28.6 + 47.7}{0.47 \times 21.4}.$$ *Answer* .... [2]

(b) Write $\frac{8}{25}$ as a decimal. *Answer* .... [1]

Answers: (a) 8 (b) 0.32 | **J14/11/Q8**

16

(a) Correct to 6 decimal places, $\sqrt{10} = 3.162\,278$, $3\frac{1}{6} = 3.166\,667$.

Find the difference between $3\frac{1}{6}$ and $\sqrt{10}$.

Give your answer correct to 2 significant figures. *Answer* .... [1]

(b) Estimate, correct to the nearest whole number, the value of $\sqrt{2.986^2 + 4.002^2}$.

Answer .... [1]

Answers: (a) 0.0044 (b) 5 | **J15/11/Q4**

17

(a) An aircraft leaves at 22 35 on a flight that takes 3 hours and 50 minutes.

Find the time when the aircraft arrives. *Answer* .... [1]

(b) The aircraft flies a distance of 3200 km, correct to the nearest 100 km.

Write down the lower bound for the distance. *Answer* .... km [1]

Answers: (a) 02 25 (b) 3150 | **J16/11/Q3**

18

By writing each number correct to 1 significant figure, estimate the value of

$$\frac{29.2 \times 8.17}{0.396}.$$

Answer .... [2]

J16/11/Q7

---

19   (a) Express, correct to **two** significant figures,

    (i) 386.71,   *Answer (a)* (i) .... [1]

    (ii) 0.02049.   (ii) .... [1]

    (b) **Hence** estimate, correct to **one** significant figure, the value of

$$386.71 \times 0.02049.$$

    (b) .... [1]

N01/Q7

20

The distance travelled by an object was 230 m, correct to the nearest 10 m.
The time taken was 7 seconds, correct to the nearest second.

    (a) Complete the statements in the answer space.

    (b) What was the least possible average speed for the journey?

Answer (a) ....m $\leqslant$ distance <.............. m [1]

..................s $\leqslant$ time   <.............. s [1]

    (b) .... m/s [1]

N01/Q12

---

21   (a) Find the fraction which is exactly halfway between $\frac{1}{7}$ and $\frac{4}{7}$.

    (b) Subtract 370 grams from 3.7 kilograms.
        Give your answer in kilograms.   *Answer (a)* .... [1]

    (b) .... kg [1]

N03/Q1

22 (a) Add together 181 centimetres and 14.85 metres.
Give your answer in metres.

(b) Express 40 000 square metres in square kilometres.

Answer (a) ....m [1]

(b) ....km² [1]

---

23 The cost of a chair, x, is $70 correct to the nearest $10.
The cost of a desk, y, is $900 correct to the nearest $50.

(a) Complete the table in the answer space.

(b) Find the lower bound of the total cost of a desk and 4 chairs.

Answer (a)

	Lower Bound	Upper Bound
x		
y		

[2]

(b) $... [1]

---

24 (a) A rectangular table top is 100 cm long and 75 cm wide.
Both lengths are correct to the nearest 5 cm.
Find the least possible perimeter of the table.

(b) The area of a rectangular room is 22 m², correct to the nearest square metre.
The width is 3 m, correct to the nearest metre.
Find the greatest possible length of the room.

Answer (a) ....cm [2]

(b) ....m [2]

---

25 **(a)** An empty tin has a mass of 330 g.
When filled with powder, the total mass is 2.10 kg.
Find the mass, in kilograms, of the powder.

(b) Express 2.45 hours in minutes.

Answer (a) ....kg [1]

(b) ....minutes [1]

N06/1/Q2

26 In a race, an athlete runs 1600 m at an average speed of 6 m/s.
The distance is given correct to the nearest 100 m and the speed correct to the nearest metre per second.

(a) Complete the two statements in the answer space.

(b) Calculate the greatest possible time the race could have taken.

Answer (a) .... ⩽ distance < ....

.... ⩽ speed < ....[2]

(b) ....seconds [1]

N06/1/Q14

27

The mass of a marble is given as 5.4 grams, correct to the nearest tenth of a gram.
The mass of a box is given as 85 grams, correct to the nearest 5 grams.

(a) Complete the table in the answer space.

(b) Find the lower bound for the total mass of the box and 20 identical marbles.

Answer (a)

	Lower bound	Upper bound
Mass of 1 marble	.... g	.... g
Mass of the box	.... g	.... g	[2]

(b) ....g [1]

N07/1/Q11

28

(a) Estimate the value, correct to one significant figure, of $\dfrac{4.03^2 \times 29.88}{\sqrt{150}}$.

Answer (a) ....[2]

(b) Sam ran 100 metres in 12 seconds.
Calculate his average speed in kilometres per hour.

Answer (b) .... km/h [2]

Answer: (a) 40 (b) 30 km/h   **N07/1/Q19**

29 A basketball stadium has 13 492 seats.
During a season a basketball team played 26 matches and every seat was sold for each match.
At each match a seat costs $18.80.

By writing each value correct to 1 significant figure, estimate the total amount of money
paid to watch these matches during the season. *Answer* $ ....[2]

Answer: 6 000 000   **N08/1/Q4**

30

A rectangular box has dimensions 30 cm by 10 cm by 5 cm.
A container holds exactly 100 of these boxes.

(a) Calculate the total volume, in cubic metres, of the 100 boxes.

(b) Each box has a mass of 1.5 kg to the nearest 0.1 kg.
The empty container has a mass of 6 kg to the nearest 0.5 kg.
Calculate the greatest possible **total** mass of the container and 100 boxes.

Answer (a) .... m^3 [1]

(b) .... kg [2]

Answers: (a) 0.15 m^3, (b) 161.25 kg.   **N08/1/Q11**

31

The length of a side of an equilateral triangle is given as 41 mm, correct to the nearest millimetre.

(a) Write down the lower bound for the length of a side.

Answer (a) .... mm [1]

(b) Giving your answer in **centimetres**, calculate the lower bound for the perimeter of the triangle.

Answer (b) .... cm [1]

Answers: (a) 40.5 (b) 12.15   **N10/11/Q6**

34 By writing each number correct to one significant figure, estimate the value of

$$\frac{0.387 \times 7.03^2}{\sqrt[3]{8.11}}$$

Answer ....[2]

Answer: 10   **N10/11/Q8**

35 *ABC* is a triangle.
Angle *A* is 62°, correct to the nearest degree.
Angle *B* is 53.4°, correct to the nearest tenth of a degree.

   (a) Write down the lower bound for angle *B*.   *Answer* *(a)* ....[1]

   (b) Calculate the upper bound for angle *C*.   *Answer* *(b)* ....[2]

Answer: **(a)** 53.35° **(b)** 65.15°   **N10/12/Q21**

36 **(a)** Express, correct to **two** significant figures,

      (i)   15 823.769,   *Answer* *(a)*(i) ....[1]

      (ii)   0.003 048 9.   *Answer* *(a)*(ii) ....[1]

   (b) Use your answers to part **(a)** to estimate, correct to **one** significant figure, the value of

$$15\,823.769 \times 0.003\,048\,9.$$ *Answer* *(b)* ....[2]

Answer: **(a)(i)** 16 000 **(ii)** 0.0030 **(b)** 50   **N10/12/Q22**

37 **(a)** Add 55 minutes to 2.4 hours, giving your answer in hours and minutes.

      Answer   .... hours   .... minutes [1]

   (b) The mass of a bag of sugar is given as 1.5 kg, correct to the nearest tenth of a kilogram.

     Write down the upper bound of this mass, giving your answer in grams.

      Answer   .... g [1]

Answers: **(a)** 3 hours 19 minutes **(b)** 1550   **N11/11/Q4**

38 By making suitable approximations, estimate the value of $\dfrac{304.3 \times \sqrt{15.98}}{0.1975}$.

      Answer   .... [2]

Answer: 6000, 6080 or 6100   **N11/11/Q6**

39 By writing each number correct to two significant figures, estimate, correct to one significant figure, the value of

$$\sqrt{110.94} - 0.2034 \times 368.62 \ .$$

Answer .... [2]

---

40 The length of a side of a square is given as 57 mm, correct to the nearest millimetre.

(a) Write down the upper bound for the length of a side. *Answer* .... mm [1]

(b) Giving your answer in **centimetres**, calculate the upper bound for the perimeter of the square.

Answer .... cm [2]

---

41 Arrange these lengths in order of size, starting with the smallest.

$$2300 \, \text{mm} \qquad 220 \, \text{cm} \qquad 0.021 \, \text{km} \qquad 2\tfrac{1}{4} \, \text{m}$$

42

(a) **Estimate**, correct to the nearest whole number, the value of $\sqrt{\dfrac{72.187}{\pi}}$.

Show clearly the approximate values you use. *Answer* .... [1]

(b) The values of three cube roots, correct to 1 decimal place, are given below.

$$\sqrt[3]{5} = 1.7 \qquad \sqrt[3]{50} = 3.7 \qquad \sqrt[3]{500} = 7.9$$

Using as much of the above information as is necessary, find the value of $\sqrt[3]{0.005}$.

Answer ....[1]

---

43 The mass of a box is 2 kilograms, correct to the nearest kilogram.

(a) Write down the lower bound for the mass of the box. *Answer* .... kg [1]

(b) The mass of a can is 350 grams, correct to the nearest 10 grams.

Giving your answer in kilograms, calculate the lower bound for the total mass of the box and 20 identical cans.

Answer .... kg [2]

44 The length of a side of a square is given as d cm, correct to the nearest 10 cm.

Find an expression in terms of d for

(a) the **upper** bound of the perimeter of the square,

Answer .... cm [1]

(b) the **lower** bound of the area of the square. *Answer* .... cm² [1]

Answers: (a) 4d + 20 (b) (d − 5)² **N13/11/Q5**

45

By making suitable **approximations**, estimate the value of $\dfrac{38.982 \times \sqrt{8.8536}}{6.0122}$.
Show clearly the approximate values you use.

Answer .... [2]

Answer: 20 **N13/11/Q7**

---

46 (a) Write the number forty one thousand and six in figures.

Answer .... [1]

(b) Write 237 400 correct to two significant figures.

Answer .... [1]

Answers: **(a)** 41 006; **(b)** 240 000. **N14/11/Q1**

---

47 (a) Write 30 682 correct to three significant figures.

Answer .... [1]

(b) Given that 538 × 210 = 112 980, evaluate 112.98 ÷ 210.

Answer .... [1]

Answers: **(a)** 30700 **(b)** 0.538 **N15/11/Q6**

---

48 An empty box has a mass of 0.8 kg correct to the nearest 0.1 kg.

(a) Write down the lower bound for the mass of the empty box.

Answer .... kg [1]

(b) The box is filled with books.
The total mass of the box and the books is 6 kg correct to the nearest kilogram.

Work out the lower bound for the mass of the books. *Answer* .... kg [2]

Answers: **(a)** 0.75 **(b)** 4.65 **N15/11/Q11**

49 **(a)** Write the number 357.864 correct to 2 significant figures.

Answer .... [1]

(b) Estimate, correct to the nearest whole number, the value of $\dfrac{\sqrt[3]{67}}{1.03}$.

Answer .... [1]

Answers: **(a)** 360 **(b)** 4   **N15/11/Q5**

50 By writing each number correct to 1 significant figure, calculate an estimate for the value of

$\dfrac{987.65}{0.0193}$.   *Answer* .... [2]

Answer: 50 000   **N17/11/Q4**

Conversion and Limits of Accuracy Paper 2

1 **(b)** Chris ran 200 m, correct to the nearest 10 metres.
He took 25 s, correct to the nearest second.

Find lower bounds for

    (i) the distance run, [1]

    (ii) his average speed. [3]

(b)(i) 195 m, **(b)(ii)** 7.65 m/s. **J09/2/Q5b**

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2 **(b)**
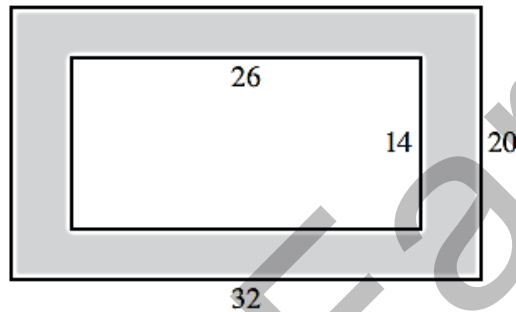
In the diagram, the shaded area represents a rectangular picture frame.
The outer rectangle is 32 cm by 20 cm.
The inner rectangle is 26 cm by 14 cm.
All measurements are given to the nearest centimetre.

    (i) Calculate the lower bound of the perimeter of the outer rectangle.

                          Answer .... cm [2]

    (ii) Calculate the upper bound of the area of the frame.

                          Answer ....cm^2 [3]

(b) (i) 102 **(ii)** 322 **J11/22Q2b**

3

(b) The length of a rectangular photo is 17.8 cm, correct to the nearest millimetre.
The width of the photo is 12.7 cm, correct to the nearest millimetre.

(i) Calculate the lower bound of the area of the photo.

Answer .... cm² [2]

(ii) Kate has a rectangular frame with length 18 cm and width 13 cm, both measured
correct to the nearest centimetre.

Will the photo definitely fit into the frame?
Explain your answer.

Answer .... because ....

.... [1]

(b)(i) 224.5 or 225 (ii) No, frame could measure 17.5 cm by 12.5 cm   **J14/21/Q6b**

Prime Factorization Paper 1

1   **(a)**   Express 99 as the product of its prime factors.   *Answer (a)* .... [1]

     (b)   Find the smallest possible integer value of *n* for which 99*n* is a multiple of 24.

                              (b) .... [1]

Answers: **(a)** 3 × 3 × 11; **(b)** 8.   J03/1/Q6

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2   25   **(a)**   **(i)**   Express 7056 as the product of its prime factors.

         (ii)   Hence evaluate $\sqrt{7056}$.   *Answer (a)(i)* .... [2]

                           (ii) .... [1]

     (b) $\sqrt{5\frac{1}{16}}$ can be expressed as the rational number $\frac{p}{q}$ where *p* and *q* are integers.

        Find the value of *p* and the value of *q*.   *Answer (b)* $p =$ .... , $q =$ .... [1]

     (c)   Write down an example of an irrational number.   *Answer (c)* .... [1]

Answers: **(a)(i)** 24 × 3² × 7², **(ii)** 84; **(b)** $p = 9$, $q = 4$.   J04/1/Q25

---

3   Green Line buses run every 10 minutes.
Red Line buses run every 20 minutes.
Purple Line buses run every 35 minutes.
One bus from each Line leaves the city centre at 0900.
After how many minutes will buses from all three Lines next leave the city centre at the same time?

                        Answer .... [2]

Answer: 140 minutes.   J05/1/Q10

---

4   **(a)**   Express 154 as the product of its prime factors.

     (b)   Find the lowest common multiple of 154 and 49.

                      Answer (a) ....[1]

                         (b) ....[1]

Answer: **(a)** 2 × 7 × 11   **(b)** 1078   J07/1/Q6

---

5   **(a)**   Write down the two cube numbers between 10 and 100.

                      Answer (a) ....[1]

     Write down the two prime numbers between 30 and 40.

                      Answer (b) .... [1]

Answer: (a) 27, 64   (b) 31, 37   **J09/1/Q3**

6   (a) Write down all the factors of 18.   *Answer* (a) ....[1]

) Write 392 as the product of its prime factors.   *Answer* (b) .... [1]

Answer: (a) 1, 2, 3, 6, 9, 18   (b) $2^3 \times 7^2$ **J09/1/Q6**

7   Written as a product of prime factors, $168 = 2^3 \times 3 \times 7$.

  (a) Express 140 as a product of its prime factors.   *Answer* (a) ....[1]

  (b) Find the highest common factor of 168 and 140.   *Answer* (b) ....[1]

  (c) Find the smallest positive integer, n, such that $168n$ is a square number.

                                                          Answer (c) ....[1]

Answer: (a) $2^2 \times 5 \times 7$ (b) 28   (c) 42   **J10/12/Q9**

8   Write down

  (a) a square number that is a factor of 75,   *Answer* .... [1]

  (b) a cube number that is a multiple of 24.   *Answer* .... [1]

Answers: (a) 1 or 25   (b) 216 etc.   **J11/11/Q4**

9   (a) Express 108 as a product of its prime factors.   *Answer* .... [1]

  (b) Written as products of their prime factors, $N = 2^p \times 5^q \times 7^r$ and $500 = 2^2 \times 5^3$.

      The highest common factor of N and 500 is $2^2 \times 5^2$.
      The lowest common multiple of N and 500 is $2^3 \times 5^3 \times 7$.

      Find p, q and r.   *Answer* $p = $ .... , $q = $ .... , $r = $ .... [2]

Answer: (a) $2^2 \times 3^3$ (b) $p = 3, q = 2, r = 1$ **J11/12/Q14**

10   An empty lorry has a mass of 4.3 tonnes, correct to the nearest tenth of a tonne.

  (a) What is the lower bound for the mass of the empty lorry?   *Answer* ....tonnes [1]

  (b) The total mass of the lorry and its load is 6.8 tonnes, correct to the nearest tenth of a tonne.

      Find the upper bound for the mass of the load.   *Answer* ....tonnes [1]

Answer: (a) 4.25 (b) 2.6   **J12/11/Q5**

11 Buses following route A leave the bus station every five minutes.
Buses following route B leave the bus station every six minutes.
Buses following route C leave the bus station every nine minutes.
Three buses, following routes A, B and C, leave together at 13 00.

What is the next time when buses following all three routes leave the bus station together?

Answer .... [2]

Answer: 14 30   J12/11/Q9

12
(a) Express 500 as the product of its prime factors.   *Answer* .... [1]

(b) $M = 2 \times 3^2$ $N = 2^4 \times 3^2$

Find the values of p and q when

(i) $M \times N = 2^p \times 3^q$,   *Answer* $p =$ .... $q =$ .... [1]

(ii) $M \div N = 2^p \times 3^q$,   *Answer* $p =$ .... $q =$ .... [1]

(iii) $N^2 = 2^p \times 3^q$.   *Answer* $p =$ .... $q =$ .... [1]

Answers: (a) $2^2 \times 5^3$ (b)(i) $p = 5$ and $q = 4$ (b)(ii) $p = -3$ and $q = 0$ (b)(iii) $p = 8$ and $q = 4$ J16/11/Q21

13
(a) Express 36 as the product of its prime factors.   *Answer* .... [1]

(b) Write down two prime numbers whose sum is 15.   *Answer* .... [1]

Answers: (a) $2 \times 2 \times 3 \times 3$ (b) 2 and 13   J17/11/Q4

14 The numbers 168 and 324, written as the products of their prime factors, are

$$168 = 2^3 \times 3 \times 7, \quad 324 = 2^2 \times 3^4.$$

Find

(a) $\sqrt{324}$,

(b) the largest integer which is a factor of both 168 and 324,

(c) the smallest positive integer value of n for which $168n$ is a multiple of 324.

Answer (a) .... [1]

(b) .... [1]

(c) .... [1]

15 Expressed as the product of prime factors,

$$198 = 2 \times 3^2 \times 11 \quad \text{and} \quad 90 = 2 \times 3^2 \times 5.$$

Use these results to find

(a) the smallest integer, k, such that $198k$ is a perfect square,

(b) the highest common factor of 198 and 90.

16

(a) Find the lowest common multiple of 12, 30 and 66.

(b) Three lightships flash simultaneously at 6 00 a.m.
The first lightship flashes every 12 seconds, the second lightship every 30 seconds and the third lightship every 66 seconds.
At what time will the three lightships next flash together?

Answer (a).. [1]

(b).. [1]

17 Written as the product of its prime factors, $360 = 2^3 \times 3^2 \times 5$.

(a) Write 108 as the product of its prime factors.

(b) Find the lowest common multiple of 108 and 360.
Give your answer as the product of its prime factors.

(c) Find the smallest positive integer k such that $360\,k$ is a cube number.

Answer (a) $108 =$ ....[1]

(b) ....[1]

(c) $k =$ ....[1]

18

(a) It is given that $p = 4 \times 10^5$ and $q = 8 \times 10^6$.
Expressing your answers in standard form, find

  (i) $\dfrac{p}{q}$, Answer (a)(i) ....[1]

  (ii) $\sqrt[3]{q}$. (ii) ....[1]

(b) The numbers 225 and 540, written as the products of their prime factors, are

$$225 = 3^2 \times 5^2, \qquad 540 = 2^2 \times 3^3 \times 5.$$

  (i) Write 2250 as the product of its prime factors.

  (ii) Find the smallest positive integer value of n for which $225n$ is a multiple of 540.

  Answer (b)(i) ....[1]

  (ii) $n =$ ....[1]

Answer: (a)(i) 5×10^{-2} (ii) 2×10^2 (b)(i) $2 \times 3^2 \times 5^3$ (ii) $n = 12$ **N07/1/Q17**

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19 The numbers 294 and 784, written as the product of their prime factors, are

$$294 = 2 \times 3 \times 7^2, \qquad 784 = 2^4 \times 7^2.$$

Find Answer (a) ....[1]

(a) the largest integer which is a factor of both 294 and 784,

(b) $\sqrt{784}$. Answer (b) ....[1]

Answers: (a) 98 (b) 28 **N09/1/Q4**

---

20 A number written as the product of its prime factors is $2^2 \times 5^2 \times 7$.

(a) Evaluate this number. Answer .... [1]

(b) The lowest common multiple of $2^2 \times 5^2 \times 7$ and another number, N, is $2^2 \times 3 \times 5^2 \times 7^2$.

Find the lowest possible value of N. Answer $N =$ .... [1]

Answers: (a) 700 (b) 147 **N15/11/Q3**

Number Sequences and Patterns Paper 1

1   Consider the sequence 1^3-2, 2^3-4, 3^3-6, 4^3-8, ...

(a)   Write down the 5th term of the sequence.   *Answer (a)* .... [1]

(b)   Write down, in terms of n, an expression for the nth term of the sequence.   *(b)* .... [1]

(c)   Evaluate the 10th term of the sequence.   *(c)* .... [1]

2   (a)   The nth term of a sequence is $7-2n$.

Write down the 23rd term in this sequence.

Answer (a) ....[1]

(b)   (i)   The first five terms of another sequence are

$$4 \quad 7 \quad 10 \quad 13 \quad 16.$$

Write down an expression, in terms of n, for the nth term of this sequence.

(ii)   The first five terms of another sequence are

$$\frac{4}{1} \quad \frac{7}{4} \quad \frac{10}{9} \quad \frac{13}{16} \quad \frac{16}{25}.$$

(a)   Write down the next term in this sequence.

(b)   Write down an expression, in terms of n, for the nth term of this sequence.

Answer (b) (i) ....[1]

(b)(ii)(a) ....[1]

(b)(ii)(b) ....[1]

Answers: (a) -39 (b) (i) $3n+1$ (ii) $\dfrac{19}{36}$ (iii) $\dfrac{3n+1}{n^2}$

3 The nth term of a sequence is $\dfrac{4}{n^2}$.

(a) Write down the first three terms of the sequence, expressing each term in its simplest form.

Answer (a)........... , ... , ...[1]

(b) The kth term in the sequence is $\dfrac{1}{100}$.

Find the value of k. *Answer (b)* $k =$ ....[2]

(c) Given that the mth term of the sequence is less than 0.0064, find the smallest value of m.

Answer (c) $m =$ ....[2]

Answer: (a) $4, 1, \dfrac{4}{9}$ (b) 20 (c) 26 **J09/1/Q21**

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4 (a) Write down, in terms of n, an expression for the nth term of the sequence

19   16   13   10 .... *Answer (a)* .... [2]

Answer: (a) $22 - 3n$ **J10/11/Q21**

---

5 The first four terms, u_1, u_2, u_3 and u_4, in a sequence of numbers are given by

$$
\begin{aligned}
u_1 &= 1 \times 2 + 3^2 = 11 \\
u_2 &= 2 \times 3 + 4^2 = 22 \\
u_3 &= 3 \times 4 + 5^2 = 37 \\
u_4 &= 4 \times 5 + 6^2 = 56.
\end{aligned}
$$

(a) Evaluate u_5. *Answer* .... [1]

(b) Write down an expression, in terms of n, for the nth term, u_n, of the sequence.

Answer .... [1]

(c) Given that $u_n = An^2 + Bn + C$, find the values of A, B and C.

Answer $A =$ ....

$B =$ ....

$C =$ .... [2]

Answers: (a) 79 (b) $n(n+1) + (n+2)^2$ (c) $A = 2$ $B = 5$ $C = 4$ **J11/11/Q18**

89

6   A sequence of diagrams is made using black and white counters.

Diagram 1   Diagram 2   Diagram 3   Diagram 4
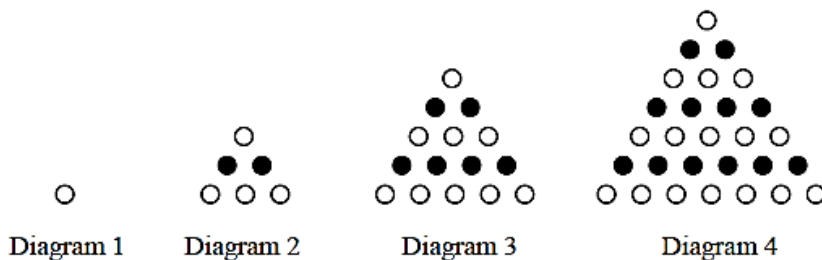
The number of black and white counters in each diagram is shown in the table below.

Diagram number	1	2	3	4	5	6
Number of white counters	1	4	9	16		
Number of black counters	0	2	6	12		

(a)   Complete the table for Diagrams 5 and 6.   [1]

(b)   Write an expression, in terms of n, for the number of **white** counters in the nth diagram.

Answer   ....[1]

   (c)   By considering the number patterns in the table, write an expression, in terms of n, for the

number of **black** counters in the nth diagram.   *Answer*   ....[1]

(d)   What is the **total** number of counters in the 20th diagram?

Answer   ....[1]

Answer: (a) 25 36 (b) n^2 (c) $n^2 - n$ (d) 780   **J12/11/Q19**
           20 30

90

7 **(a)** Here are the first four terms of a sequence.

$$7 \quad 11 \quad 15 \quad 19$$

Write down an expression, in terms of n, for the nth term of this sequence.

Answer .... [1]

(b) u_n is the nth term of another sequence.
Here is the formula connecting the nth and $(n+1)$th terms of this sequence.

$$3u_n - 4 = u_{n+1}$$

The value of u_3 is 11.

Answer $u_2 = $ ....

Find u_2 and u_4.

$u_4 = $ .... [2]

Answers: (a) $4n + 3$ (b) 5 and 29   **J14/11/Q12**

---

8   A pattern of numbers is given below.

Row 1 $\dfrac{1}{1 \times 2} = \dfrac{1}{1} - \dfrac{1}{2}$

Row 2 $\dfrac{1}{2 \times 3} = \dfrac{1}{2} - \dfrac{1}{3}$

Row 3 $\dfrac{1}{3 \times 4} = \dfrac{1}{3} - \dfrac{1}{4}$

Row 4 $\dfrac{1}{4 \times 5} = \dfrac{1}{4} - \dfrac{1}{5}$

(a) Write down Row 10.

Answer.. [1]

(b) Adding the first two rows gives the result $\dfrac{1}{1 \times 2} + \dfrac{1}{2 \times 3} = \dfrac{1}{1} - \dfrac{1}{3} = \dfrac{2}{3}$.

Adding the first three rows gives the result $\dfrac{1}{1 \times 2} + \dfrac{1}{2 \times 3} + \dfrac{1}{3 \times 4} = \dfrac{1}{1} - \dfrac{1}{4} = \dfrac{3}{4}$.

(i) Write down the result of adding the first four rows.

Answer .... [1]

(ii) Use **the pattern** to write down

(a) the value of $\dfrac{1}{1 \times 2} + \dfrac{1}{2 \times 3} + \dfrac{1}{3 \times 4} + ... + \dfrac{1}{19 \times 20}$,

Answer .... [1]

91

(b) the number of rows that add up to $\frac{109}{110}$,

Answer .... [1]

(c) an expression, in terms of n, for the result of adding the first n rows.

Answer .... [1]

Answers: **(a)** $\frac{1}{10} - \frac{1}{11}$ **(b)(i)** $\frac{1}{1} - \frac{1}{5} = \frac{4}{5}$ **(ii)(a)** $\frac{19}{20}$ **(b)** 109 **(c)** $\frac{n}{n+1}$

9 A sequence of diagrams is made using counters.

Diagram 1   Diagram 2   Diagram 3   Diagram 4
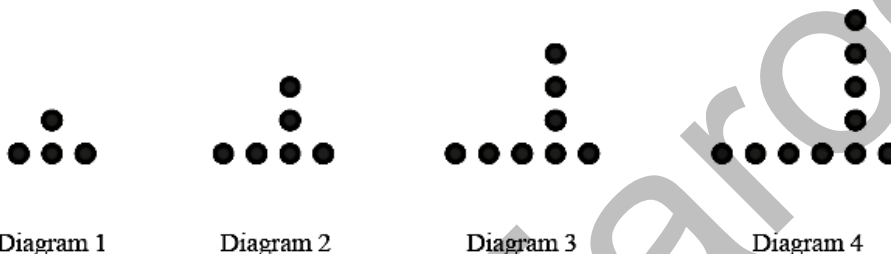
(a) Complete the table.

Diagram number	1	2	3	4	5
Number of counters	4	6	8		

[1]

(b) Find an expression, in terms of n, for the number of counters in Diagram n.

Answer .... [1]

(c) In this sequence, Diagram p has 200 counters.

Find the value of p.   *Answer* p = ... [2]

Answers: **(a)** 10, 12 **(b)** $2n + 2$ **(c)** 99

10 Find

(a) the missing number in the sequence 1, 3, 6, ..., 15, 21, 28,

(b) the 7th term in the sequence whose nth term is $3n - 1$,

(c) an expression, in terms of n, for the nth term of the sequence 5, 9, 13, 17, 21, ...

Answers: **(a)** 10; **(b)** 20; **(c)** $4n + 1$.

11 Look at this pattern

$$1^2 - 0^2 = 1$$
$$2^2 - 1^2 = 3$$
$$3^2 - 2^2 = 5$$
$$4^2 - 3^2 = 7$$
$$\vdots \quad \vdots \quad \vdots$$

(a) Write down

    (i) the 8th line of the pattern,

    (ii) the nth line of the pattern.

(b) Use the pattern to find

    (i) $340^2 - 339^2$,

    (ii) the integers x and y such that $x^2 - y^2 = 701$.

Answers: **(a)(i)** $8^2 - 7^2 = 15$, **(ii)** $n^2 - (n-1)^2 = 2n-1$; **(b)(i)** 679, **(ii)** $x = 351$, $y = 350$.   N03/Q18

12

(a) Write down the next two terms in the sequence $20, 16\frac{1}{2}, 13, 9\frac{1}{2}, 6, \ldots\ldots$

(b) Write down an expression, in terms of n, for the nth term of the sequence

$$1, \ 4, \ 7, \ 10, \ 13, \ \ldots\ldots\ldots\ldots$$

           Answer (a) ...., ... [1]

           (b) .... [1]

Answers: **(a)** $2\dfrac{1}{2}, -1$; **(b)** $3n - 2$ N04/Q4

13

(a) The first five terms of a sequence are $1, \ 3, \ 6, \ 10, \ 15$.
The nth term of this sequence is $\frac{1}{2}n(n+1)$.
Find the 19th term.   *Answer (a)* .... [1]

(b) Write down an expression, in terms of n, for the nth term of the sequence

$$3, \ 6, \ 10, \ 15, \ 21, \ \ldots\ldots\ldots$$

           (b) .... [1]

Answers: **(a)** 190; **(b)** $\dfrac{1}{2}(n+1)(n+2)$ or equivalent expressions.   N05/1/Q6

14

24 A series of diagrams, using three types of triangle, is shown below.
The triangles are grey, white or black.

Diagram 1   Diagram 2   Diagram 3   Diagram 4

The table below shows the numbers of each type of triangle used in the diagrams.

Diagram	1	2	3	4		n
Grey triangles	2	4	6			x
White triangles	1	4	9			y
Black triangles	0	2	6			z

(a) Complete the column for Diagram 4.   [1]

(b) By considering the number patterns in the table, find, in terms of n, expressions for x, y and z.

Answer (b) $x =$ ....

$y =$ ....

$z =$ ....[4]

Answer. (a) (4), 8, 16, 12   (b) $x = 2n$; $y = n^2$; $z = n^2 - n$ N07/1/Q24

15   The diagrams below show small black, grey and white triangles forming a pattern.

Diagram   1   2   3   4   5

94

The table below shows the number of triangles in each diagram.

Answer (a)

Diagram (n)	1	2	3	4	5	6
Small triangles	1	4	9	16	25	
Black triangles	1	3	5	7	9	
Grey triangles	0	1	3	6	10	
White triangles	0	0	1	3	6	10

(a) Complete the column for Diagram 6. [2]

(b) Write an expression, in terms of n, for the number of

    (i) small triangles in Diagram n,   *Answer (b)(i)* ....[1]

    (ii) black triangles in Diagram n.   *Answer (b)(ii)* ....[1]

Answers: **(a)** 36, 11, 15   **(b)(i)** n^2 **(ii)** $2n - 1$ **N09/1/Q22**

---

16 Look at this pattern.

$$2^2 - 0^2 = 4 \times 1$$
$$3^2 - 1^2 = 4 \times 2$$
$$4^2 - 2^2 = 4 \times 3$$
$$5^2 - 3^2 = 4 \times 4$$

(a) Write down the 7th line of the pattern.

Answer (a) ....[1]

(b) Write down the nth line of the pattern.

Answer (b) ....[1]

(c) Use the pattern to find $521^2 - 519^2$.   *Answer (c)* ....[1]

(d) Use the pattern to find the positive integers x and y such that $x^2 - y^2 = 484$.

Answer (d) $x =$ ....

$y =$ ....[1]

Answers: **(a)** $8^2 - 6^2 = 4 \times 7$ **(b)** $(n+1)^2 - (n-1)^2 = 4n$ **(c)** 2080 **(d)** $x = 122$ and $y = 120$ **N10/11/Q23**

17 A series of shapes, made of matchsticks, is shown below.

Shape 1   Shape 2   Shape 3   Shape 4
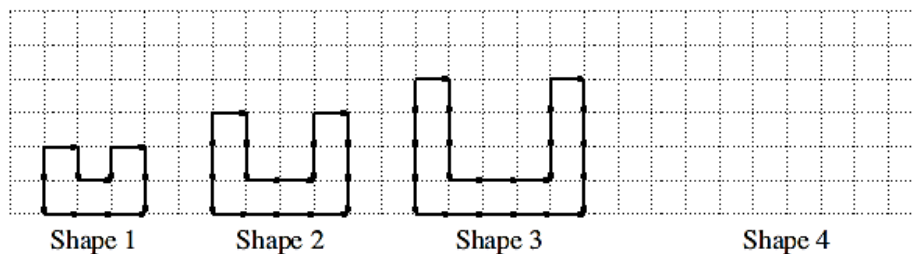
(a) Draw Shape 4.   [1]

(b) The table shows the numbers of matchsticks used to make Shapes 1 and 2.

Shape	1	2	3	4
Number of matchsticks	12	18		

Complete the table for Shapes 3 and 4.   [1]

(c) Find an expression, in terms of n, for the number of matchsticks used to make Shape n.

                                        Answer .... [1]

(d) Explain why there is not a shape that is made of 100 matchsticks.

     Answer ....[1].........................

18 The first four terms of a sequence are $55, 53, 49, 41$.
The nth term of this sequence is $57 - 2^n$.

(a) Calculate the fifth term.   Answer .... [1]

(b) Write down the nth term of the sequence $56, 55, 52, 45 \ldots$ Answer .... [1]

19 The sequence of positive integers is arranged in the pattern below.

Row 1	1	2	3
Row 2	4	5	6
Row 3	7	8	9
Row 4	10	11	12

           .   .   .   .

     Row n .... $3n - 1$ ....

96

(a) Complete Row n. [1]

(b) The table shows some results obtained from this pattern.

Row number	1	2	3	4		n
Square of the middle number in the row	4	25	64			x
Product of the first and the last number in the row	3	24	63			y

    (i) Complete the column for Row number 4. [1]

    (ii) Find an expression, in terms of n, for y.   *Answer* ....[1]

    (iii) Show that $x - y$ is always equal to 1.

20   The **first** and **second** terms of a sequence are 15 and 11 respectively.

The nth term of the sequence is $10 + An + \dfrac{B}{n}$.

(a) Show that $A + B = 5$ and $4A + B = 2$. [2]

(b) Solve the simultaneous equations.

$$A + B = 5$$
$$4A + B = 2$$

    Answer $A = $ ....

    $B = $ ....[2]

(c) Hence find the third term of the sequence.   *Answer* ....[1]

21   **(a)** The term-to-term rule for a sequence is

> multiply the previous term by 3 and subtract 1 .

The first three terms in this sequence are 1, 2 and 5 .

Write down the next two terms in this sequence.

    Answer   .... , ... [1]

(b) The nth term of a second sequence is given by the expression $4n - 3$.

Find the number in this sequence that is closest to 150.   *Answer* ....[1]

(c) The nth term of a different sequence is given by the expression $n^2 + 1$.

    (i) Write down the first four terms of this sequence.

                    Answer .... , ... , ... , ... [1]

    (ii) Hence write down an expression, in terms of n, for the nth term of the following sequence.

$$0 \quad\quad 3 \quad\quad 8 \quad\quad 15 \quad\quad .....$$

                    Answer .... [1]

Answers: (a) 14 41; (b) 149; (c)(i) 2 5 10 17, (ii) $n^2 - 1$.

22 A sequence of patterns is made using dots and lines.
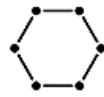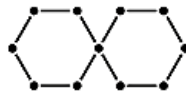
Pattern 1   Pattern 2   Pattern 3

Pattern number (p)	1	2	3	4
Number of dots (d)	6	11	16	

(a) Complete the table for Pattern 4.   [1]

(b) Find a formula for the number of dots, d, in Pattern p.   *Answer* $d = $ .... [2]

Answers: (a) 21 (b) $5p + 1$

23
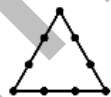
Triangle 1   Triangle 2   Triangle 3   Triangle 4
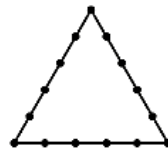
The diagrams show a sequence of triangles made up of identical sticks.
Each triangle has two more sticks on each edge than its previous triangle.
The table shows information relating to this sequence.

Triangle number	1	2	3	4		n
Number of sticks on each side	1	3	5			x
Number of sticks in the triangle	3	9	15			y

98

(a) Complete the column for triangle 4. [1]

(b) Find an expression, in terms of *n*, for *x*. *Answer* x = ... [1]

(c) Find an expression, in terms of *n*, for *y*. *Answer* y = ... [1]

(d) The total number of sticks in the first triangle = 3
The total number of sticks in the first two triangles = 12
The total number of sticks in the first three triangles = 27

   (i) Write down the total number of sticks in the first four triangles.

                                   Answer .... [1]

   (ii) Find an expression, in terms of *n*, for the total number of sticks in the first *n* triangles.

                                   Answer .... [2]

Answers: (a) 7, 21 (b) $2n-1$ (c) $6n-3$ (d)(i) 48 (ii) $3n^2$ **N16/11/Q23**

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24 Mary makes pendants, of the same design, from small beads.
The sequence of diagrams shows the pendants she makes.

Diagram 1   Diagram 2   Diagram 3   Diagram 4
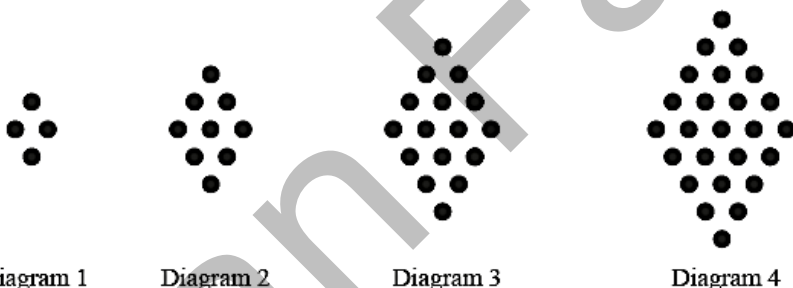
(a) Complete the table.

Diagram number	1	2	3	4	5
Number of rows	3	5	7	9	
Number of beads	4	9	16	25	

[1]

(b) Find an expression, in terms of *N*, for

   (i) the number of rows in Diagram *N*, *Answer* .... [1]

   (ii) the number of beads in Diagram *N*. *Answer* .... [1]

(c) Julia asks Mary to make her a pendant with 25 rows.

How many beads are used to make this pendant? *Answer* .... [2]

Answers: (a) 11, 36 (b)(i) $2N + 1$ (ii) $(N + 1)^2$ (c) 169   **N17/11/Q25**

Number Sequences and Patterns Paper 2

1
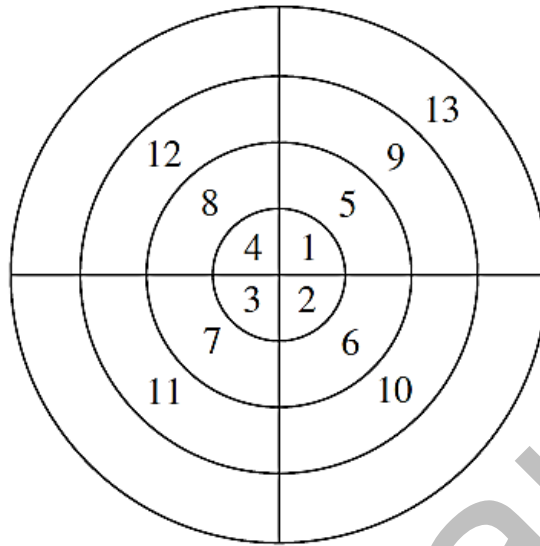
The natural numbers $1, 2, 3, \ldots$ are written, in a clockwise direction, on a circular grid as shown in the diagram.

There are four numbers in each ring.

The numbers 1, 2, 3, and 4 are in the first ring.

The numbers 5, 6, 7 and 8 are in the second ring.

The following numbers fill up the other rings in the same way.

(a) Write down the numbers in the fourth ring. [1]

(b) Write down the largest number in the tenth ring. [1]

(c) The sum, S_n, of the four numbers in the nth ring, where $n = 1, 2$ and 3, is given in the table below.

n	1	2	3	4
S_n	10	26	42	

(i) Write down the value of S_4. [1]

(ii) Find, in its simplest form, an expression, in terms of r, for S_r. [2]

(iii) In which ring is the sum of the four numbers equal to 1018? [1]

Answers: **(a)** (13), 14, 15, 16; **(b)** 40; **(c)(i)** 58, **(ii)** $16r - 6$, **(iii)** 64th.

2 Bob makes fences using identical metal rods one metre long.
The rods are bolted together at their ends.

Some fences, with different lengths, are shown below.
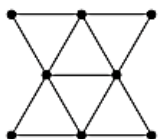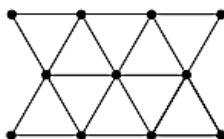
Length = 1 m   Length = 2 m   Length = 3 m   Length = 4 m
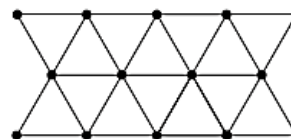
• shows the position of a bolt.

The table shows the numbers of bolts and rods used for various lengths of fence.

Length (metres)	1	2	3	4	.....	n
Number of bolts	5	8	11	p	.....	B
Number of rods	6	13	20	q	.....	R

(a) Write down the values of p and q.   [1]

(b) Given that B = 3n + k, where k is a constant, find the value of k.   [1]

(c) Find an expression for R in terms of n.   [2]

(d) Bob has 200 bolts and 400 rods.

How many complete fences can he make which have a length of 6 m?   [2]

Answers: (a) p = 14, q = 27; (b) 2; (c) 7n − 1; (d) 9.   **J04/2/Q6**

3
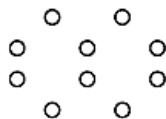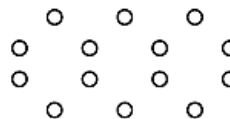
Pattern 1   Pattern 2   Pattern 3

Counters are used to make patterns as shown above.
Pattern 1 contains 6 counters.
The numbers of counters needed to make each pattern form a sequence.

(a) Write down the first four terms of this sequence.   [1]

(b) The number of counters needed to make Pattern n is An + 2.
Find the value of A.   [1]

(c) Mary has 500 counters.
She uses **as many of** these counters as she can to make **one** pattern.

Given that this is Pattern m, find

    (i) the value of m, [1]

    (ii) how many counters are **not** used. [1]

Answers: (a) 6, 10, 14, 18; (b) 4 ; (c)(i) 124, (ii) 2. **J06/2/Q5**

---

4 **(a)** Express as a single fraction in its simplest form $\dfrac{7}{6a} - \dfrac{5}{9a}$. [2]

    (b) Simplify $3b(b-1) - 2(b-2)(b+2)$. [2]

    (c) The nth term of a sequence, S, is $n^3 + 2$.
    The first four terms are 3, 10, 29 and 66.

      (i) Find the fifth term of S. [1]

      (ii) The first four terms of another sequence, T, are 4, 12, 32 and 70.
      By comparing S and T, write down

        (a) the fifth term of T, [1]

        (b) an expression, in terms of n, for the nth term of T. [1]

Answer. (a) $\dfrac{11}{18a}$; (b) $b^2 - 3b + 8$; (c)(ii) 127, (iii)(a) 132, (b) $n^3 + n + 2$; **J07/2/Q3**

5 The diagram shows the first four rows of a pattern of numbers.

Row 1   1   2   1

Row 2   2   3   2   3   2

Row 3   3   4   3   4   3   4   3

Row 4   4   5   4   5   4   5   4   5   4

The table shows some results obtained from this pattern.

Row number	1	2	3	4	5		n
Number of numbers in the row	3	5	7	9	p		x
Product of the first two numbers in the row	2	6	12	20	q		y
Sum of all the numbers in the row	4	12	24	40	r		z
Middle number in the row	2	2	4	4	s		

(a) Find the values of p, q, r and s.   [2]

(b) Find expressions, in terms of n, for x, y and z.   [3]

(c) Write down the middle number in Row 101.   [1]

Answers: (a) $p = 11$, $q = 30$, $r = 60$, $s = 6$;   (b) $x = 2n + 1$, $y = n(n + 1)$, $z = 2n(n + 1)$;   (c) 102.   **J09/2/Q6**

6

Flowerbed 1   Flowerbed 2   Flowerbed 3   Flowerbed 4
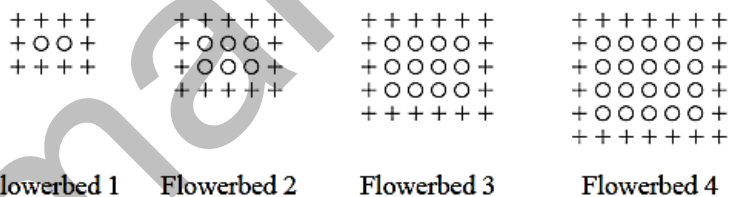
The diagrams above show the first four flowerbeds in a sequence.
Each flowerbed contains two types of plant, pansies (+) and primroses (○).

The table shows the number of plants in the first three flowerbeds.

Flowerbed number (n)	1	2	3	4	5
Number of pansies	10	14	18		
Number of primroses	2	6	12		
Total number of plants	12	20	30		

(a) Copy and complete the columns for flowerbeds 4 and 5.   [2]

(b) Find an expression, in terms of n, for

    (i) the number of pansies in flowerbed n,   [1]

    (ii) the number of primroses in flowerbed n.   [1]

(c) **Hence** show that the total number of plants in flowerbed n can be expressed in the form

$$(n+2)(n+3).$$ [2]

(d) Calculate the total number of plants in flowerbed 10.   [1]

(e) There are 306 plants in flowerbed k.

    (i) Show that k satisfies the equation

$$k^2 + 5k - 300 = 0.$$ [2]

    (ii) Solve the equation $k^2 + 5k - 300 = 0.$ [2]

    (iii) Hence find the number of pansies in flowerbed k.   [1]

Answers: **(a)** 22, 20, 42 and 26, 30, 56;   **(b)(i)** $4n+6$,   **(ii)** n^2+n,   **(d)** 156;   **(e)(ii)** 15   **J10/22/Q10**
and -20,   **(iii)** 66.

---

7 u_n is the nth term of the sequence   4, 7, 10, 13, ...

    (a)   **(i)**   Write down an expression, in terms of n, for u_n.   *Answer* ....[1]

    (ii)   Hence find the 20th term of the sequence.   *Answer* ....[1]

    (b) v_n is the nth term of the sequence 15, 13, 11, 9, ...

        (i)   Write down an expression, in terms of n, for v_n.   *Answer* ....[1]

        (ii) w_n is the nth term of another sequence that is obtained by multiplying u_n by v_n.

           Given that $w_n = 17 + kn - 6n^2$, find k.   *Answer* ....[1]

Answers: **(a) (i)** $3n+1$ **(ii)** 61   **(b) (i)** $17-2n$ **(ii)** 49   **J11/22/Q4**

---

8   **(a)** The diagrams show parallelograms made from small triangles.

Parallelogram   1   2   3   4   5
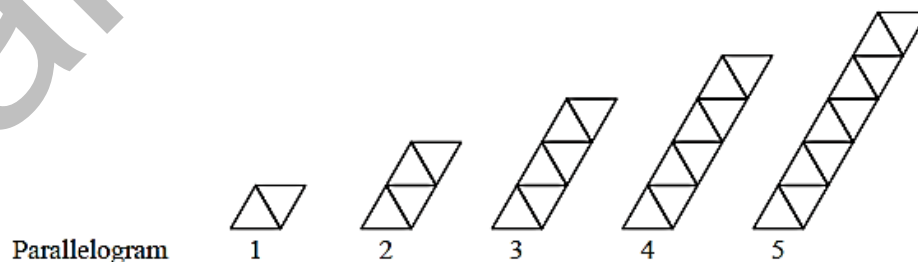
    (i) Complete the table below.

Parallelogram m	1	2	3	4	5	6
Number of small triangles	2	4	6	8		

[1]

(ii) Find an expression, in terms of m, for the number of small triangles used to make Parallelogram m.

Answer .... [1]

(b) The diagrams show triangles made from the same small triangles.
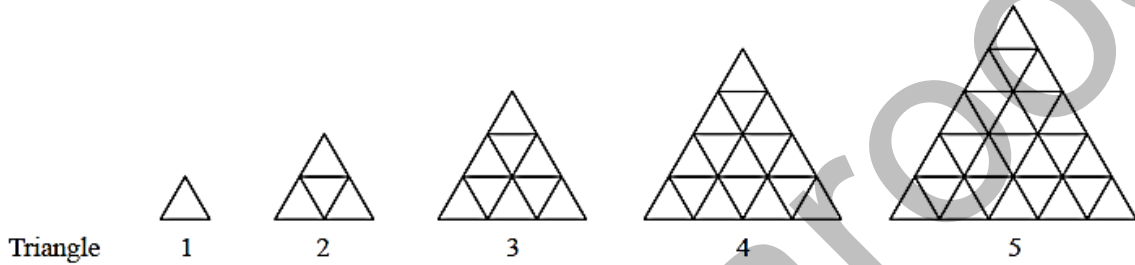
Triangle   1   2   3   4   5

(i) Complete the table below.

Triangle n	1	2	3	4	5	6
Number of small triangles	1	4	9	16		

[1]

(ii) Find an expression, in terms of n, for the number of small triangles used to make Triangle n.

Answer .... [1]

(iii) Triangle q is made from 324 small triangles.
Find q.

Answer .... [1]

(c) The diagrams show trapeziums made from the same small triangles.

Trapezium   1   2   3
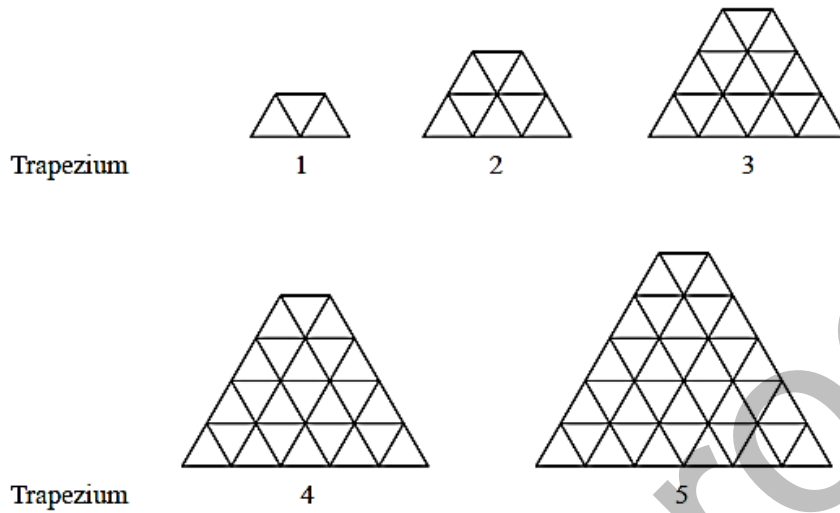
Trapezium   4   5

    (i) By comparing the diagrams with those in parts **(a)** and **(b)**, find an expression, in terms of t, for the number of small triangles used to make Trapezium t.

Answer .... [1]

    (ii) How many small triangles are used to make Trapezium 25?

Answer .... [1]

Answer: **(a)(i)** 10,12 **(ii)** 2m **(b)(i)** 25,36 **(ii)** n^2 **(iii)** 18 **(c)(i)** $t^2 + 2t$ **(ii)** 675   **J13/21/Q3**

---

9   Part of a pattern of numbers is shown in the table below.

1	2	3	4	5		n
5	8	11	14	p		x
4	9	16	25	q		y
1	8	27	64	r		z
10	25	54	103	s		t

(a) Study the patterns and write down the value of p, the value of q, the value of r and the value of s.   [4]

(b) Find expressions, in terms of n, for each of x, y, z and t.   [4]

Answers: **(a)** 17, 36, 125, 178; **(b)** $3n + 2$, $(n + 1)^2$, n^3, $n^3 + n^2 + 5n + 3$.   **N01/2/Q2**

10

Three integers, a, b and c, are such that $a < b < c$.
The three integers are said to form a Pythagorean Triple if $c^2 = a^2 + b^2$ or $c^2 - b^2 = a^2$.

For example
    3, 4, 5 form a Pythagorean Triple because $5^2 - 4^2 = (5 - 4)(5 + 4) = 1 \times 9 = 9 = 3^2$
and 5, 12, 13 form a Pythagorean Triple because $13^2 - 12^2 = (13 - 12)(13 + 12) = 1 \times 25 = 25 = 5^2$.

(a)   In the same way, show that 7, 24 and 25 form a Pythagorean Triple.   [1]

(b)   Form a Pythagorean Triple

    (i)   in which the last two integers are 40 and 41,   [1]

    (ii)   in which the first integer is 11.   [1]

(c)   (i)   Simplify $(n + 1)^2 - n^2$.   [1]

    (ii)   Hence form a Pythagorean Triple in which the first integer is 101.   [2]

(d)   It is also possible to form Pythagorean Triples in which the last two integers differ by 2.

    For example
    8, 15, 17 form a Pythagorean Triple because $17^2 - 15^2 = (17 - 15)(17 + 15) = 2 \times 32 = 64 = 8^2$.

    (i)   Copy and complete the following statements:

        … , 35, 37 form a Pythagorean Triple because $37^2 - 35^2 = (\quad)(\quad) = \ldots \times \ldots = \ldots = \ldots$.
                                                                                          [1]

        16, … , … form a Pythagorean Triple because $\ldots - \ldots = (\quad)(\quad) = 2 \times \ldots = \ldots = 16^2$. [2]

    (ii)   Simplify $(4n^2 + 1)^2 - (4n^2 - 1)^2$ and hence express it as a perfect square. [2]

    (iii) Form a Pythagorean Triple in which the first integer is 400 and the other two integers differ by 2.   [1]

Answers: **(b)(i)** 9, **(ii)** 60, 61; **(c)(i)** $2n + 1$, **(ii)** 5100, 5101; **(d)(i)** 12, 63, 65, **(ii)** $(4n)^2$, **(iii)** 39 999, 40 001.   **N04/2/Q9**

11 The terms T_1, T_2, T_3, T_4, T_5 of a sequence are given as follows:

$$T_1 = 1 = 1$$
$$T_2 = 3 = 1 + 2$$
$$T_3 = 6 = 1 + 2 + 3$$
$$T_4 = 10 = 1 + 2 + 3 + 4$$
$$T_5 = 15 = 1 + 2 + 3 + 4 + 5$$

(a) **(i)** Write down the next two terms, T_6 and T_7, in the sequence

$$1, 3, 6, 10, 15, \ldots\ldots$$ [1]

  (ii) The nth term in the sequence is given by $T_n = \dfrac{1}{2}n(n+1)$.

  Show that this formula is true when $n = 7$. [1]

  (iii) Use the formula to find T_{100}. [1]

  (iv) Use your answer to part **(iii)** to find $5 + 10 + 15 + \ldots\ldots + 500$. [1]

  (v) Hence find the sum of all the whole numbers from 1 to 500 which are **not** multiples of 5. [2]

(b) The terms S_1, S_2, S_3, S_4, S_5 of a different sequence are given as follows:

$$S_1 = 1 = 1 \times 1$$
$$S_2 = 4 = 1 \times 2 + 2 \times 1$$
$$S_3 = 10 = 1 \times 3 + 2 \times 2 + 3 \times 1$$
$$S_4 = 20 = 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1$$
$$S_5 = 35 = 1 \times 5 + 2 \times 4 + 3 \times 3 + 4 \times 2 + 5 \times 1$$

  (i) Find S_6 and S_7. [2]

  (ii) The nth term in this sequence is given by $S_n = \dfrac{1}{6}n(n+1)(n+2)$.

  Show that this formula is true when $n = 7$. [1]

  (iii) Find $1 \times 20 + 2 \times 19 + 3 \times 18 + \ldots\ldots + 20 \times 1$. [1]

(c) $S_2 - S_1 = (1 \times 2 + 2 \times 1) - 1 \times 1 = 1 \times 2 + 1 \times 1 = 2 + 1 = T_2$
  $S_3 - S_2 = (1 \times 3 + 2 \times 2 + 3 \times 1) - (1 \times 2 + 2 \times 1) = 3 + 2 + 1 = T_3$

  Show that

  (i) $S_4 - S_3 = T_4$, [1]

  (ii) $S_{n+1} - S_n = T_{n+1}$. [1]

Answer: **(a)(i)** 21, 28   **(iii)** 5050   **(iv)** 25 250   **(v)** 100 000   **N06/2/Q8**

(b)(i) 56, 84   **(iii)** 1540

109

12   **(a)**   Each diagram in the sequence below consists of a number of dots.

Diagram number	1	2	3	4
Number of dots	T_1	T_2	T_3	T_4
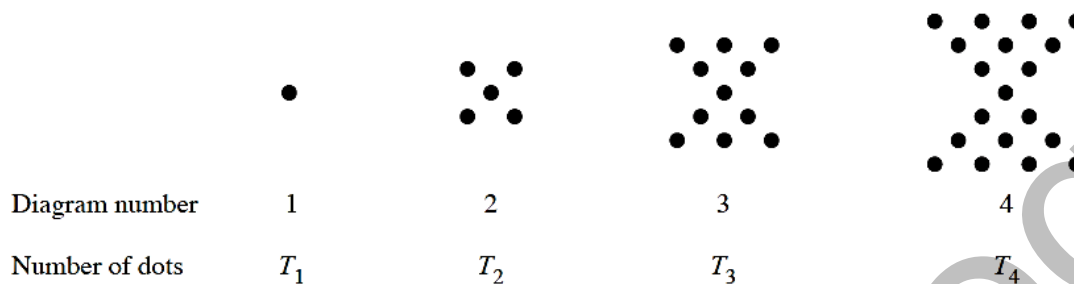
Diagram number (n)	1	2	3	4	5
Number of dots (T_n)	1	5	11	p	q

     (i)   Write down the value of p.   [1]

     (ii)   Find the value of q.   [1]

(b)   Another sequence of patterns of dots is shown below.

Diagram number	1	2	3	4
Number of dots	S_1	S_2	S_3	S_4
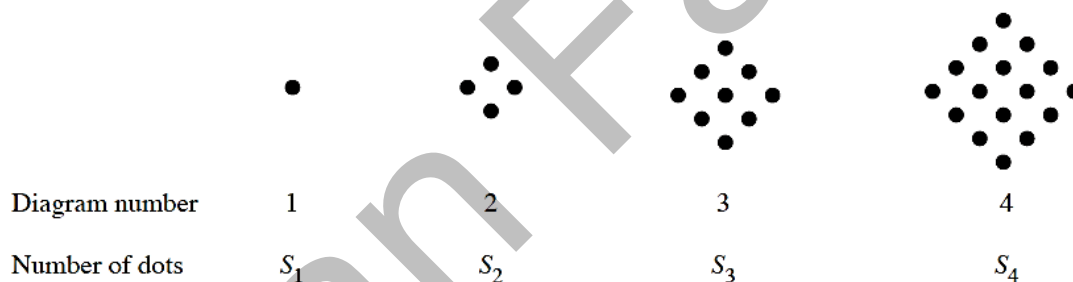
Diagram number (n)	1	2	3	4	5
Number of dots (S_n)	1	4	9	j	k

     (i)   Write down the value of j.   [1]

     (ii)   Find the value of k.   [1]

     (iii)   Express S_n in terms of n.   [1]

(c)   Another sequence is formed whose nth term U_n is $T_n - S_n$.

$$U_1 = T_1 - S_1 = 1 - 1 = 0, \qquad U_2 = T_2 - S_2 = 5 - 4 = 1, \qquad U_3 = T_3 - S_3 = 11 - 9 = 2, \qquad \ldots\ldots\ldots$$

     (i)   Evaluate U_4 and U_5.   [1]

     (ii)   Express U_n in terms of n.   [1]

     (iii)   Hence, using your expression for S_n found in **(b)(iii)**, express T_n in terms of n.   [1]

Answers: **(a)(i)** 19,   **(ii)** 29,   **(b)(i)** 16,   **(ii)** 25,   **(iii)** n^2,   **(c)(i)** 3,4,   **(ii)** $n-1$,   **N08/2/Q6**
(iii) $n^2 + n - 1$.

Volume 2

Time Paper 1

1 Local time in Singapore is 11 hours ahead of Trinidad.

(a) Look at the two clocks in the answer space.
One shows the local time in Singapore.
Show the local time in Trinidad on the other clock.

Answer (a)
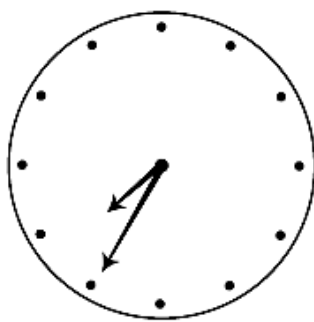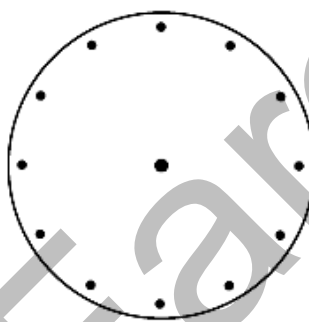
Singapore   Trinidad   [1]

(b) It is 9.15 a.m. in Trinidad.
Using the 24 hour clock, write down the local time in Singapore.

Answer (b).. [1]

J03/1/Q4

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2 (a) A TV programme list shows that a film begins at 21 55.
The film lasts for 100 minutes.
At what time will it end?
Express your answer using the 24 hour clock.

(b) The times taken by an athlete to run three races were 3 minutes 59.1 seconds,
4 minutes 3.8 seconds and 4 minutes 1.6 seconds.
Calculate the mean time.

Answer (a) .... [1]

(b) .... minutes .... seconds [2]

J05/1/Q12

3 (a) The time difference between Brunei and London is 7 hours.
So, when it is 1900 in Brunei, it is 1200 in London.
When it is 0330 in Brunei, what time is it in London?

(b) An aircraft leaves Brunei at 6 30 p.m. local time.
It arrives in Dubai at 10 p.m. local time.
The flight took $7\frac{1}{2}$ hours.
Calculate the time difference between Dubai and Brunei.

Answer (a) ....[1]

(b) ....hours [1]

Answer: (a) 20 30   (b)   4 hours

---

4) Five clocks at a hotel reception desk show the local times in five different cities at the same moment.

LONDON	MOSCOW	SYDNEY	TOKYO	NEW YORK
0738	1038	1638	1538	0238

(a) Rosidah has breakfast at 08 00 in Moscow.
What is the local time in Sydney?   *Answer* (a) ....[1]

(b) Elias catches a plane in London and flies to New York.
He leaves London at 1130 local time.   *Answer* (b) .... [2]
The flight time is 8 hours 10 minutes.
What is the local time in New York when he lands?

Answer: (a)   14 00   (b)   14 40

---

5 The times taken for a bus to travel between five stops *A, B, C, D* and *E* are shown below.

A_____B_____C_____D_____E
     4 minutes $1\frac{1}{2}$ minutes   75 seconds   2 minutes 35 seconds

Expressing each answer in minutes and seconds, find

(a) the total time for the journey from *A* to *E*,

Answer (a) ....minutes...................... seconds [1]

(b) the mean time taken between the stops,

Answer (b) ....minutes...................... seconds [2]

(c) the range of times taken between the stops.

Answer (c) ....minutes...................... seconds [1]

Answer: (a) 9   20   (b) 2   20   (c) 2   45

---

6 The table shows the record minimum monthly temperatures, in °C, in Vostok and London.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Vostok	−36	−47	−64	−70	−71	−71	−74	−75	−72	−61	−45	−35
London	−10	−9	−8	−2	−1	5	7	6	3	−4	−5	−7

Find

(a) the difference between the temperatures in Vostok and London in July,

Answer *(a)* .... °C [1]

(b) the difference between the temperatures in Vostok in February and June.

Answer *(b)* .... °C [1]

Answer: **(a)** 81   **(b)** 24   **J10/12/Q8**

7 **(a)** One morning the temperature was 5 °C.
By the evening the temperature had dropped 9 °C.

Write down the temperature in the evening.   *Answer* .... °C
[1]

(b) The times of some buses from Aytown to Deetown are shown.

Aytown	07 04	08 04	08 56	09 00	09 32	10 56
Beetown	-	-	09 05	-	09 41	11 05
Ceetown	07 18	08 18	09 14	-	-	11 14
Deetown	07 35	08 35	09 31	09 28	10 05	11 31

(i) Maryam lives in Ceetown and has to be in Deetown by 09 30.

What time is the latest bus from Ceetown that she can catch?

Answer .... [1]

(ii) Aadil catches the 09 32 from Aytown to Deetown.

How long does his journey take?   *Answer* .... minutes [1]

Answers: **(a)** −4 **(b)(i)** 0818 **(ii)** 33   **J14/11/Q10**

8   On four occasions a bus takes the following times to complete a journey.

$1\frac{2}{3}$ hours,   1.7 hours,   1 hour 39 minutes, $1\frac{3}{4}$ hours.

By first expressing these times in minutes, write them in order of size, starting with the smallest.

Answer .... minutes < .... minutes < .... minutes < .... minutes [2]

N01/Q3

9   (a)   The local time in Singapore is 7 hours ahead of the local time in London.
          A flight to London leaves Singapore at 03 00 local time.
          The flight takes 12 hours and 45 minutes.
          What is the local time in London when it arrives?

Answer (a) ....[1]

(b)   Mai changes £250 into dollars.
          The exchange rate is £1 = $3.10.
          How many dollars does she receive?   *Answer* (b) $ ....[1]

N09/1/Q5

Percentages Paper 1

1 In 2000 Esther went to a tennis tournament.
Her ticket cost Rs700.
At the tournament she bought a programme costing Rs60 and an ice cream costing Rs40.

   (a) This information is to be shown on a pie chart.
   Calculate the angle of the sector which represents the amount she spent on ice cream. [2]

   (b) In 2001 the cost of a ticket was Rs735.
   Calculate the percentage increase in the cost of a ticket. (b) ....% [2]

2 (a) Express 0.03 as a fraction.   Answer (a)... [1]

   (b) Express $150 as a percentage of $500.   (b)... % [1]

Answers: (a) $\frac{3}{100}$; (b) 30%.

3 (a) Express $\frac{7}{100}$ as a decimal.   Answer (a) .... [1]

   (b) Express 0.08 as a percentage.   (b) .... % [1]

Answers: (a) 0.07; (b) 8 %.

4 (a)

SALE
$400 NOW $280

Calculate the percentage reduction in the price of the camera.

Answer (a) .... % [2]

(b) Matthew invested $500 at 6% simple interest per year.
Calculate how much interest had been earned after 8 months.

Answer (b) $ .... [2]

5

(a) Express $\dfrac{13}{20}$ as a decimal.   Answer (a) .... [1]

(b) In a test, Rose scored 56 marks out of 70.   (b) ....% [1]
Express this score as a percentage.

6

(a) Express 0.527 as a percentage.   Answer (a) .... % [1]

(b) Evaluate 5.6 ÷ 0.08.   (b) ....[1]

7

A dealer sold a painting for $800.
She made a profit of 25% on the price she paid for it.
Calculate the price she paid for the painting.   [2]

8

(a) Calculate 5% of $280000.   Answer (a) $..[1]

(b) A single carton of juice costs $4.20.
A special offer pack of 3 cartons costs $9.45.
Ali bought a special offer pack instead of 3 single cartons.

Calculate his percentage saving.   (b) .... % [2]

9

(a) A jar contained 370 g of jam.
Usman ate 30% of the jam.

What mass of jam remained in the jar?

(b) In 2006 the population of a town was 30 000.
This was 5000 more than the population in 1999.

Calculate the percentage increase in population.

Answer (a) .... g [1]

(b) .... % [2]

Answers: (a) 259 g   (b) 20 %

10 **1** **(a)** Evaluate $17 - 5 \times 3 + 1$.   *Answer (a)* ....[1]

(b) Express 0.82 as a percentage.   *Answer (b)* .... % [1]

Answer: (a) 3   (b) 82

11 **(a)** Evaluate $10 - 8 \div 2 + 3$.   *Answer (a)* ....[1]

(b) Find 20% of 60 cm.   *Answer (b)* .... cm [1]

Answer: (a) 9   (b) 12

12 **(a)** In a town, 11 000 people out of the total population of 50 000 are aged under 18.

What percentage of the population is aged under 18?

Answer (a) .... % [1]

(b) A company employing 1200 workers increased the number of workers by 15%.

How many workers does it now employ?   *Answer (b)* .... [1]

Answer: (a) 22   (b) 1380

13 **(a)** Express 18% as a fraction in its lowest terms.   *Answer* .... [1]

) Write down an irrational number between 3 and 4.   *Answer* .... [1]

Answer: (a) $\frac{9}{50}$ (b) π or √10 ...etc.

14 **(a)** It is given that $\frac{1}{5} < n < \frac{1}{4}$.

Write down a decimal value of n that satisfies this inequality. *Answer* .... [1]

(b) Express $\frac{48}{60}$ as a percentage.   *Answer* .... % [1]

Answer: (a) Any decimal value of n such that $0.2 < n < 0.25$ (b) 80

15 **(a)** Evaluate $8 - 5 \times 4 + 3$.   *Answer* ....[1]

(b) Express 1.03 as a percentage of 1.   *Answer* ....% [1]

118

16

(a) The profits of a company were $5 million in 2009 and $8 million in 2010.

Find the percentage increase in profits from 2009 to 2010.

Answer ....% [1]

(b) Another company had an income of $20 million in 2008.
In 2009 this income decreased by 10%.
In 2010 the income increased by 15% from the 2009 income.

Find the income in 2010.   Answer $ .... million [2]

Answer: (a) 60 (b) 20.7   **J12/11/Q16**

17

(a) Complete the statement in the answer space using one of these symbols.

$$\leqslant \qquad < \qquad = \qquad > \qquad \geqslant$$

Answer   0.65 .... $\frac{27}{40}$ [1]

(b) Express 7% as a decimal.   Answer ....[1]

Answer: (a) < (b) 0.07   **J13/11/Q4**

18

Using the information given in the advertisement shown, find the **sale price** of the table.

swer $ .... [2]

SALE
All prices reduced by 30%

Save $180 on this table

Answer: $420.   **N01/Q5**

19 The population of a country was 13.1 million.

(a) Nine hundred thousand of the population were at least 2 metres tall. How many were less than 2 metres tall?

(b) 0.2% of the population were over 95 years old. How many were over 95?

Answer (a)... [1]

(b)... [2]

Answers: (a) 12.2 million; (b) 26 200.

20 (a) Express $\frac{17}{40}$ as a percentage.

(b) Find the decimal number which is exactly halfway between 4.7 and 5.0.

21 Express $7\frac{1}{2}\%$

(a) as a decimal, Answer (a).. [1]

(b) as a fraction in its simplest form. (b) .... [1]

Answers: (a) 0.075; (b) $\frac{3}{40}$.

22 In an examination, Alan obtained 32 out of 40 marks. In another examination Ben obtained $\frac{5}{8}$ of the total marks.

Express the mark of each candidate as a percentage.

Answer Alan ....% [1]

Ben ....% [1]

Answer: (a) 80%; (b) $62\frac{1}{2}\%$

23

(a) Write the following numbers in order of size, starting with the **smallest**.

$$0.7, \quad 0.7^2, \quad \frac{7}{11}, \quad \frac{7}{9}.$$

Answer (a) .... , ... , ... , ... [1]

(b) In a school election, John received 220 votes.
This was 55% of the total number of votes.
Find the total number of votes.

Answer (b) ....[1]

Answer: (a) 0.7^2, $\frac{7}{11}$, 0.7, $\frac{7}{9}$; (b) 400.

24

(a) Express $22\frac{1}{2}$ % as a fraction in its lowest terms.

(b) Evaluate 0.9×0.02.

Answer (a) .... [1]

(b) ....[1]

Answer: (a) $\frac{9}{40}$ (b) 0.018

25

(a) Express 0.45 as a fraction, giving your answer in its lowest terms.

(b) Express $\frac{13}{40}$ as a percentage.

Answer (a) ....[1]

(b) .... % [1]

26

(a) Express 60% as a fraction, giving your answer in its lowest terms.

Answer (a) ....[1]

(b) The mass of a jar and its contents is 1.6 kg.
The contents have a mass of 875 grams.

Calculate the mass, in grams, of the jar.

Answer (b) .... g [1]

Answers: (a) 3/5 (b) 725

---

27 In a sale, a shopkeeper reduced the marked price of his goods by 20%.

(a) The marked price of a book was $20.

Calculate its price in the sale.

Answer (a) $ ....[1]

(b) The price of a camera in the sale was $60.

Calculate its marked price.

Answer (b) $ ....[2]

28   (a)   Write the following numbers in order of size, starting with the smallest.

$$0.67 \qquad \frac{7}{9} \qquad \frac{2}{3} \qquad 66\%$$

Answer .... , ... , ... , ... [1]

smallest

(b)   During one month, the volume of perfume in a bottle decreased from 5 ml to 4 ml.

Calculate the percentage decrease.   *Answer* .... % [1]

Answers: (a) 66%, $\frac{2}{3}$, 0.67, $\frac{7}{9}$ (b) 20   **N11/11/Q3**

29   The length of a rectangle is 8 cm.
It is increased by 150%.

Calculate the new length.   *Answer* .... cm [2]

Answer. 20   **N11/12/Q12**

30   (a)   A television priced at $500 is sold for $400.

Find the percentage discount.   *Answer* .... % [1]

(b)   Tax on the original price of a radio is charged at 20% of the original price.
After tax was included, a customer paid $60 for the radio.

Calculate the tax charged.   *Answer* $ .... [2]

Answers: (a) 20 (b) 10   **N13/11/Q9**

Percentages Paper 2

1   (d) Michelle bought a watch for $135.
         This price included a tax of 8%.
         Calculate the **tax** that was paid.   [3]

2   In 2001 the price of one litre of petrol was 72 cents.

   (a) 65% of this price is 'tax' and the remainder is 'other costs'.

      (i) Find, in its simplest form, the ratio of tax to other costs.
         Give your answer in the form $m : n$, where m and n are integers.   [1]

      (ii) Calculate how much tax is paid on one litre of petrol.   [1]

   (b) Maureen bought as many complete litres of petrol as she could with a $20 note ($1 = 100 cents).

      (i) Calculate how many litres she bought.   [1]

      (ii) Calculate how much change she received.   [1]

   (c) In 2002 the price of one litre of petrol was 81 cents.

      Calculate the percentage increase in the price of petrol from 2001 to 2002.   [2]

   (d) The price of petrol in 2001 was 10% less than the price in 2000.

      Calculate the price of one litre of petrol in 2000.   [3]

   (e) Andrew's car will travel 480 km on a full tank of petrol.
      He starts a journey of 620 km with a tank which is **half** full.
      He wants to stop only once for petrol.

      Between what distances from the start of his journey must he stop for petrol?   [2]

Answers: (a)(i) 13 : 7, (ii) 46.8c; (b)(i) 27, (ii) 56c; (c) 12.5; (d) 80c; (e) between 140 and 240 km.

3   (b) During a sale, a shop sold packets of tea for 20% less than the price shown on their labels.
      Elizabeth and Peter each bought a packet of tea in the sale.

      (i) Elizabeth's packet had a label price of $4.50.

         How much did she pay?   [1]

      (ii) Peter paid $6.20 for his packet.

         Calculate the price shown on its label.   [2]

(b)(i) $3.60, (ii) $7.75.

4 Wasim owns a shop.
The table shows the cost price and selling price of three items in his shop.

Item	Cost Price ($)	Selling Price ($)
Trampoline	48	66
Swing	x	19.50
Bicycle	82	110

(a) Calculate his percentage profit when he sells a trampoline. [2]

(b) Wasim makes a profit of 30% when he sells a swing.

Calculate the cost price, x, of a swing. [3]

(c) In a sale, the selling price of a bicycle is reduced by 20%.

   (i) Find the sale price of a bicycle. [1]

   (ii)

   ### THURSDAY SPECIAL

   Save 10% off the **sale price**.

   Jaspreet bought a bicycle on a Thursday.

   Calculate the difference between the amount Jaspreet paid and the cost price. [2]

Answers: (a) 37.5%   (b) $15   (c)(i) $88   (ii) $2.80   **J10/21/Q2**

---

5 (b) Greg buys 60 garden plants at a cost price of $2.00 each to sell in his shop.
He sells 25 of them at a profit of 75% and 18 of them at a profit of 35%.
He sells the rest of the plants for $\frac{4}{5}$ of the cost price.

   (i) Calculate the profit or loss he makes from selling these 60 plants, stating if it is a profit or loss.

   Answer   Greg makes a .... of $..................... [3]

   (ii) Find the percentage profit or loss.   *Answer*   .... % [1]

(b)(i) Profit, $43.30 (ii) 36 to 36.1%   **J13/21/Q5b**

---

6 (a) Zara owns a hairdressing salon.
She buys a pack of 60 bottles of shampoo from a warehouse for $240.
She plans to sell the bottles of shampoo to her customers for $5.50 each.

   (i) Calculate the percentage profit Zara makes on each bottle she sells for $5.50 .

   Answer   .... [2]

   (ii) Zara sells 45 bottles at the full price then sells the rest with a 20% discount.

Calculate the total profit she makes selling all 60 bottles.

Answer $ .... [2]

(iii) When the warehouse sells a pack of shampoo for $240 it makes a profit of 15%.

Calculate the price paid for the pack of shampoo by the warehouse.

Answer $ .... [2]

(iv) Zara borrows $2500 from a bank to make improvements to her salon.
She is charged 4.5% per year simple interest.
She pays the money back after 3 years.

Calculate the **total** amount Zara must pay to the bank.

Answer $ .... [2]

Answers: (a)(i) 37.5 (ii) 73.50 (iii) 208.70 (iv) 2837.50   **J14/21/Q3**

7 **(a)** A furniture salesman earned $36 200 last year.

(i) He had to pay 22% of this amount as tax.

How much was left after paying tax?   *Answer* $ .... [2]

(ii) His earnings of $36 200 were made up of $25 000 basic salary plus 8% of the value of the furniture that he sold.

Calculate the value of the furniture that he sold.   *Answer* $ .... [3]

(iii) He bought a bookcase from the shop where he worked.
Its marked price was $1080 but because he worked there, he only paid $756.

Calculate the percentage discount on the marked price that he had been given.

Answer .... % [2]

(b) George opened an account and invested a sum of money at 4.5% simple interest per year for 3 years. At the end of the 3 years he closed the account, withdrawing a total of $681.

Calculate the amount that George invested.

Answer $ .... [3]

J15/21/Q1

8 **(a)** In 2005, the cost of posting a letter was 28 cents.
A company posted 1200 letters and was given 4% discount on the cost.

Calculate the total discount.   [1]

(b) In 2006, the cost of posting a letter was increased from 28 cents to 35 cents.

Calculate the percentage increase in the cost of posting a letter.   [2]

(c) After the price increase to 35 cents, the cost to the company of posting 1200 letters was $399.

Calculate the percentage increase in the cost of posting a letter. [2]

(c) After the price increase to 35 cents, the cost to the company of posting 1200 letters was $399.

Calculate the percentage discount that the company was given in 2006. [2]

(d) In 2006, it cost $4.60 to post a parcel.
This was an increase of 15% on the cost of posting the parcel in 2005.

Calculate the cost of posting this parcel in 2005. [3]

Answers: **(a)** $13.44,   **(b)** 25%,   **(c)** 5%,   **(d)** $4.   **N07/2/Q4**

---

9   **(a)**   In 2006 the cost of fuel was 91.8 cents per litre.

   (i)   Calculate the maximum number of whole litres that could be bought for $15. [1]

   (ii)   In 2007 the cost of fuel was increased by 4 cents per litre.

   (a)   Calculate the percentage increase in the cost of fuel in 2007. [2]

   (b)   On average, a car travelled 21 km on 1 litre of fuel.
During 2006 this car travelled 19 200 km.
In 2007 the car travelled the same distance.

Calculate the extra cost for fuel in 2007.
Give your answer to the nearest dollar. [2]

   (iii)   In 2006 the cost of fuel was 10% less than the cost in 2005.

Calculate the cost, in cents, per litre in 2005. [2]

(b)   A service station sells unleaded, super-unleaded and diesel fuel.
The pie chart represents the amounts of these fuels sold during one week.
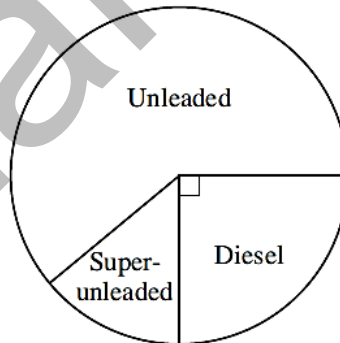
The total amount of fuel sold during this week was 54 000 litres.

   (i)   How many litres of diesel were sold? [1]

   (ii)   The amount of unleaded fuel sold was $\frac{2}{3}$ of the total for the week.

How many litres of super-unleaded fuel were sold? [2]

Answers: **(a)(i)** 16 litres,   **(ii)(a)** 4.36%,   **(b)** $ 37,   **(iii)** 102 cents,   **(b)(i)** 13 500 litres, **N08/2/Q1**
(ii) 4 500 litres.

10 Archie bought a car from a dealer.
The dealer sold the car for $7040.
Archie paid a deposit of $1760.

(a) Calculate the deposit as a percentage of the dealer's selling price. [1]

(b) Archie took out a loan agreement to repay the outstanding balance of $5280.
He paid $212.67 per month for 36 months.

Calculate

(i) the amount of interest that he paid, [2]

(ii) the rate of simple interest per year that had been charged. [3]

(c) When the dealer sold the car, he made a profit of 30%.

Calculate the profit, correct to the nearest dollar, that the dealer made on the sale of this car. [3]

Answers: (a) 25 (b)(i) 2376.12 (ii) 15 (c) 1625   **N10/21/Q5**

---

11 Matthew makes pieces of furniture and sends them to a shop where they are sold.
When a piece is sold, the shopkeeper receives 15% of the selling price, and Matthew receives
the rest.

(a) A table is sold for $200.

(i) Calculate the amount the shopkeeper receives.

Answer $ .... [1]

(ii) The cost of making this table was $131.80.

Calculate the percentage profit that Matthew makes when this table is sold.

Answer   .... % [3]

(b) Matthew made a bookcase.
The cost of making the bookcase was $647.50.
After the bookcase is sold and the shopkeeper has received 15% of the selling price,
Matthew makes a profit of $160.

Calculate the selling price of the bookcase.   *Answer* $ ....[3]

Answer: (a)(i) 30 (ii) 29.0 (b) 950   **N12/21/Q3**

Currency, Ratios and Proportions Paper 1

1 It is given that $x = -3.5$, $y = 1.5$ and $z = 4.5$.

(a) Find the value of $x - z$.

(b) Given also that $(y + z) : t = 4 : 15$, find the value of t.

Answer (a) $x - z =$ .... [1]

(b) $t =$ .... [2]

Answers: **(a)** -8; **(b)** 22.5.   J04/11/Q12

2 (a) When Peter went to Hong Kong, he changed £50 into $616.
Calculate what one British pound (£) was worth in Hong Kong dollars ($).

(b) It takes 8 hours for 5 people to paint a room.
How long would it take 4 people?

Answer (a) £1 = $..[1]

(b) ....h [1]

Answers: **(a)** $12.32; **(b)** 10 h.   J05/1/Q5

3 The rate of exchange between pounds (£) and dollars ($) was £1 = $2.80.

Calculate

(a) the number of dollars received in exchange for £120,

(b) the number of pounds received in exchange for $224.

Answer (a) $..[1]

(b) £..[1]

                                                      J05/1/Q3

4 (a) The ratio of Sayed's age to his mother's age is $2 : 7$.
Sayed is 14 years old.

How old is his mother?   *Answer* *(a)* ....years [1]

(b) The ratio of Fatima's age to her father's age is $3 : 8$.
The total of their ages is 66 years.

How old is Fatima?   *Answer* *(b)* ....years [1]

Answer: **(a)** 49   **(b)** 18   J10/11/Q8

5 Ahmed goes shopping.

  (a) In one shop he buys shorts for $26.84 and a shirt for $13.97.

    How much does Ahmed spend altogether?   *Answer*   *(a)* $................................... [1]

  (b) In another shop he buys 15 postcards for 46 cents each.

    (i) Calculate the total cost, in dollars, of the postcards.

              Answer   *(b)(i)* $............................... [1]

    (ii) The rate of exchange between pounds (£) and dollars ($) was £1 = $2.50.

        Calculate the total cost of the postcards in pounds.

              Answer   *(b)(ii)* £............................ [2]

Answer: (a) 40.81   (b)(i) 6.90   (b)(ii) 2.76   **J10/11/Q15**

---

6   **(a)** Jane and Ken share some money in the ratio 5 : 3.
       Ken's share is $16 less than Jane's share.

      Find each person's share.   *Answer*   *(a)* Jane $ ....

  (b) The scale of a map is 1 : 25 000.
       The distance between two villages is 10 cm on the map.

      Find the actual distance, in kilometres, between the villages.   Ken $ .... [2]

              Answer   *(b)* .... km [1]

Answer: (a) 40   24   (b) 2.5   **J10/12/Q10**

Answers: (a) 63   (b) 60   **J11/11/Q8**

7

  (a) A bag contains red and blue pens in the ratio 2 : 7.
There are 18 red pens in the bag.

     How many blue pens are in the bag?   *Answer* ....[1]

  (b) A box contains apples and oranges.   *Answer* ....% [1]
The ratio of apples to oranges is 2 : 3.

     What percentage of the fruit are oranges?

  (a) The ratio of boys to girls in a class is 4 : 5 .

     What fraction of the class are boys?   *Answer* ....[1]

  (b) The ratio of boys to girls in a school is 3 : 4 .
There are 120 more girls than boys.

                                    Answer ....[1]
     How many students are in the school?

Answer. (a) $\frac{4}{9}$ (b) 840   **J11/12/Q7**

8

  (a) Jane puts some red balloons and some blue balloons into a bag.
The ratio of red balloons to blue balloons is 3 : 4.
There are 84 balloons in the bag.

     How many blue balloons are in the bag?   *Answer* ....[1]

  (b) At a party the ratio of boys to girls is 5 : 4.
There are 40 boys at the party.

     Find the total number of children at the party. *Answer* ....[1]

Answer. (a) 48 (b) 72   **J12/11/Q7**

---

9 On a map the length of a lake is 4.5 centimetres.
The actual length of the lake is 2.7 kilometres.

  Write the scale of the map as a ratio in the form 1 : *n*.   *Answer* 1 : ....[2]

Answer. 1 : 60 000   **J13/11/Q7**

---

10 (a) The angles of a triangle are in the ratio 3 : 4 : 5.

     Calculate the smallest angle in the triangle. *Answer* ....[1]

  (b) The ratio of boys to girls in a class is 4 : 5.
There are 3 more girls than boys.

     Calculate the total number of students in the class. *Answer* ....[1]

Answers: (a) 45° (b) 27   **J14/11/Q6**

11 A car manufacturer states that a particular car

- uses 5 litres of fuel in travelling 100 km,

- produces 110 grams of CO_2 for each kilometre travelled.

Use this information to calculate the mass of CO_2 produced by 1 litre of fuel.
Give your answer in kilograms.

Answer .... kg [2]

Answer: 2.2   **J15/11/Q6**

---

12 A map is drawn to a scale of 2 cm to 5 km.

(a) Two towers are 9 km apart.

Calculate the distance between them on the map.   *Answer* .... cm [1]

(b) On the map, a town covers an area of 4cm^2.   *Answer* .... km^2 [1]

Calculate its actual area.

(c) Express the scale of the map in the form 1 : n.   *Answer*   1 : .... [1]

Answers: **(a)** 3.6 **(b)** 25 **(c)** 1: 250 000   **J16/11/Q12**

---

13 (a) 6 square metres of carpet cost $258.

Work out the cost of 10 square metres.   *Answer* $ .... [1]

(b)

$$\boxed{1 \text{ dirham} = \$0.30}$$

Amin changes $90 into dirhams.

Calculate the number of dirhams that Amin receives.   *Answer*   .... [1]

(c) Sabah is filling a tank with water.
It takes 20 minutes to fill the empty tank when the water flows at a rate of 2.4 litres/minute.

Calculate the time it will take to fill the empty tank if the water flows at a rate of 4 litres/minute.

Answer   ....minutes [2]

Answers: **(a)** 430 **(b)** 300 **(c)** 12   **J17/11/Q19**

---

14 (a) Evaluate $15.05 \div 0.5$.

(b) The rate of exchange between Swiss francs (F) and British pounds (£) was 2.4F = £1.
Calculate the number of pounds received in exchange for 60F.

Answers: **(a)** 30.1; **(b)** £25.

---

15   **(a)** A sum of money is divided in the ratio 4 : 3.
The larger part is $7.20.
Find the smaller part.

      (b) Express the ratio 700 g to 1.75 kg in its lowest terms.
Give your answer in the form $m : n$, where m and n are integers.

Answers: **(a)** $5.40; **(b)** 2:5.

16   **(a)** Three numbers are given in the answer space.
Write L against the largest, M against the next largest and S against the smallest.

Answer (a) 0.7 million   ....

                687 000   ....

          eight hundred and four thousand   .... [1]

      (b) An amount of money is divided into two parts in the ratio 1: 4.
Find the smaller part as a percentage of the whole amount.

Answer (b) ....% [1]

Answers: *M, S, L;* **(b)** 20%.

17   **(a)** The number 3002.05 can be written as $3 \times 10^3 + 2 \times 10^x + 5 \times 10^y$.
Given that x and y are integers, find the values of x and y.

      (b) A bank exchanged Japanese yen and Singapore dollars ($) at a rate of 66 yen = $1.

        (i)   Calculate, in yen, the amount received for $200.

       (ii)   Calculate, in dollars, the amount received for 33 000 yen.

Answer (a) x = ... [1]

              y = ...[1]

          (b)(i) ....yen [1]

           (ii) $ .... [1]

Answers: **(a)** x = 0, y = –2; **(b)** 13 200 yen, $500.

18   A wooden plank is cut into three pieces in the ratio 2 : 5 : 1.
The length of the longest piece is 125 cm.

Find   *Answer (a)* .... cm [1]

(a)   the length, in centimetres, of the shortest piece,

(b)   the total length, in metres, of the plank.   *(b)* .... m [1]

Answers: **(a)** 25 cm,   **(b)** 2 m.   N08/1/Q6

---

19   (a)   The mass of a container and its contents is 2.4 kg.
The mass of the contents is 750 g.

Calculate the mass, in kilograms, of the container. *Answer* *(a)* ....kg [1]

(b)   Express the ratio 24 cm to 3 m in its lowest terms.
Give your answer in the form $p : q$, where p and q are integers.

Answer *(b)* .... : ....[1]

Answer. **(a)** 1.65 **(b)** 2 : 25   N10/12/Q5

---

20   A map has a scale of 2 cm to 5 km.

(a)   Express this scale in the form 1 : n.   *Answer*   1 : .... [1]

(b)   The actual distance between two places is 35 km.

Calculate the distance on the map between these two places.

Answer   .... cm [1]

(c)   On the map, the area of a lake is 8 cm².

Calculate the actual area of the lake.   *Answer*   .... km² [1]

Answers: **(a)** 250 000 **(b)** 14 **(c)** 50   N11/11/Q13

---

21   (a)   Ali and Ben share $30 such that   Ali's share : Ben's share = 3 : 2.

Calculate Ali's share.   *Answer* $ .... [1]

(b)   Write the following times in order of size, starting with the smallest.

6 500 seconds   110 minutes $1\frac{3}{4}$ hours

Answer   ...., ..., ... [1]
                smallest

Answer. **(a)** 18 **(b)** $1\frac{3}{4}$ hours, 6 500 seconds, 110 minutes   N11/12/Q2

133

22 Exactly 9 litres of liquid filled 60 identical bottles.

(a) How many litres filled 40 of these bottles?   *Answer* .... [1]

(b) How many of these bottles are filled using 750 ml of liquid?

Answer .... [1]

Answer: (a) 6 (b) 5   **N11/12/Q3**

23 A model of a car is made to a scale of $\frac{1}{40}$.

(a) The height of the actual car is 1.5 m.

Find the height, in **centimetres**, of the model.

Answer .... cm [1]

(b) The luggage capacity of the model is 5 millilitres.

Find the luggage capacity, in **litres**, of the actual car.

Answer .... litres [2]

Answers: (a) 3.75 or 3¾ (b) 320   **N13/11/Q11**

24
(a) Express as a single fraction $\frac{2}{3} \div \frac{3}{4}$.   *Answer* .... [1]

(b) A bag of sweets contains mints and toffees only.
There are 21 mints in the bag.
One quarter of the sweets are toffees.

Calculate the total number of sweets in the bag.   *Answer* .... [1]

(c) $360 is shared in the ratio   3 : 5 .

Calculate the difference between the larger share and the smaller share.

Answer $ .... [1]

N14/11/Q6

Answers: (a) $\frac{8}{9}$; (b) 28; (c) 90.

25 The exchange rate between pounds (£) and dollars ($) is £1 = $1.60 .

(a) Amit changes £200 to dollars.

Calculate the number of dollars he receives.   *Answer* $...................................... [1]

(b) Ayesha changes $240 to pounds.

Calculate the number of pounds she receives.   *Answer* £...................................... [1]

134

Currency, Ratios and Proportions Paper 2

1 Denise, Elaine and Michelle went on holiday to America.

(a) The exchange rate between American dollars ($) and British pounds (£) was $1.60 = £1.
Denise changed £400 into dollars.
Calculate how many dollars she received. [1]

(b) To change the money, the bank charged a fixed amount of £3 plus 2% of the number of pounds changed.

(i) Calculate the amount the bank charged Denise to change her £400. [1]

(ii) The bank then charged Elaine £15 to change her money.
Calculate the number of pounds she changed. [2]

(c) While in America they spent a total of $450 on food.
The amounts spent by Denise, Elaine and Michelle were in the ratio 7 : 6 : 5.
Calculate how much Denise spent on food. [2]

J02/2/Q1

2 (a) Two varieties of tea, 'High Blend' and 'Normal Blend', are made by mixing Grade A leaves and Grade B leaves.

(i) In High Blend, the ratio of the masses of Grade A leaves to Grade B leaves is 3 : 2.

Find the mass of Grade A leaves used in making 250 g of High Blend. [1]

(ii) 1 kg of Normal Blend is made by using 450 g of Grade A leaves.

Find, **in its simplest form**, the ratio of the masses of Grade A to Grade B leaves in Normal Blend.
Give your answer in the form $m : n$, where m and n are integers. [2]

(iii) 250 g of High Blend is mixed with 1 kg of Normal Blend.

Calculate the percentage of the mass of this mixture that consists of Grade A leaves. [2]

Answers: **(a)(i)** 150 g, **(ii)** 9:11, **(iii)** 48%;

J05/2/Q2

3 (a) On average, Jim's heart beats 75 times per minute.

Calculate the number of times his heart beats during 50 weeks.
Give your answer in standard form. [2]

(b) After an exercise, Ali and Ben measured their heart rates.
The ratio of their heart rates was 15 : 17.
Ben's heart beat 18 times per minute more than Ali's.

Calculate Ali's heart rate. [2]

Answers: **(a)** 3.78×10^7;   **(b)** 135 ;

J08/2/Q3

4 **(a) (i)**

Exchange rate

$1 = €0.72

Eddie travels from the USA to Germany.
He changes $300 into euros (€).

How many euros does he receive?   *Answer* €... [1]

(ii) When Eddie returns to the USA he has €51 left that he exchanges for $75.

What exchange rate has been used in this case?

Answer $1 = € .... [1]

Answer: (a)(i) €216 (ii) €0.68   **J13/21/Q5a**

5 **(b)** The exchange rate between dollars ($) and pounds (£) is $1 = £0.64 .
The exchange rate between dollars ($) and euros (€) is $1 = €0.78 .

(i) Luke changes $250 into pounds.

Calculate how many pounds he receives.   *Answer* £ .... [1]

(ii) Complete the statement to show the exchange rate between pounds and euros.

Exchange rate

£1 = € ....

[2]

(b)(i) 160 (ii) 1.21875 to 1.22   **J14/21/Q3b**

6 **(a)** The rate of exchange between pounds (£) and dollars ($) is £1 = $1.87.
The rate of exchange between pounds (£) and euros (€) is £1 = € 1.21.

(i) Catherine changes £500 into dollars.

Calculate how many dollars she receives.   [1]

(ii) Esther changes € 726 into pounds.

Calculate how many pounds she receives. [1]

(iii) Rose changes $850 into euros.

Calculate how many euros she receives. [2]

(b) Matthew changes $770 into rupees.
He receives 40 000 rupees.

How many rupees did he receive for each dollar? [2]

(c) (i) Lily bought a car for $13 500.
She paid for it in 36 equal monthly payments.

Calculate the amount she paid each month. [1]

(ii) George bought a car for $27 000.
He borrowed the $27 000 at 15% per year simple interest for 3 years.
He repaid the total amount in 36 equal monthly payments.

Calculate the amount he paid each month. [3]

---

7 *Answers:* **(a)(i)** 935 **(ii)** 600 **(iii)** 550 **(b)** 51.95 **(c)(i)** $ 375 **(ii)** $ 1087.50 **N10/22/Q2**

(a) The rate of exchange between dollars ($) and pounds (£) is $1.56 = £1.
The rate of exchange between euros (€) and pounds is €1.10 = £1.

(i) Amy changes £300 into dollars.

Calculate how many dollars Amy receives. *Answer* $ ....[1]

(ii) Ben changes €770 into pounds.

Calculate how many pounds Ben receives.

Answer £ ....[1]

(iii) Chris changes $780 into euros.

Calculate how many euros Chris receives.

Answer € ....[2]

3

(b) Debbie changed some dollars into Japanese yen.
The rate of exchange was 81 dollars = 1 yen.

Emma changed the same number of dollars into yen.
The rate of exchange for Emma was 82 dollars = 1 yen.

Emma received 3 fewer yen than Debbie.

Given that the number of dollars changed each time is x, find x.

Answer ....[3]

N13/21/Q1

Word Problems Paper 1

1 The number of hours worked each day by Adam and Brenda is shown in the table.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
Adam	7	5	8	9	8	0
Brenda	0	8	9	9	7	5

The number of hours for which they are paid is calculated in the following way.
On each of days 1 to 5, every hour worked after the first 7 hours is counted as $1\frac{1}{2}$ hours.
On day 6, every hour worked is counted as 2 hours.

(a) Calculate the total number of hours for which Adam was paid.

(b) The rate of pay is $14.50 per hour.
How much did Brenda earn on day 6?

Answer (a) ....h [1]

(b) $... [1]

Answers: **(a)** 39 hours; **(b)** $145.   **J05/1/Q7**

---

2 (a) A journey of 170 kilometres took $4\frac{1}{4}$ hours.
Calculate the average speed in kilometres per hour.

(b) Potatoes cost 75 cents per kilogram.
John paid $1.20 for a bag of potatoes.
How many kilograms did he buy?

Answer (a) .... km/h [1]

(b) .... kg [1]

J05/1/Q5

---

3 Ed goes on a car journey.
The first 60 km of the journey takes 45 minutes.
The remaining 20 km of the journey takes 30 minutes.

Calculate his average speed, in kilometres per hour, for the whole journey.

Answer ....km/h [2]

Answer: 64   **J10/11/Q4**

---

4 10 (a) Sara sets out on a journey at 10 50.
She travels 65 km at an average speed of 20 km/h.

At what time does she complete her journey?

Answer .... [2]

(b) Kevin takes T minutes to travel to work.
Jane travels the same distance to work but goes 10% faster than Kevin.

Find an expression, in terms of T, for the number of minutes that Jane takes to travel to work.

Answer .... [1]

5

(a) Imran is paid \$16 per hour.

    (i) One week he works 35 hours.

       Calculate the amount he is paid for the week.   *Answer* \$.......................................[1]

    (ii) Imran is paid 20% extra per hour for working at weekends.

       Work out the total amount Imran is paid for working 4 hours at the weekend.

                                                Answer \$.......................................[2]

(b) The exchange rate between pounds and dollars is £1 = \$1.80.
Anna converts \$270 into pounds.

    Calculate the number of pounds Anna receives.   *Answer* £.......................................[2]

6

(a) The diagram shows the fuel gauge in Abid's car.
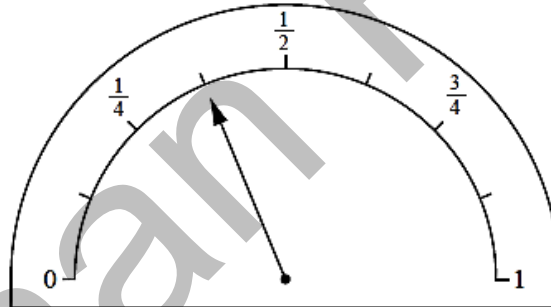
The tank contains 50 litres when it is full.

Estimate the number of litres in the tank.   *Answer* ....litres [1]

(b) The diagram shows the fuel gauge in Ben's car.
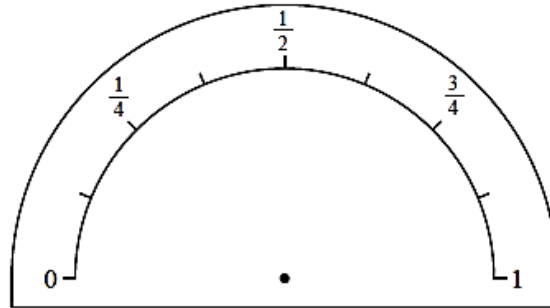
Draw an arrow on the gauge above to indicate that the tank is approximately $\frac{4}{5}$ full. [1]

Answer: **(a)** 18.75 **(b)** arrow between $\frac{3}{4}$ and $\frac{7}{8}$

---

7 Gill swims lengths of the swimming pool.
The pool is 25 m long and she swims a total of 1.6 km.

(a) How many lengths of the pool does she swim? *Answer* ....[1]

(b) Gill swims for $1\frac{1}{4}$ hours and ends her swim at 11 05.

   (i) At what time did she begin her swim? *Answer* ....[1]

   (ii) What is her average speed, in kilometres per hour?

                                                   Answer ....km/h [1]

Answer: **(a)** 64 **(b)** (0)9 50 **(c)** 1.28

---

8 Ahmed pays a total of $81 for wood, paint and a hammer.

(a) The amounts he pays for the wood, paint and hammer are in the ratio 4 : 3 : 2.

   Calculate how much Ahmed pays for the hammer. *Answer* $ ....[1]

(b) When Ahmed paid $81 he had received a 10% discount on the normal price.

   Calculate the normal price. *Answer* $ ....[2]

Answer: **(a)** 18 **(b)** 90

---

9 A bottle full of liquid has a total mass of 1.27 kg.
When the bottle is half-full of liquid the total mass is 900 grams.

Calculate the mass of the bottle. *Answer* .... grams [2]

Answer: 530

10   Carl spent *t* minutes on his English homework.
He spent three times as long on his Mathematics homework as on his English homework.
He spent a total of 2 hours 20 minutes on his English and Mathematics homework.

Write down an equation to represent this information and hence find the value of *t*.

Answer t = ... [2]

11   A mark, M, on a cliff is 1.96 m below sea level at high tide.
At low tide the sea level is 4.18 m lower than at high tide.
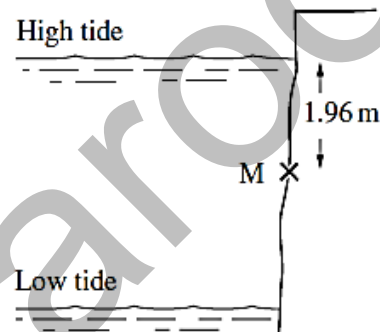
(a) How far is M above sea level at low tide?

(b) At a certain time the sea level is exactly half way between high tide and low tide.
How far is the sea level below M at this time?

High tide

1.96 m

M ✕

Low tide

Answer (a).. m [1]

(b).. m [1]

12   The temperature at 0900 is −4 °C.
The temperature at 1500 is 14 °C.

(a) Find the difference between the two temperatures.

(b) Assuming that the temperature rises at a steady rate, find

    (i) the temperature at 1300,

    (ii) the time when the temperature is 12.5 °C.

13   The highest air temperature recorded is 58.8 °C.
The lowest air temperature recorded is −89.2 °C.

(a) What is the difference between these two temperatures?

(b) The lowest air temperature recorded in Britain is 62 °C higher than −89.2 °C.
Find the lowest air temperature recorded in Britain.

14   (a)   A coach left London at 20 45 and arrived in Edinburgh the next day at 05 05.
How long did the journey take?

(b)   The distance between London and Edinburgh is 660 km.

(i)   A train took 5 hours 30 minutes to complete the journey.

Calculate its average speed.

(ii)   The average speed of another train was 150 km/h.

How long did this train take for the journey?
Give your answer in hours and minutes.

Answer (a) ....h............minutes [1]

(b) (i) ....km/h [2]

(ii) ....h ....minutes [2]

15   The diagram shows a fuel gauge in a car.

(a)   What fraction does the gauge show?

(b)   The fuel tank holds 48 litres when it
is full.
How many litres must be added to
fill the tank?
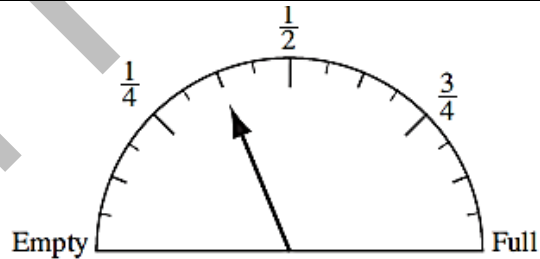
Answer (a) .... [1]

(b) ....litres [1]

16   Ann, Brian and Carol share the cost of a car.
Ann pays $\frac{2}{5}$ of the cost, Brian pays $\frac{1}{3}$ and Carol pays the rest.

(a)   What fraction of the cost does Carol pay?

Answer (a) ....[1]

(b)   Ann pays $1600 more than Brian.
Find the total cost of the car.

(b) $ ....[2]

17 (a) The rate of exchange between dollars and euros was $0.8 to 1 euro.
Calculate the number of euros received in exchange for $300.

Answer (a) ....[1]

(b) Find the simple interest on $450 for 18 months at 4% per year.

(b) $ ....[1]

Answer: (a) 375   (b) $27

18

The foot of a mountain is at sea level.   *Answer (a)* .... °C [1]
The temperature at the foot of the mountain was 16°C.
The temperature at a height of 3000 m on the mountain was −4°C.

(a) Find the difference between these temperatures.

(b) Given that the temperature fell at a constant rate, find

(i) the temperature at a height of 1800 m,   (b)(i) .... °C [1]

(ii) the height at which the temperature was 0°C, (ii) .... m [1]

(iii) an expression, in terms of x, for the temperature, in °C, at a height of x metres.

(iii) ....[2]

Answer: (a) 20 °C   (b)(i) 4 °C   (ii) 2400 m   (iii) $16 - \dfrac{x}{150}$

145

19
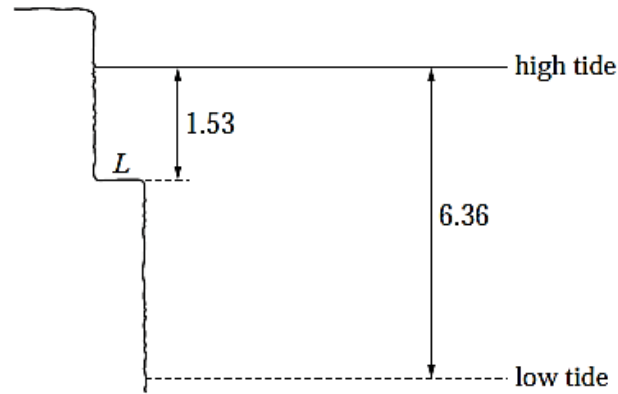
The sea level at high tide is 1.53 m above a ledge, L, on a cliff.
At low tide the sea level is 6.36 m below the sea level at high tide.

(a) How far below L is the sea level at low tide?   *Answer (a)* .... m [1]

(b) On a certain day, high tide is at 07 32.
After 2 hours and 34 minutes, the sea level has dropped $\frac{1}{3}$ of the distance between high tide and low tide.

   (i) At what time does the sea reach this level?   *Answer (b)(i)* ....[1]

   (ii) How far below L is the sea level at this time?   *Answer (b)(ii)* .... m [1]

Answers: (a) 4.83 m   (b)(i) 10 06   (ii) 0.59 m   **N09/1/Q10**

146

20 **(a)**

Paris to Creil		Paris to Creil
Adult 25 euros		Child 17.50 euros
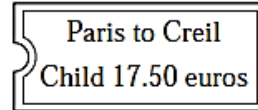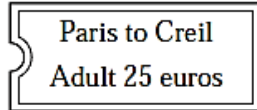
During a visit to France, a family took a train from Paris to Creil.
The cost of an adult ticket was 25 euros and the cost of a child ticket was 17.50 euros.

(i) How much did it cost for a family of 2 adults and 3 children?

Answer (a)(i) .... euros [1]

(ii) Express the cost of a child ticket as a percentage of the cost of an adult ticket.

Answer (a)(ii) .... % [2]

(b)

Creil to Clermont		Creil to Clermont
Adult		Child 12 euros
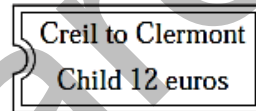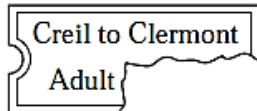
At Creil the family changed trains and travelled to Clermont.
The cost of a child ticket was 12 euros.
The cost of a child ticket was 60% of the cost of an adult ticket.
What was the cost of an adult ticket?

Answer (b) .... euros [2]

Answers: **(a)(i)** 102.5   **(ii)** 70   **(b)** 20   **N09/1/Q23**

147

21

The diagram shows a thermometer, with a circular dial, that records temperatures in °C.
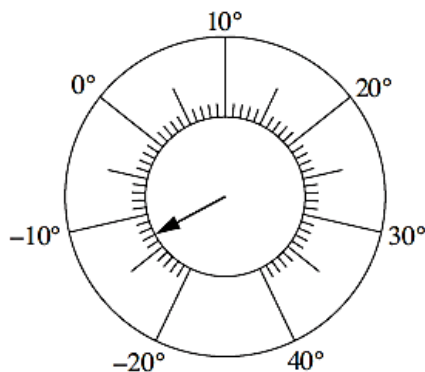
(a) Write down the temperature indicated by the pointer.

Answer .... °C [1]

(b) When the temperature increases from –20 °C to 40 °C, the pointer turns through an angle of 300°.

Calculate the angle through which the pointer turns when there is a 7 °C rise in temperature.

Answer .... [1]

(c) On one particular day, the temperature at 1 a.m. was 4 °C. By 6 a.m. it had fallen by 9 °C.

Calculate the temperature at 6 a.m.

Answer .... °C [1]

Answers: (a) -13 (b) 35 (c) -5   N11/11/Q12

22

Renata went on a journey that took $7\frac{1}{2}$ hours.

(a) The journey started at 22 48 on Monday.

At what time on Tuesday did it finish?   *Answer* .... [1]

(b) In the first part of the journey Renata travelled 150 km in 5 hours. She travelled at an average speed of 20 km/h for the rest of the journey.

Calculate her average speed for the whole journey.   *Answer* .... km/h [2]

Answer: (a) 06 18 (b) $26\frac{2}{3}$ N11/12/Q18

23

(a) Alice and Brenda share $300 so that Alice's share : Brenda's share = 3 : 7.

How much more does Brenda receive than Alice?   *Answer* $ ....[1]

(b) Find the simple interest on $200 for 4 years at 2% per year.

Answer $ ....[1]

Answers: (a) 120 (b) 16   N12/11/Q3

24 Pierre goes on a holiday from France to the UK.

    (a) His journey takes 4 hours and 43 minutes.
        It ends at 02 13 on Saturday.

        At what time on Friday does his journey start?   *Answer* ....[1]

    (b) Pierre changes 400 euros into pounds (£).
        The exchange rate is 1 euro = £0.845.

        How many pounds does he receive?   *Answer* £ ....[1]

Answers: **(a)** 21 30 or 9.30 pm **(b)** 338   **N12/11/Q5**

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25   **(a)** Express the ratio   30 minutes to $2\frac{1}{4}$ hours   in its lowest terms.
        Give your answer in the form $m : n,$ where m and n are integers.

                          Answer .... : ....[1]

  (b) Find the simple interest on $200 for 4 years at 0.6% per year.

                          Answer $ ....[1]

Answers: **(a)** 2:9 **(b)** 4.8   **N13/11/Q3**

---

26 A shopkeeper sells fruit at the prices shown in the table below.

Oranges	35 cents each
Apples	$2.40 per kg
Melons	$1.85 each

  (a) Sabah buys 750 g of apples and one melon.

     Calculate how much she pays.   *Answer* $ .... [1]

  (b) The shopkeeper buys

           100 oranges for $25,
           50 kg of apples for $80 and
           20 melons for $15.

     He sells all of these oranges, apples and melons at the prices shown in the table.

     Calculate his percentage profit.   *Answer* ....% [3]

Answers: **(a)** 3.65; **(b)** 60.   **N14/11/Q14**

27 The paper on a roll is 4.5 metres long.
Mary cuts as many pieces as possible, each of length 60 cm, from the roll.

(a) Calculate the number of pieces. *Answer* .... [1]

(b) Calculate the length of paper that remains on the roll. *Answer* .... cm [1]

Answers: **(a) 7 (b) 30**   **N16/11/Q2**

---

28 **(a)** One kilogram of tea costs $16.

Calculate the cost of 300 grams of tea. *Answer* $ .... [1]

(b) Find the simple interest on $400 for 3 years at 2% per annum.

Answer $ .... [1]

Answers: **(a) 4.80 (b) 24**   **N16/11/Q4**

---

29 **(a)** Find the simple interest on $200 for 3 years at 4% per year.

Answer $ .... [1]

(b) Two brothers share $200 in the ratio 2 : 3.

Calculate the larger share. *Answer* $ .... [1]

Answers: **(a) 24 (b) 120**   **N17/11/Q3**

---

30 As part of her training, Samantha runs for 2 hours.
For the first $1\frac{1}{2}$ hours she runs at an average speed of 10 km/h.
She runs 7 km in the remaining $\frac{1}{2}$ hour.

Calculate her average speed for the 2 hours. *Answer* .... km/h [2]

Answer: **11**   **N17/11/Q6**

150

Word Problems Paper 2

1   Two villages, *P* and *Q*, are joined by a straight road 6000 m long.

   (a)   Ann left *P* and ran to *Q* at a steady speed of 3 m/s.
   At the same instant that Ann left *P*, Ben left *Q* and cycled to *P* at a steady speed of 7 m/s.

   (i)   **(a)**   How far, in metres, did Ann travel in the first 2 minutes?   [1]

   (b)   Calculate the distance between Ann and Ben at the end of the first 2 minutes.   [1]

   (ii)   Ann and Ben passed each other at *M*.

   Calculate the distance *PM*.   [2]

   (iii)   Calculate the time that Ben took to cycle from *Q* to *P*.
   Give your answer in minutes and seconds, correct to the nearest second.   [2]

   (b)   The villages appear on a map which has a scale of 2 cm to 5 km.

   (i)   Express this scale in the form 1 : *n*.   [1]

   (ii)   Calculate the length of the road joining *P* and *Q* on the map.   [2]

Answers: **(a)(i)(a)** 360 m, **(b)** 4800 m, **(ii)** 1800 m, **(iii)** 14 min 17 s; **(b)(i)** 1:250 000, **(ii)** 2.4 s.   **J04/2/Q1**

2   **(a)**   These are the prices for a ride in an amusement park.

Adult	$3.60
Child	$2.25

   (i)   A family of two adults and three children went on the ride.
   They paid with a $20 note.

   Calculate the change they received.   [1]

   (ii)   Express $2.25 as a percentage of $3.60.   [1]

Answers: **(a)(i)** $6.05, **(ii)** 62.5%;   **J06/2/Q4**

3   James and Dan are partners in a small company.
From each year's profit, James is paid a bonus of $15 000 and the remainder is shared between James and Dan in the ratio 2 : 3.

   (a)   In 1996 the profit was $20 000.

   Show that Dan's share was $3000.   [1]

   (b)   In 1997 the profit was $21 800.

   Calculate

   (i)   the percentage increase in the profit in 1997 compared to 1996,   [2]

(ii) the total amount, including his bonus, that James received in 1997. [2]

(c) In 1998 Dan received $7500.

Calculate the profit in 1998. [3]

(d) In 1999, the profit was $x, where $x > 15\,000$.

    (i) Write down an expression, in terms of x, for the amount Dan received. [1]

    (ii) Given that Dan received half the profit, write down an equation in x and hence find the amount that Dan received. [3]

Answers: (b)(i) 9%, (ii) $17720; (c) $27500; (d)(i) $\dfrac{3}{5}$ (x − 15000), (ii) $ 45000.   **J06/2/Q7**

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4   **(a)** The table shows the fares charged by a taxi company.

> $1.20 per kilometre for the first 10 km
> then
> 80 cents for each additional kilometre after the first 10 km

    (i) Calculate the fare for a journey of

        (a) 8 km, [1]

        (b) 24 km. [1]

    (ii) Find the length of the journey for which the fare was $16. [2]

(b) The table gives the times of high tides at a harbour.

Date	May 5	May 6	May 7
Times	1000	1120	0036
	2256		1250

    (i) Calculate, in hours and minutes, the length of time between the high tide on May 6 and the morning high tide on May 7. [1]

    (ii) Given that low tides occurred midway between high tides, calculate the time of the low tide on the afternoon of May 5. [2]

(c) The height of a mountain is 1800 metres.
It is suggested that this mountain has been worn away at an average rate of 0.15 mm per year.
Assuming that the suggestion is correct, calculate the height of the mountain 20 million years ago. [2]

Answer: (a)(i)(a) $9.60,   (b) $23.20,   (ii) 15 km;   (b)(i) 13 hr 16 m,   (ii) 16 28;   (c) 4800 m.   **J07/2/Q1**

5

(a) Anne's digital camera stores its images on a memory card.
The memory card has 128 units of storage space.
When 50 images were stored, there were 40 units of **unused** storage space on the memory card.

    (i)   Calculate the percentage of **unused** storage space on the memory card. [1]

    (ii) Calculate the average amount of storage space used by each image. [2]

(b) Shop A charged 60 cents for each photograph.
Shop B charged 63 cents for each photograph and gave a discount of $1 on all purchases more than $10.

    (i)   Anne bought 24 photographs from Shop A and paid with a $20 note.

       Calculate the change she received. [1]

    (ii) Find how much cheaper it was to buy 24 photographs from Shop B than from Shop A. [2]

    (iii) Find the smallest number of photographs for which it was cheaper to use Shop B. [2]

Answers: (a) (i) 31.25%,   (ii) 1.76;   (b)(i) $5.60,   (ii) $0.28,   (iii) 16.   **J08/2/Q2**

6

(a) During a 20 week period in 2007, a bank made a profit of $378 million.

    (i)   Calculate the average profit it made each second. [2]

    (ii) During the same 20 week period in 2008, the profit was $945 million.

       For this 20 week period, calculate the percentage increase in the profit from 2007 to 2008. [2]

    (iii) Find the ratio of $378 million to $945 million.
       Give your answer in the form $m : n$, where m and n are the smallest possible integers. [2]

(b) Mary changed 480 euros into dollars.
The exchange rate was $1 = 0.6 euros.
The bank took, as commission, 2% of the amount that had been changed.

    Calculate the number of **dollars** the bank took as commission. [2]

Answers: **(a)(i)** $31.25,   (ii) 150%,   (iii) 2 : 5;   (b) $16.   **J09/2/Q2**

7

(a) Sarah bought some soup, apples and mushrooms from her local shop.
The table shows some of the amounts and prices.

Items	Price ($)
p cans of soup at 90 cents per can	6.30
1.5 kilograms of apples at $$q$ per kilogram	4.35
r kilograms of mushrooms at $6.40 per kilogram	1.60

    (i)   Find the values of p, q and r. [2]

(ii) Sarah gives the shopkeeper $20.00 to pay for all these items.

How much change does she receive? [1]

(b)

Washing Machine	Finance offer
$980	Pay a 20% deposit _and_ 24 monthly payments of $36 each

Lavin decides to buy this washing machine.

How much more would it cost Lavin if he paid for the washing machine using the finance offer instead of paying the $980 immediately? [2]

(c) Asif deposits $650 into a bank paying simple interest.
He leaves the money there for 5 years.
At the end of the 5 years, the amount in the bank is $763.75.

Calculate the percentage rate of interest the bank paid per year. [3]

Answers: (a)(i) $p = 7$, $q = 2.90$, $r = 0.25$, (ii) $7.75; (b) $80; (c) 3.5%. **J10/22/Q1**

---

8 **(a)** Ahmed's internet provider offers two payment schemes.

Scheme A : $30 per month for unlimited use.
Scheme B : $0.05 per minute on weekdays and $0.03 per minute at the weekend.

Each month Ahmed uses the internet for a **total** of $5\frac{1}{4}$ hours at the weekday rate and a **total** of 12 hours at the weekend rate.

Find the cost per month, in dollars, for Scheme B and decide which payment scheme is cheaper.

Answer Scheme B costs $ ....

Scheme ....is cheaper [2]

(b) Ahmed's printer can use large or small black cartridges.
A large cartridge costs $48.50 and prints 1000 pages.
A small cartridge prints 650 pages.
2 small cartridges cost $65.

(i) Find the cost per page, in dollars, if Ahmed buys 2 small cartridges.

Answer $ .... [1]

(ii) Is it cheaper per page for Ahmed to buy 2 small cartridges or a large cartridge?
Show your working. [1]

154

(c)

Monitor — $375

Keyboard — $70

Computer

Ahmed buys a new monitor, keyboard and computer.
He is given a 15% discount off the total price.
The discounted price that Ahmed pays is $1134.75.
The price of the monitor before the discount was $375.
The price of the keyboard before the discount was $70.

Calculate the price of the computer before the discount.

Answer $...................................... [3]

Answers: (a) 37.35 and A (b)(i) 0.05 (ii) Large (c) 890   **J11/21/Q1**

9 (b)

Exchange Rate
$1 = £0.45

Jean-Pierre bought a watch for $110.
Simon bought an identical watch for £46.62.

Find the difference, in dollars, between the amount Jean-Pierre paid and the amount Simon paid.

Answer $...................................... [2]

(c) The time taken to build a brick wall is inversely proportional to the number of workers.
3 workers took 30 hours to build a wall.

Calculate the time it would have taken 5 workers to build this wall.

155

10

(a) Sunil needs to hire a digger from Monday to Thursday one week and on Monday and Tuesday the following week.
The Hire company charges $48 each time the digger is hired plus $13 per day.
He has two options.

    Option 1:   Hire the digger for four days, return it and then hire it again for two days.

    Option 2:   Hire it continuously from the first Monday to the second Tuesday.

Which is the cheaper option and by how much?

          Answer   Option .... is cheaper by $ ....[2]

(b) Tina invests some money in an account that earns simple interest at 3% per year.
At the end of one year the investment is worth $2781.

How much money did she invest?

          Answer $..[2]

Answers: (a) $\frac{3}{5}$ (b) 9 or -9 J12/22/Q2

11

(a) (i) In 2000 the cost of a packet of nails was $4.20.
In 2001 the price had increased to $4.50.

    Calculate the percentage increase in the cost of a packet of nails.   [2]

(ii) A new cereal packet contains 20% more than the old packet.
The new packet contains 264 grams of cereal.

    Calculate the mass of cereal in one of the old packets.   [3]

(iii) A builder bought a large number of bags of cement.
As a result he was given a discount of 7%.
The discount was $910.

    Calculate how much the builder paid for the cement.   [3]

(b) On 1 May 1998 William invested $900 for 4 years at 6% per annum simple interest.

(i) Calculate the interest he received on his investment.   [2]

(ii) He invested another $900 for 3 years at 6% per annum simple interest on 1 May 1999,
             then $900 for 2 years at 6% per annum simple interest on 1 May 2000,
      and a final $900 for 1 year at 6% per annum simple interest on 1 May 2001.

William withdrew all of his money on 1 May 2002.

Calculate the total sum that he withdrew.   [2]

Answers: (a)(i) 7.14%, (ii) 220 g, (iii) $12 090; (b)(i) $216, (ii) $4140.   N02/2/Q6

12 (a) John opened a bank account.
He deposited $800 in his account.
The account pays simple interest at the rate of 5% per year.

Calculate the total amount in his account after 3 years. [2]

(b) Two telephone companies have different ways of charging their customers.

(i) Michael uses Company A which charges 6 cents for each unit or part unit of time.
A unit of time is 200 seconds.
He makes a call lasting 1 hour 22 minutes.

Calculate the cost of his call. [2]

(ii) Norman uses Company B which charges 5 cents for each of the first 400 units.
The charge for each additional unit is reduced by one quarter.
There is also a fixed charge of $27 for the use of the equipment.

He is charged for 629 units.

Calculate, correct to the nearest cent, the total sum that he has to pay. [3]

Answers: **(a)** $920; **(b)(i)** $1.50 or 150 cents, **(ii)** $55.59. **N03/2/Q1**

---

13 A company manufactures biscuits.

(a) One batch of biscuits contains 300 grams of dried fruit.
This consists of sultanas and currants, with masses in the ratio 2 : 3.

Find the mass of the sultanas. [2]

(b) The mixture used to make one batch of biscuits has a mass of 18 kg.
The mixture loses 12% of its mass when it is cooked to make the biscuits.

(i) Calculate the mass of one batch of biscuits. [2]

(ii) Each biscuit has a mass of 12 grams.
One batch of biscuits is put into packets.
Each packet contains 16 biscuits.

Find how many packets can be filled, and the number of biscuits remaining. [2]

(iii) The total mass of each packet, including packaging, is 201 grams.

Express the mass of the packaging as a percentage of the total mass of a packet. [2]

(c) A trader sells one packet of biscuits for 80 cents.
He makes a profit of 25% of his cost price.

Calculate the price he paid for a packet of biscuits. [2]

Answers: **(a)** 120 g; **(b)(i)** 15.84 kg, **(ii)** 82 packets, 8 biscuits, **(iii)** 4.48 %; **(c)** 64 cents. **N03/2/Q2**

14 Mr Smith bought three companies, Alpha, Beta and Gamma, for a total of $80 000 000.

The amounts he paid for these companies were in the ratio 4 : 5 : 7.

(a) Calculate how much he paid for each company. [2]

(b) When he sold the companies, he made a profit of 12% on the $80 000 000 he paid for them.

Calculate the profit he made on the sale of the three companies. [1]

(c) When he sold the companies, he made a profit of 25% on Alpha and a loss of 10% on Beta.

Calculate

(i) the profit he made on Alpha, [1]

(ii) the percentage profit that he made on Gamma. [3]

(d) When the previous owner, Mr Jones, sold the companies to Mr Smith for $80 000 000, he made a profit of 60%.

Calculate the total amount Mr Jones had paid for the companies. [2]

Answers: **(a)** $20 000 000, $25 000 000, $35 000 000; **(b)** $9 600 000; **(c)(i)** $5 000 000, **(ii)** 20.3%; **N04/2/Q3**
(d) $50 000 000.

15 The cost of parking in a car park is 10 cents for each hour.
When he parked his car, John had only a large number of 10 cents coins and 20 cent coins to put into the ticket machine.
The table shows how he can pay to park his car.

Parking time (hours)	Ways of paying (amounts in cents)	Number of ways of paying to park
1	10	1
2	10 then 10 20	2
3	10 then 10 then 10 10 then 20 20 then 10	3

(a) Show that there are

(i) 5 ways to pay for 4 hours, [1]

(ii) 8 ways to pay for 5 hours. [1]

(b) The table below shows the number of ways John can pay when parking for various times.

Time (hours)	1	2	3	4	5	6	7		n	$n+1$	$n+2$
Number of ways	1	2	3	5	8	a	b		x	y	z

158

(i) Find the values of *a* and *b*. [3]

(ii) Write down an equation connecting *x*, *y* and *z*. [1]

Answers: **(b)(i)** $a = 13$, $b = 21$, **(ii)** $z = x + y$. **N05/2/Q5**

16

(a) One day the rate of exchange between pounds (£) and United States dollars ($) was £1 = $1.65. On the same day, the rate of exchange between pounds (£) and euros was £1 = 1.44 euros.

   (i) Alan changed £500 into dollars.

     Calculate how many dollars he received. [1]

   (ii) Brenda changed 900 euros into pounds.

     Calculate how many pounds she received. [1]

   (iii) Clare changed $792 into euros.

     Calculate how many euros she received. [2]

(b) The cost of manufacturing a television was $15 000.

   (i) It was sold to a wholesaler at a profit of 8% of the cost.

     Calculate the price the wholesaler paid for the television. [1]

   (ii) The wholesaler sold the television to a shop at a profit of 8% of the price he paid for it.

     The shop then sold the television to John at a profit of 8% of the price it paid.

     Calculate how much the television cost John. [2]

   (iii) Calculate the percentage increase in the cost of the television from its manufacture till John owns it. [2]

(c) The shop sold a Home Entertainment system to Leslie for $46 480. The shop made a profit of 12% on the price it paid for the system.

   Calculate how much the shop paid for the system. [2]

Answer: **(a)(i)** $825   **(ii)** £625   **(iii)** 691.2 euros **N06/2/Q5**

  (b)(i) $16 200   **(ii)** $18 895.68   **(iii)** 26.0%   **(c)** $41 500

17

(a) 100 g of spaghetti contains 3.6 g of fibre.

Express

mass of fibre : mass of spaghetti

as the ratio of two integers in its simplest form. [1]

(b) A tin contains 210 g of beans.

    (i) 100 g of beans contains 4.5 g of protein.

       Calculate the mass of protein in the tin. [1]

    (ii) 100 g of beans contains 0.3 g of fat.

       (a) What percentage of the beans is fat? [1]

       (b) The recommended daily amount of fat is 70 g.

          Calculate what percentage of the recommended daily amount is in the tin. [3]

    (iii) The mass of salt in 100 g of beans is 1.0 g, correct to 1 decimal place.

       Calculate an upper bound for the mass of salt contained in the tin. [2]

(c) A tin of soup contains 166 calories.
This is 8.3% of the recommended daily number of calories.

Calculate the recommended daily number of calories. [2]

Answers: **(a)** 9 : 250 **(b)(i)** 9.45 g **(ii)(a)** 0.3% **(b)** 0.9% **(iii)** 2.205 g **(c)** 2000 **N09/2/Q6**

18

(a) Ada and Bill own a company.
In 2008 Ada invests \$22 500 in the company and Bill invests \$37 500.

(i) Express $22\,500 : 37\,500$ in the form $m : n$, where m and n are the smallest possible integers.

Answer .... : .... [1]

(ii) The profit made by the company in 2008 is shared in the ratio of the amounts invested.

Given that Ada's share of the profit is \$3 600, calculate the total profit made by the company.

Answer \$.................................. [1]

(iii) Ada's investment in 2008 is $12\frac{1}{2}\%$ more than the amount she invested in 2007.

Calculate the amount that Ada invested in 2007. *Answer* \$.................................. [2]

(b)

LAWNMOWER

\$2395

Plan A: Deposit \$595 and
12 monthly payments of \$171.04

Plan B: Deposit \$395 and
24 monthly payments of \$

(i) Rashid buys one of these lawnmowers for \$2395.
Sayeed buys one of these lawnmowers using Plan A.

In total, how much **more** than Rashid will Sayeed pay?

Answer \$.................................. [1]

(ii) When one of these lawnmowers is bought using Plan B, the total cost is \$3054.20.

Answer \$.................................. [2]

Calculate the monthly payment.

(iii) In a sale, the price of the lawnmower is reduced from \$2395 to \$1595.

Calculate the percentage discount.

Answer .... % [2]

Answers: (a)(i) 3 : 5 (ii) 9 600 (iii) 20 000 (b)(i) 252.48 (ii) 110.80 (iii) 33.4   **N11/21/Q6**

161

19 **(a)** A shopkeeper buys some plates from a manufacturer for $10 each.

    (i) **(a)** The shopkeeper sells a plate for $12.

        Calculate the percentage profit.   *Answer* ....% [1]

        (b) The shopkeeper buys 240 plates and sells 180 at $12 each.
            The rest were sold to a café for a total of $540.

            Calculate the percentage discount given to the café.

                                        Answer ....% [2]

    (ii) The manufacturer made a profit of 60% when he sold each plate for $10.

        Calculate the cost of manufacturing each plate. *Answer* $................................. [2]

(b) Another shopkeeper bought 100 pans at $5 each.
    He sold 63 at $6 each and x at $4 each.
    He did not sell all the pans nor enough to make an overall profit.

    (i) Form an inequality in x.   *Answer* .... [1]

    (ii) Hence find the greatest possible number of pans that were sold.

                        Answer .... [2]

(c) One day, the rate of exchange between American dollars ($) and British pounds (£)
    was $1.45 = £1.

    (i) Alan changed £300 into dollars.

        Calculate how many dollars he received.   *Answer* $................................. [1]

    (ii) On the same day, the rate of exchange between South African rands (R) and pounds
        was R10.44 = £1.

        Calculate the number of rands received in exchange for one dollar.

                        Answer R................................. [2]

Answers: (a)(i)(a) 20 (b) 25 (ii) 6.25 (b)(i) 63×6 + 4x ≤ 500 (ii) 93 (c)(i) 435 (ii) 7.20   **N11/22/Q4**

162

20 (a) The table shows some of the nutritional information for a 300 g tin of soup.

Carbohydrate	18 g
Fat	20.1 g
Fibre	0.6 g
Sodium	1.38 g

(i) What percentage of the 300 g tin of soup is carbohydrate?

Answer ....% [1]

(ii) What fraction of the 300 g tin of soup is fibre?
Give your answer as a fraction in its lowest terms. *Answer* .... [1]

(iii) Of the carbohydrates, 15% are sugars.

How many grams of sugars are in one tin of soup?

Answer .... g [1]

(b) I need 2500 g of soup.

How many 300 g tins of soup do I need to buy? *Answer* .... [1]

(c) During March there is a special promotion and the soup is on sale in tins containing 20% extra.

(i) These tins of soup each contain 4.2 g of protein.

How much protein was contained in each original 300 g tin of soup?

Answer .... g [2]

(ii) The special promotion tins cost $0.80 .
The soup can also be bought in larger tins containing 500 g for $1.12 .

Is it better value to buy the 500 g tin or the special promotion tin?
Show your working. [2]

Answers: (a)(i) 6; (ii) $\frac{1}{500}$; (iii) 2.7; (b) 9; (c)(i) 3.5; (ii) The special promotion tin with for example N14/21/Q1
both 0.222 cents per gram and 0.224 cents per gram stated.

21 **(a)** Tim invests $2500 in a bank paying simple interest at 2.3% per year.
What is the total amount of money in the bank at the end of 4 years?

Answer $ .... [2]

(b)

TABLET	FINANCE OFFER
$750	Pay 15% of $750 as deposit <u>and</u> 36 monthly payments of $25.

Chris buys the tablet using the finance offer.
How much more does he pay than if he had paid $750 for it?

Answer $ .... [2]

(c) Lavin buys some sweets, pens and paper at her local shop.
The shop is offering 20% discount on all items.
This is her receipt.

Items and prices	Cost ($)
0.3 kg of sweets at $15.50 per kg 6 pens at $x per pen Paper	w 4.50 z
Total before discount	y
Total after discount	32.40

Find the missing values w, x, y and z.

Answer $w =$ ....

$x =$ ....

$y =$ ....

$z =$ .... [5]

Answers: (a) 2730; (b) 262.50; (c) $w = 4.65$, $x = 0.75$, $y = 40.50$, $z = 31.35$

N15/21/Q1

164

22 **(a)** In 2016, the price of a television is $1995.

    (i) Afzal pays the $1995 with a deposit of $399 and 12 equal monthly payments.

    Calculate Afzal's monthly payment.

                                            Answer $ .... [1]

    (ii) What percentage of $1995 is $399?

                                            Answer .... % [1]

    (iii) The price of the television in 2016 is 5% more than the price in 2015.

    Calculate the price in 2015.

                                            Answer $ .... [2]

(b) Afzal watched a programme that lasted 2 hours 53 minutes.
It ended at 01 15.

  At what time did it start?

                                            Answer .... [1]

(c) A company paid a quarter of a million dollars for an advertisement that lasted 38 seconds.

  Calculate the cost, correct to the nearest hundred dollars, for each second of the advertisement.

                                            Answer $ .... [2]

(d) The programme showed an athlete running 100 metres, measured correct to the nearest metre.
The time the athlete took was 11.3 seconds, measured correct to the nearest 0.1 second.

  Calculate the upper bound of the athlete's average speed.

                                            Answer .... m/s [2]

Answers: (a)(i) 133 (ii) 20 (iii) 1900 (b) 22 22 (c) 6600 (d) 8.93   **N16/21/Q1**

23 **(a) (i)** Jasmine earns $12.50 for each hour she works.
She works for 38 hours each week.
She is given a pay increase of 6%.

Calculate the total amount Jasmine earns each week after the pay increase.

Answer $ .... [3]

(ii) Abdul earns $525 each week.
He moves to a new job where he earns $462 each week.

Calculate the percentage reduction in his earnings in his new job.

Answer .... % [2]

(iii) Maria is given a pay increase of 3%.
After the pay increase, she earns $2472 each month.

Calculate her monthly pay before the pay increase. *Answer* $ .... [2]

(b) The exchange rate between dollars ($) and pounds (£) is $1 = £0.65 .
The exchange rate between euros (€) and pounds is €1 = £0.74 .

Dan changes €520 into pounds.
He spends £260 and then changes the rest into dollars.

Work out how many dollars he receives. *Answer* $ .... [3]

Answers: (a)(i) 503.50 (ii) 12 (iii) 2400 (b) 192

Algebra Word Problems Paper 1

1 A stone is thrown vertically upwards from the ground so that its height above the ground after t seconds is $(20t - 5t^2)$ metres.

    (a) (i) Show that the values of t when the stone is 15 metres above the ground satisfy the equation

$$t^2 - 4t + 3 = 0.$$

                                                      [1]

       (ii) Find the values of t when the stone is 15 metres above the ground.

                                  Answer *(a)*(ii) $t = $ .... and .... **[2]**

    (b) Find the value of t when the stone hits the ground.

                                        Answer *(b)* $t = $ .... **[2]**

Answer: **(a)(ii)** 1, 3   **(b)** 4

---

2 The diagram shows a rectangle with length $(2x + 3)$ cm and width $(x - 1)$ cm.

    (a) The area of the rectangle is $12\,\text{cm}^2$.

        Form an equation in x and show that it reduces to $2x^2 + x - 15 = 0$.   **[2]**

    (b) Solve $2x^2 + x - 15 = 0$.   *Answer* $x = $ .... or .... **[2]**

    (c) Find the perimeter of the rectangle.   *Answer* .... cm **[1]**

Answer: **(b)** 2.5, - 3 **(c)** 19

3

    (a) A car travels 144 km in h hours.
        Write down, in its simplest form, an expression in terms of h for its average speed in metres per second.

    (b) Solve the equation $3(2x - 7) = 6 - 4(2 - x)$.

                            Answer (a)..m/s **[2]**

                            (b) $x = $ .... **[2]**

Answers: **(a)** $40 \div h$; **(b)** $9\frac{1}{2}$

4 A group of 15 adults and 12 children are going on a coach to a concert.
The tickets for the coach cost \$$a$ for each adult and \$$c$ for each child.
The total cost for the coach tickets is \$324.

(a) Show that $5a + 4c = 108$. [1]

(b) For a different group of 2 adults and 3 children the cost is \$53.

Solve the simultaneous equations.

$$5a + 4c = 108$$
$$2a + 3c = 53$$

Answer $a = $ ....

$c = $ .... [4]

(c) Find the cost for a group of 4 adults and 5 children to travel on the coach.

Answer \$ .... [1]

Algebra Word Problems Paper 2

1   **(a)** Factorise completely $20t^2 - 5$.   [2]

     (b) Express as a single fraction in its simplest form

$$\frac{7}{2x} - \frac{5}{3x}.$$ [2]

     (c) Tickets for a concert were priced at \$5, \$8 and \$12.

        The number of \$5 tickets sold was twice the number of \$8 tickets.
        The number of \$12 tickets sold was 80 more than the number of \$8 tickets.

        The number of \$8 tickets sold was x.

        (i) Find an expression, in terms of x, for the total sum of money received from the sale of the tickets.   [1]

        (ii) Given that \$9360 was received from the sale of the tickets, form an equation in x.

           Solve this equation and hence find the **total** number of tickets that were sold.   [3]

Answers: **(a)** $5(2t-1)(2t+1)$; **(b)** $\dfrac{11}{6x}$; **(c)(i)** $30x + 960$, **(ii)** 1200.   **J03/2/Q2**

---

2   The distance between two houses, P and Q, is 200 km.
     Joe travelled by car from P to Q at an average speed of x km/h.

     (a) Write down an expression, in terms of x, for the number of hours he took to travel from P to Q.   [1]

     (b) He returned from Q to P at an average speed of $(x + 5)$ km/h.
        Write down an expression, in terms of x, for the number of hours he took to travel from Q to P.   [1]

     (c) The **total** time he took to go from P to Q and to return from Q to P was 8 hours.

        (i) Write down an equation in x and show that it simplifies to

$$x^2 - 45x - 125 = 0.$$ [4]

        (ii) Solve the equation $x^2 - 45x - 125 = 0$,   giving each answer correct to 2 decimal places.   [4]

        (iii) Calculate, correct to the nearest minute, the time he took to travel from P to Q.   [2]

J02/2/Q10

3

Diagram I   **Diagram II**

Diagram I shows a quadrilateral, $ABCD$, in which $DA = AB = x$ centimetres and $BC = CD = y$ centimetres.

$A\hat{B}C = C\hat{D}A = 90°$.

(a) Show that the area of this quadrilateral is xy square centimetres.   [1]

(b) Five of these quadrilaterals are joined together to make the shape shown in Diagram II. The total area of this shape is 80 cm^2.

　　(i) Show that the outside perimeter, P centimetres, of this shape is given by

$$P = 10x + \frac{32}{x}.$$ [2]

　　(ii) (a) In the case when $P = 38$, show that $5x^2 - 19x + 16 = 0$.   [2]

　　　　　(b) Solve the equation $5x^2 - 19x + 16 = 0$, giving both answers correct to two decimal places.   [4]

　　　　　(c) Find the two possible values of y when $P = 38$.   [1]

　　(iii) (a) Calculate the value of P when $x = y$.   [1]

　　　　　(b) What is the special name given to the quadrilateral $ABCD$ when $x = y$?   [1]

Answers: **(b)(ii)(b)** $x = 2.54$ or 1.26, **(c)** $y = 6.30$ or 12.7; **(iii)(a)** 48, **(b)** square.   J03/2/Q10

4　　**Read these instructions.**

A　Choose two **different** digits from $1, 2, 3, 4, 5, 6, 7, 8$ and 9.
B　Write down the larger two-digit number which can be formed from the chosen digits.
C　Write down the smaller two-digit number which can be formed from the chosen digits.
D　Subtract the smaller number from the larger and note the result.

Example:　　A　Choose 2 and 8.
　　　　　　　B　Larger number is eighty-two (82).
　　　　　　　C　Smaller number is twenty-eight (28).
　　　　　　　D　Subtract:　　82
　　　　　　　　　　　　　　　－28
　　　　　　　　　　　　　　　───
　　　　　　　　　　　　　　　　54
　　　　　　　Result = 54

(a) The digits 3 and 7 are chosen.
Follow the instructions to find the result. [1]

(b) Choose three other different pairs of digits.
Follow the instructions to find the result in each case. [1]

(c) What do you notice about all these results? [1]

(d) The digits x and y, where $x > y$, are chosen.

Find expressions, in terms of x and y, for the value of

(i) the larger number, [1]

(ii) the result. [2]

5   A route up a mountain is 20 km long.
John followed this route at an average speed of x km/h.

(a) Write down an expression, in terms of x, for the number of hours he took to walk up the mountain.
[1]

(b) He came down the mountain by a different route.
The length of this route was 25 km.
His average speed coming down the mountain was 2 km/h greater than his average speed going up the mountain.

Write down an expression, in terms of x, for the number of hours he took to walk down. [1]

(c) It took John $1\frac{1}{2}$ hours less to come down than to go up.

Write down an equation in x, and show that it simplifies to

$$3x^2 + 16x - 80 = 0.$$

[3]

(d) Solve the equation $3x^2 + 16x - 80 = 0$, giving both answers correct to 3 decimal places. [4]

(e) Calculate, correct to the nearest minute, the **total** time John took to go up and come down the mountain. [3]

6 **(d)** On Monday, two girls, Jane and Susan, collected some seashells.
Jane collected x shells and Susan collected 22 **more** than Jane.
On Tuesday, Susan gave 60 of her shells to Jane.
The table shows the numbers of shells each girl had on the two days.

	Jane	Susan
Monday	x	$x + 22$
Tuesday	$x + 60$	y

    (i) Write down an expression for y in terms of x. [1]

    (ii) Given that, on Tuesday, Jane had three times as many shells as Susan,

        (a) write down and solve an equation in x, [2]

        (b) find the total number of shells the girls collected. [1]

(d)(i) x − 38, (ii)(a) x + 60 = 3(x − 38) giving x = 87, (b) 196.   **J07/2/Q3d**

7

In the diagram, $ABCD$ is a rectangle.
$AB = 12$ cm and $BC = 8$ cm.
$AP = BQ = CR = DS = x$ centimetres.

(a) Find an expression, in terms of x, for

    (i) the length of QC, [1]

    (ii) the area of triangle CRQ. [1]

(b) Hence show that the area, in square centimetres, of the quadrilateral $PQRS$ is $2x^2 - 20x + 96$. [3]

(c) When the area of quadrilateral $PQRS$ is 60 cm^2, form an equation in x and show that it simplifies to

$$x^2 - 10x + 18 = 0.$$ [1]

(d) Solve the equation $x^2 - 10x + 18 = 0$, giving each answer correct to 2 decimal places. [3]

(e) It is given that $2x^2 - 20x + 96 = 2(x - 5)^2 + K$.

    (i) Find the value of K. [1]

    (ii) Hence **write down** the smallest possible area of the quadrilateral $PQRS$ and the value of x at which it occurs. [2]

8

Ahmed throws a ball to John.
The ball travels 10 metres at an average speed of x metres per second.

(a) Write an expression, in terms of x, for the time taken, in seconds, for the ball to travel from Ahmed to John. [1]

(b) John then throws the ball to Pierre.
The ball travels 15 metres.
The ball's average speed is 0.5 metres per second greater than the ball's average speed from Ahmed to John.

Write an expression, in terms of x, for the time taken, in seconds, for the ball to travel from John to Pierre. [1]

(c) The time taken between John catching the ball and then throwing it to Pierre is 2 seconds.
The total time taken for the ball to travel from Ahmed to Pierre is 7 seconds.

Write down an equation in x, and show that it simplifies to

$$2x^2 - 9x - 2 = 0.$$ [3]

(d) Solve the equation $2x^2 - 9x - 2 = 0$, giving each answer correct to 2 decimal places. [4]

(e) (i) Find the average speed, in metres per second, of the ball as it travels from John to Pierre. [1]

(ii) How much longer does it take for the ball to travel from John to Pierre than from Ahmed to John?
Give your answer in seconds. [2]

9

Rectangular paving slabs measure x centimetres by y centimetres, where $x < y$.
Fourteen of these slabs form a path around each of two different rectangular gardens.
These two gardens are shaded in the designs below.

Design 1   Design 2

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(a) The **outside** perimeter of the path in **Design 1** is 10.2 metres.
The **total** perimeter of the path in **Design 2** is 13.6 metres.

Show that $4x + 5y = 510$ and $5x + 2y = 340$. [2]

(b) Solve the simultaneous equations.

$$4x + 5y = 510$$
$$5x + 2y = 340$$

Answer x = ...

y = ... [3]

(c) Find the difference between the areas of the two gardens.
Give your answer in square metres.

Answer .... m² [2]

Answers: **(b)** x = 40, y = 70 **(c)** 0.56

J11/21/Q4

10 **(a)**

A is a point on the circle, centre M, and AB is a tangent at A.
AM = y centimetres, AB = (y + 5) centimetres and MB = (y + 9) centimetres.

(i) Show that $y^2 - 8y - 56 = 0$.

[2]

(ii) Solve the equation $y^2 - 8y - 56 = 0$, giving each answer correct to 1 decimal place.

Answer y = ... or .... [3]

(iii) Find the length of the longest side of triangle ABM.

Answer .... cm [1]

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(b)

Q, R and S are points on a circle, centre O.
QT is the tangent at Q and $Q\hat{T}O = x°$.

(i)   (a) Show that $Q\hat{R}S$ is $\frac{1}{2}(90 - x)$.   [1]

    (b) Find an expression, in terms of x, for $O\hat{Q}S$.   *Answer* .... [2]

(ii)   It is given that three times $Q\hat{R}S$ is twice $O\hat{Q}S$.

    (a) Show that $180 + 2x = 270 - 3x$.   [2]

    (b) Hence find $Q\hat{T}O$.   *Answer* .... [1]

Answers: (a)(ii) 12.5 or – 4.5 (iii) 21.5 (b)(i)(b) ½ (90 + x) (ii)(b) 18°   **J11/21/Q8**

---

11   **(a)** Factorise $9x^2 - 64y^2$.   *Answer* ....[1]

   (b) The product of three numbers 4, x and $(x + 3)$ is 55.

   Form an equation in x and solve it to find the possible values of x.

                                   Answer $x =$ .... or ....[3]

   (c)   **(i)** Given that $\dfrac{x-1}{3} - \dfrac{5}{x+2} = 1$ show that $x^2 - 2x - 23 = 0$.   [2]

    (ii) Solve $x^2 - 2x - 23 = 0$.
         Give your answers correct to one decimal place.

                                   Answer $x =$ .... or ....[3]

Answers: **(a)** $(3x - 8y)(3x + 8y)$ **(b)** 2.5, −5.5   **(c)(ii)** 5.9, −3.9   **J12/21/Q3**

---

12 (a) In an athletics match Ben won the 100 m race in 9.98 s and Calvin won the 200 m race in 19.94 s.

What is the difference in their average speeds?
Give your answer in metres per second, correct to two decimal places.

Answer .... m/s [2]

(b) Two cars each complete a journey of 120 km.
The first car is driven at an average speed of x km/h.
The second car is driven at an average speed 3 km/h faster than the first car.
The first car takes 6 **minutes** longer to complete the journey.

(i) Write down an equation in x and show that it simplifies to $x^2 + 3x - 3600 = 0$. [3]

(ii) Solve the equation $x^2 + 3x - 3600 = 0$, giving each answer correct to one decimal place.

Answer $x =$ .... or .... [3]

(iii) How many **minutes** does the first car take to travel the 120 km?

Answer .... minutes [2]

Answer: **(a)** 0.01 **(b)(i)** Correct equation shown **(ii)** 58.5 or − 61.5 **(iii)** 123   **J13/21/Q7**

13 (a) Here is part of a number grid.

A square can be placed anywhere on the grid outlining four numbers.

The numbers in opposite corners of the square are multiplied together and the difference between the products is found.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20				

$$9 \times 14 - 8 \times 15 = 126 - 120$$
$$= 6$$

(i) The grid is continued downwards.

If n represents the number of the top left of the square, complete this square with expressions for the other numbers.

n	$n + 1$

[1]

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(ii) Use your answer to **part (a)(i)** to prove that the difference between the products of the opposite corners is always 6. [2]

(b) Here is part of a different number grid.
A cross can be placed anywhere on the grid outlining five numbers.
The numbers in the cross are added together.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36				
41	42								

(i) Find and simplify an expression, in terms of n, for the sum of the numbers in the cross below.

n

Answer .... [2]

(ii) The sum of the numbers in the cross below is 330.

Complete the cross with the correct numbers.

[2]

Answers: (a)(i) $n + 6$, $n + 7$ (ii) $n^2 + 7n + 6 - n^2 - 7n = 6$ (b)(i) $5n + 50$ or $5(n + 10)$ (ii) 56, 65, 66, 67, 76 **J14/21/Q5**

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13 Imran drives a distance of 180 km on a business trip.
He drives the first 100 km at an average speed of x km/h.
He drives at an average speed 5 km/h slower than this for the remainder of the journey.

(a) Find, in terms of x, an expression for the time taken, in hours, for the first 100 km.

Answer .... hours [1]

(b) Given that the journey takes a total of 2 hours 30 minutes, form an equation in x and [4]
show that it simplifies to $x^2 - 77x + 200 = 0$.

(c) Solve the equation $x^2 - 77x + 200 = 0$, giving each answer correct to 2 decimal places.

Answer $x =$ .... or .... [4]

(d) Which of the solutions in **part (c)** represents the speed for the first 100 km of Imran's trip?
Give a reason for rejecting the other solution.

Answer .... km/h because .... [1]

(e) Find the difference between the times taken for the first and second parts of the journey.
Give your answer in minutes, correct to the nearest minute.

Answer .... minutes [2]

Answers: **(a)** 100/x **(b)** $x^2 - 77x + 200 = 0$ **(c)** 74.31 and 2.69 **(d)** 74.31 since 2.69 would give a negative speed value for second part **(e)** 11   **J14/21/Q11**

---

14 (a) (i) Evaluate $\dfrac{8\sin 54°}{\sin 18°}$.

Answer .... [1]

(ii) Evaluate $\sqrt{4.73^2 - 1.65\sin 43°}$.

Answer .... [1]

(b)

In the triangle ABC, $BC = 16\,\text{cm}$ and $B\hat{A}C = 60°$.
$AB = x\,\text{cm}$ and $AC = 2x + 3\,\text{cm}$.

   (i)   Form an equation in x and show that it simplifies to $3x^2 + 9x - 247 = 0$.   [4]

  (ii)   Solve the equation $3x^2 + 9x - 247 = 0$,   giving your answers correct to 2 decimal places.

                                      Answer $x =$ .... or .... [3]

 (iii) Hence write down the lengths of AB and AC.

                                  Answer $AB =$ ....cm $AC =$ ....cm [1]

  (iv) Find the area of triangle ABC.   *Answer*   .... cm² [2]

Answers: (a)(i) 20.9 to 21.(0) (ii) 4.6(0) to 4.61 (iii) 7.70 and –10.70 (iv) 61.3 to 62.(0)   J15/21/Q7

15   (a) Given that $v = u + \dfrac{Ft}{m}$,

        express t in terms of F, m, u and v.   [2]

   (b) An elastic string hangs from a nail N.

        When a mass of m grams is attached to its lower end, the elastic is
        stretched so that its total length is x cm, as shown in the diagram.

        The table below shows the results of two experiments.

Length (x cm)	43	49
Mass (m grams)	50	80

        It is known that x and m are connected by the equation $x = c + dm$,
        where c and d are constants.

     (i)   Use this information to write down two equations in c and d.   [1]

    (ii)   Solve your equations to find the value of c and the value of d.   [3]

   (iii) Find the mass at the end of the string when its length is 40 cm.   [2]

   (iv) What does the value of c represent?   [1]

Answers: (a) $t = \dfrac{m(v - u)}{F}$; (b)(ii) $c = 33$, $d = 0.2$, (iii) 35.   N01/2/Q4

16   **(a)** Express as a single fraction in its simplest form

$$\frac{200}{x} - \frac{200}{x+4}.$$ [2]

   (b) When driven in town, a car runs x kilometres on each litre of petrol.

     (i) Find, in terms of x, the number of litres of petrol used when the car is driven 200 km in town. [1]

     (ii) When driven out of town, the car runs $(x + 4)$ kilometres on each litre of petrol.
It uses 5 litres less petrol to go 200 km out of town than to go 200 km in town.

     Use this information to write down an equation involving x, and show that it simplifies to

$$x^2 + 4x - 160 = 0.$$ [3]

   (c) Solve the equation $x^2 + 4x - 160 = 0$, giving both answers correct to two decimal places. [4]

   (d) Calculate the total volume of petrol used when the car is driven 40 km in town and then
120 km out of town. [2]

Answers: **(a)** $\dfrac{800}{x(x+4)}$; **(b)(i)** $\dfrac{200}{x}$; **(c)** 10.81 and −14.81cm; **(d)** 11.8 litres.   **N01/2/Q7**

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17   A trader bought some paraffin for \$500.
He paid \$$x$ for each litre of paraffin.
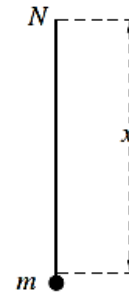
   (a) Find, in terms of x, an expression for the number of litres he bought. [1]

   (b) Due to a leak, he lost 3 litres of paraffin.
He sold the remainder of the paraffin for \$1 per litre more than he paid for it.

   Write down an expression, in terms of x, for the sum of money he received. [2]

   (c) He made a profit of \$20.

     (i) Write down an equation in x to represent this information, and show that it reduces to

$$3x^2 + 23x - 500 = 0.$$ [3]

     (ii) Solve the equation $3x^2 + 23x - 500 = 0$, giving both answers correct to one decimal place. [4]

   (d) Find, correct to the nearest whole number, how many litres of paraffin he sold. [2]

Answers: **(a)** $\dfrac{500}{x}$; **(b)** $\left(\dfrac{500}{x} - 3\right)(x + 1)$; **(c)(ii)** 9.6 and -17.3; **(d)** 49 litres.   **N02/2/Q7**

18   A polar explorer is planning an expedition.
He investigates three possible routes.

(a) If he travels on route A, which is 800 km long, he expects to cover x km per day.

Route B, which is the same distance as route A, has more difficult ice conditions and he would only expect to cover $(x-5)$ km per day.

Route C, which is 100 km longer than route A, has easier conditions and he would expect to cover $(x+5)$ km per day.

Write down an expression, in terms of x, for the number of days that he expects to take on

(i)   route A,

(ii) route B,

(iii) route C.   [2]

(b) He estimates that route C will take 20 days less than route B.

Form an equation in x, and show that it reduces to $x^2 + 5x - 450 = 0$.   [4]

(c) Solve the equation $x^2 + 5x - 450 = 0$,   giving both answers correct to 1 decimal place.   [4]

(d) Calculate the number of days that he expects to take on route A.   [2]

Answers: (a)(i) $\dfrac{800}{x}$, (ii) $\dfrac{800}{x-5}$, (iii) $\dfrac{900}{x+5}$; (c) 18.9 and −23.9; (d) 42.4.   N03/2/Q8

19
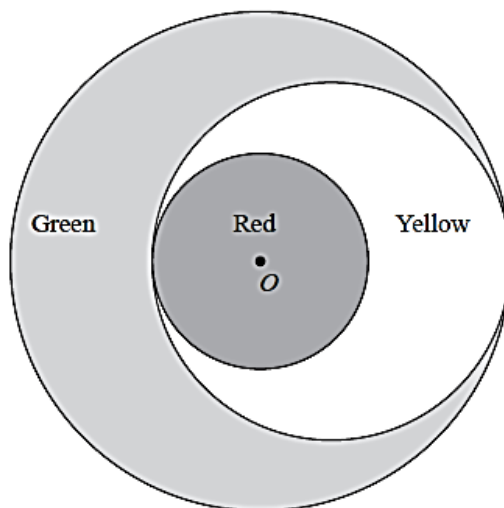
The diagram shows the design of a company symbol.

It consists of three circles.
The smallest circle has centre O and radius $2x$ centimetres.
The largest circle has centre O and radius $2y$ centimetres.
The third circle touches both the other two circles as shown.
The three regions formed are coloured red, yellow and green as shown.

(a) Explain fully why the radius of the third circle is $(x + y)$ centimetres. [2]

(b) Write down, in terms of π, x and y, expressions for the area of the region that is coloured

   (i) yellow, [1]

   (ii) green. [1]

(c) The area of the green region is twice the area of the yellow region.
Use this information to write down an equation involving x and y, and show that it simplifies to

$$y^2 - 6xy + 5x^2 = 0.$$ [3]

(d) (i) Factorise $y^2 - 6xy + 5x^2$. [1]

   (ii) Solve the equation $y^2 - 6xy + 5x^2 = 0$, expressing y in terms of x. [2]

(e) Calculate the fraction of the design that is coloured yellow. [2]

Answers: **(b)(i)** $\pi(x+y)^2 - \pi(2x)^2$, **(ii)** $\pi(2y)^2 - \pi(x+y)^2$; **(d)(i)** $(y-x)(y-5x)$, **(ii)** $y = x$, $y = 5x$; **N04/2/Q8**
(e) $\frac{8}{25}$ or 0.32.

20   A road tanker holds 24 tonnes of oil.

(a) In cold weather it can pump out x tonnes of oil per minute.

Write down an expression, in terms of x, for the number of minutes it takes to empty the tanker in cold weather.   [1]

(b) In hot weather it can pump out $(x + 0.5)$ tonnes of oil per minute.

Write down an expression, in terms of x, for the number of minutes it takes to empty the tanker in hot weather.   [1]

(c) It takes 2 minutes longer to empty the tanker in cold weather than in hot weather.

Write down an equation in x, and show that it simplifies to

$$2x^2 + x - 12 = 0.$$ [3]

(d) Solve the equation $2x^2 + x - 12 = 0$, giving the solutions correct to 3 decimal places.   [4]

(e) Find the time taken, in minutes and seconds, correct to the nearest second, to empty the tanker in cold weather.   [2]

Answers: **(a)** 24/x; **(b)** 24/($x + 0.5$); **(d)** 2.212 and –2.712; **(e)** 10 minutes 51 seconds.   **N05/2/Q6**

---

21   (a) Solve the equation $7a^2 + 12a - 11 = 0$, giving your answers correct to two decimal places.   [4]

(b) Ann drove for 4 hours at an average speed of x km/h and then for 6 hours at an average speed of y km/h.

She drove a total distance of 816 km.

(i) Write down an equation in terms of x and y, and show that it simplifies to

$$2x + 3y = 408.$$ [1]

(ii) Ken drove for 3 hours at an average speed of x km/h and then for 5 hours at an average speed of y km/h.

He drove a total distance of 654 km.

Write down an equation, in terms of x and y, to represent this information.   [1]

(iii) Solve these two equations to find the value of x and the value of y.   [3]

Answer: **(a)** 0.66 and – 2.38   **(b)(ii)** $3x + 5y = 654$ **(iii)** $x = 78$ $y = 84$ **N06/2/Q6**

183

22   A light aircraft flew from Maseru to Nata and returned to Maseru.

(a)   The distance from Maseru to Nata is 1080 km.

(i)   On the outward flight, the average speed of the aircraft was x kilometres per hour.
Write down an expression, in terms of x, for the time taken in hours. [1]

(ii)   On the return flight, the average speed was 30 km/h greater than the average speed on the outward flight.
Write down an expression, in terms of x, for the time taken, in hours, on the return flight. [1]

(b)   The time taken on the return flight was half an hour less than the time taken on the outward flight.
Form an equation in x and show that it reduces to $x^2 + 30x - 64800 = 0$. [3]

(c)   Solve the equation $x^2 + 30x - 64800 = 0$. [4]

(d)   Calculate

(i)   the time taken, in hours, on the outward flight, [1]

(ii)   the average speed for the whole flight from Maseru to Nata and back to Maseru. [2]

Answers: (a)(i) $\dfrac{1080}{x}$, (ii) $\dfrac{1080}{x+30}$, (b) The given equation correctly obtained, (c) 240 and **N08/2/Q7**
-270 , (d)(i) 4.5 , (ii) 254.

---

23   A gardener uses *No-Weed* to kill weeds in his lawn.

(a)   He mixes 15 ml of *No-Weed* with 5 litres of water.

Calculate the ratio of *No-Weed* to water in the mixture.
Express the ratio in the form $m : n$, where m and n are the smallest possible whole numbers. [1]

(b)   The gardener's lawn is a rectangle of length 27 metres and width 25 metres.

(i)   15 ml of *No-Weed* are required to kill weeds in 10 m² of lawn.
No-Weed is sold in 500 ml bottles.

(a)   How many bottles does the gardener need to buy? [3]

(b)   How many millilitres are not used? [1]

(ii)   The lawn is to be made larger.
The length is increased by $3x$ metres and the width by x metres.
The area of the larger lawn is twice that of the original lawn.

(a)   Form an equation in x and show that it reduces to $x^2 + 34x - 225 = 0$. [2]

(b)   Solve the equation $x^2 + 34x - 225 = 0$, giving each answer correct to 2 decimal places. [4]

(c)   Hence find the length of the larger lawn. [1]

Answers: (a) 3 : 1000 (b)(i)(a) 3 (b) 487.5 (ii)(b) 5.67, -39.67 (c) 44.0 **N10/21/Q11**

24
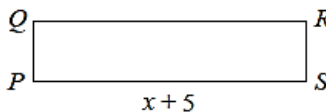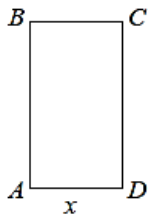
ABCD and PQRS are rectangles.
Each rectangle has an area of 13 cm².
$AD = x$ centimetres and $PS = (x + 5)$ centimetres.

(a) Find, in terms of x, an expression for

    (i) *AB*, [1]

    (ii) *PQ*. [1]

(b) Given that *AB* is 3 cm greater than *PQ*, form an equation in x and show that it simplifies to

$$3x^2 + 15x - 65 = 0.$$ [3]

(c) Solve the equation $3x^2 + 15x - 65 = 0$, giving each answer correct to 2 decimal places. [4]

(d) (i) Show that the perimeter of *ABCD* is 14.9 cm, correct to 3 significant figures. [1]

    (ii) Find the difference between the perimeters of the two rectangles. [2]

Answers: (a)(i) $\dfrac{13}{x}$ (ii) $\dfrac{13}{x+5}$ (c) 2.78 and – 7.78   (d)(ii) 4 cm.   **N10/22/Q7**

25

A piece of wire, 28 cm in length, is cut into two parts.
One part is used to make a rectangle and the other a square.

The length of the rectangle is three times its width.
The width of the rectangle is x centimetres.

(a) (i) Write down an expression, in terms of x, for the length of the rectangle.

    Answer .... cm [1]

    (ii) Find, and simplify, an expression, in terms of x, for the length of a side of the square.

    Answer .... cm [2]

185

(b) It is given that the area of the rectangle is equal to the area of the square.

    (i) Form an equation in x and show that it reduces to $x^2 - 28x + 49 = 0$.   **[2]**

<div align="center">21</div>

    (ii) Solve the equation $x^2 - 28x + 49 = 0$, giving each solution correct to 3 significant figures.   *Answer* $x = $ .... or .... **[4]**

  (iii) Which solution represents the width of the rectangle?
       Give a reason for your answer.

       Answer The width of the rectangle is ....cm because ....

       ....**[2]**

  (iv) Calculate the area of the square.   *Answer* ....cm² **[1]**

Answers: (a)(i) $3x$ (ii) $7 - 2x$ (b)(ii) 1.88 and 26.1 (iii) 1.88 (iv) 10.6

---

26   London is 320 km from York.
A train travels from York to London at an average speed of x kilometres per hour.

  (a) Write down an expression, in terms of x, for the time taken, in hours, for this train to travel from York to London.
                                       Answer .... h **[1]**

  (b) A car travels from York to London.
     The average speed of the car is 80 km/h slower than the average speed of the train.

     Write down an expression, in terms of x, for the time taken, in hours, for the car to travel from York to London.

                                         Answer ....h **[2]**

  (c) The car took $2\frac{1}{2}$ hours longer than the train.

     Form an equation in x and show that it simplifies to $x^2 - 80x - 10240 = 0$.   **[3]**

<div align="center">21</div>

  (d) Solve this equation, giving each solution correct to 1 decimal place.

                               Answer $x = $ .... or .... **[4]**

  (e) Hence find the time taken by the **train** to travel from York to London.
     Give your answer in hours and minutes, correct to the nearest minute.

                       Answer .... hours .... minutes **[2]**

Answer: (a) $\dfrac{320}{x}$ (b) $\dfrac{320}{x-80}$ (d) 148.8 -68.8 (e) 2 h 9 mins

---

27 (a) Solve

(i) $\dfrac{4x}{3} = 1$,

Answer $x =$ .... [1]

(ii) $4y - 3(2y + 1) = 5$.

Answer $y =$ .... [2]

(b) Simplify $\dfrac{15w^2 - 30w}{5w^2 - 20}$.

Answer .... [3]

(c)
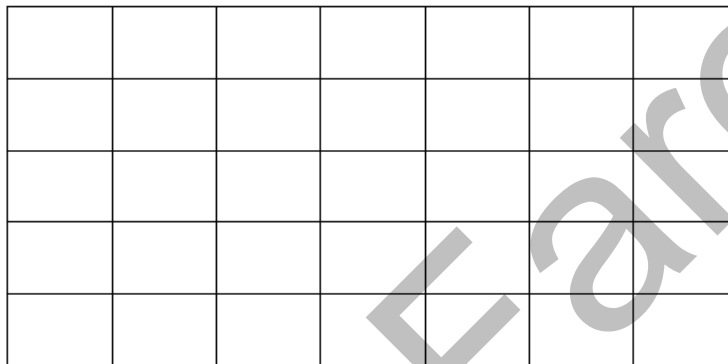
The diagram shows the plan of a patio made from rectangular paving slabs.
The width of each paving slab is p cm.
The length of each paving slab is 20 cm longer than its width.

(i) Find an expression, in terms of p, for the area, in cm^2, of **one** paving slab.

Answer .... cm^2 [1]

(ii) Given that the area of the patio is 12.25 m^2, show that p satisfies the equation

$$p^2 + 20p - 3500 = 0 .$$
[2]

(iii) (a) Solve by factorisation $p^2 + 20p - 3500 = 0$. Answer $p =$ .... or .... [2]

(b) Hence state the **length** of each paving slab.

Answer .... cm [1]

Answers: (a)(i) 0.75 (ii) −4 (b) $\dfrac{3w}{w+2}$ (c)(i) $p(p+20)$ (ii) The given equation correctly derived. **N14/21/Q7**

(iii)(a) $p = 50$ $q = -70$ (b) 70

28 **(a)**
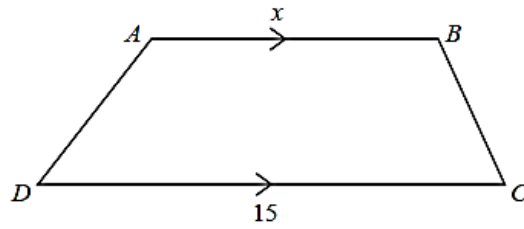
$ABCD$ is a trapezium with AB parallel to DC.
$DC = 15$ cm and $AB = x$ cm.
The perpendicular distance between AB and DC is 3 cm less than the length of AB.
The area of $ABCD$ is 75 cm².

(i) Show that $x^2 + 12x - 195 = 0$.

[2]

(ii) Find AB, giving your answer correct to 1 decimal place.

Answer .... cm [3]

(iii) AD is 0.8 cm longer than BC.

Given that the perimeter of the trapezium is 38.0 cm, calculate AD.

Answer .... cm [2]

(b) Another trapezium, $LMNO$, has LM parallel to ON.
The reflex angle $LMN = 252°$.

(i) Calculate $M\hat{N}O$.

Answer .... [2]
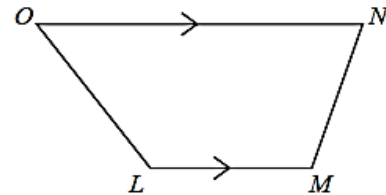
(ii) The ratios of the angles inside the trapezium are $L\hat{O}N : L\hat{M}N = 1:2$ and $O\hat{L}M : M\hat{N}O = 1:k$.

Find k, giving your answer as a fraction in its simplest form.

Answer .... [3]

Answers: (a)(i) $x^2 + 12x - 195 = 0$ correctly derived; (ii) 9.2; (iii) 7.3; (b)(i) 72°; (ii) $\frac{4}{7}$

29 On Monday, Abdul sold 140 boxes of matches at 30 cents per box.

(a) Calculate the income, in dollars, Abdul received on Monday.

Answer $ .... [1]

(b) On Tuesday, the price per box decreased by 10% and the number of boxes sold increased by 30%.

Calculate the percentage change in the income.

Answer .... % [3]

(c) On Wednesday, the price of a box was y cents less than it was on Monday.
Abdul sold $4y$ more boxes on Wednesday than he did on Monday.

(i) Write down an expression, in terms of y, for the income received on Wednesday.
Give your answer in dollars.

Answer $ .... [2]

(ii) Given that this income is equal to $40, write down an equation in y and show that it simplifies to

$$y^2 + 5y - 50 = 0.$$

[2]

(iii) Solve the equation $y^2 + 5y - 50 = 0$.

Answer $y =$ .... or .... [3]

(iv) Hence find the number of boxes sold on Wednesday.

Answer .... [1]

Answers: (a) 42 (b) 17 (c)(i) $\dfrac{(30-y)(140+4y)}{100}$ (ii) $y^2 + 5y - 50 = 0$ correctly derived (iii) −10 or 5 **N16/21/Q9**
(iv) 160

30 (a) A pump takes 12 minutes to add 3000 litres of water to a pond.

How long will it take the same pump to add 1750 litres of water to a pond?

Answer .... minutes [2]

(b) A tank holds 2500 litres of oil.
A small pump can add oil to the tank at a rate of x litres per minute.
A large pump can add oil to the tank at a rate of $(x + 20)$ litres per minute.

(i) Write down an expression, in terms of x, for the number of minutes the small
pump takes to fill the empty tank.

Answer .... [1]

(ii) It takes 15 minutes longer to fill the empty tank using the small pump than it
does with the large pump.

Form an equation in x and show that it simplifies to $3x^2 + 60x - 10\,000 = 0$. [3]

(iii) Solve the equation $3x^2 + 60x - 10\,000 = 0$.
Give each answer correct to 2 decimal places.

Answer $x =$ .... or .... [3]

(iv) Find the length of time the large pump takes to fill the empty tank.
Give your answer in minutes and seconds, correct to the nearest second.

Answer .... minutes .... seconds [3]

Answers: **(a)** 7 **(b)(i)** $\dfrac{2500}{x}$ **(ii)** $3x^2 + 60x - 10\,000 = 0$ correctly derived **(iii)** 48.59 and −68.59;

(iv) 36 minutes 27 seconds

Volume 3

Lines and Angles Paper 1

1

In the diagram, *EAF* is a straight line and *AB* is parallel to *CD*.

AB bisects $F\hat{A}C$ and $C\hat{A}B = 58°$.

Find the value of

(a) *x*,   *Answer (a) x =* .... [1]

(b) *y*.   *(b) y =* .... [1]
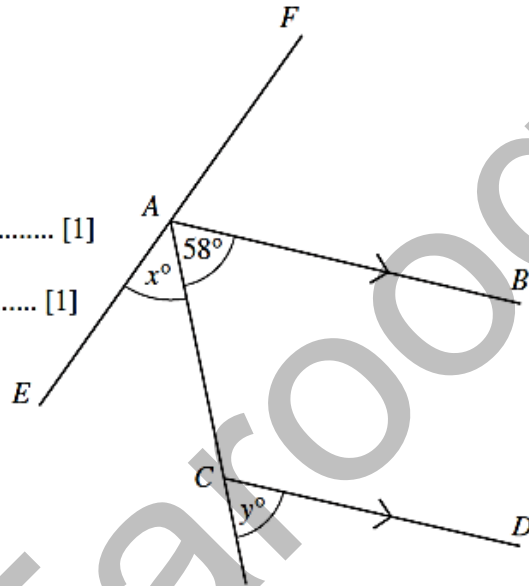
Answers: **(a)** 64°; **(b)** 58°.   **J04/1/Q5**

2

In the diagram, the lines *BA*, *DC* and *EF* are parallel.
$A\hat{B}C = 140°$ and $B\hat{C}F = 115°$.

Find

(a) $D\hat{C}B$,

(b) $D\hat{C}F$,

(c) $E\hat{F}C$.
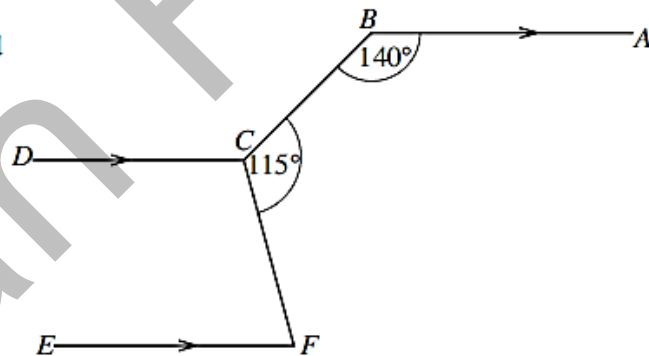
Answer (a) $D\hat{C}B =$ ....[1]

(b) $D\hat{C}F =$ ....[1]

(c) $E\hat{F}C =$ ....[1]

Answer: **(a)** 140° **(b)** 105° **(c)** 75°   **J07/1/Q17**

192

3
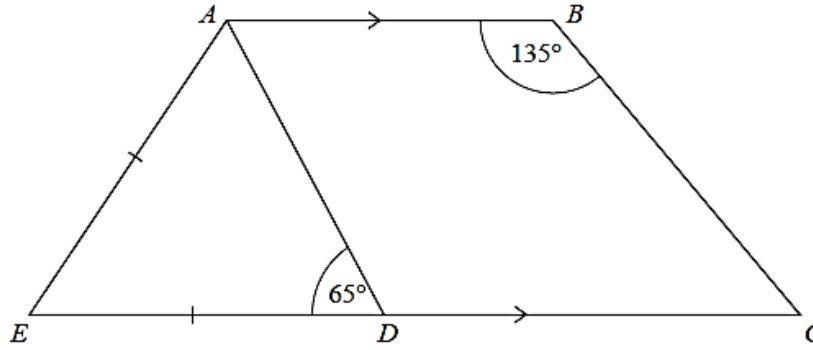
In the diagram, *AB* is parallel to *EC*.
D is the point on *EC* such that *ED* = *EA*.
$A\hat{B}C = 135°$ and $A\hat{D}E = 65°$.

Find

(a) $A\hat{E}D$,

(b) $D\hat{A}B$,

(c) $B\hat{C}D$,

(d) reflex $A\hat{B}C$.

Answer (a) $A\hat{E}D$ = ... [1]

Answer (b) $D\hat{A}B$ = ... [1]

Answer (c) $B\hat{C}D$ = ... [1]

Answer (d) reflex $A\hat{B}C$ = ... [1]

Answer: (a) 50°   (b) 65°   (c) 45°   (d) 225°

J10/11/Q18

4
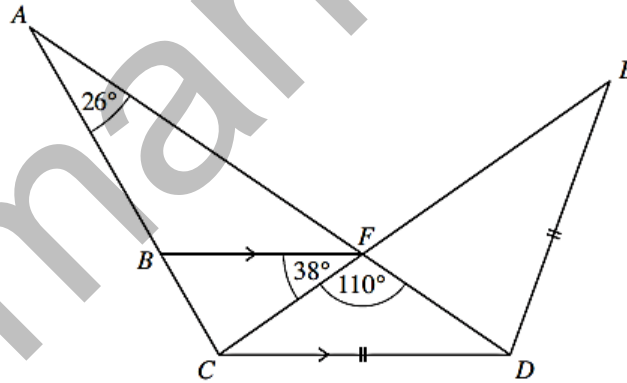
In the diagram, *ACD* and *ECD* are triangles with *CD* = *DE*.
AD and *EC* intersect at *F*.
B is the point on *AC* such that *BF* is parallel to *CD*.
$B\hat{A}F = 26°$, $B\hat{F}C = 38°$ and $C\hat{F}D = 110°$.

193

Find

(a) $F\hat{C}D$,   *Answer* $F\hat{C}D =$ ....[1]

(b) $E\hat{D}C$,   *Answer* $E\hat{D}C =$ ....[1]

(c) $A\hat{B}F$,   *Answer* $A\hat{B}F =$ ....[1]

(d) $B\hat{C}F$.   *Answer* $B\hat{C}F =$ ....[1]

Answers: **(a)** 38 **(b)** 104 **(c)** 122 **(d)** 84   **J11/11/Q17**

5
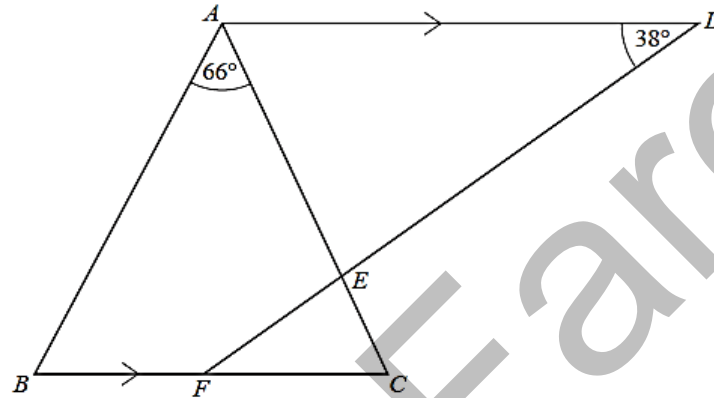
In the diagram $AB = AC$ and AD is parallel to BC.
A line from D intersects AC at E and BC at F.
$A\hat{D}E = 38°$ and $B\hat{A}C = 66°$.

Find

(a) $D\hat{F}C$,   *Answer* $D\hat{F}C =$ ....[1]

(b) $A\hat{B}C$,   *Answer* $A\hat{B}C =$ ....[1]

(c) $A\hat{E}D$.   *Answer* $A\hat{E}D =$ ....[1]

Answers: **(a)** 38° **(b)** 57° **(c)** 85°   **J14/11/Q16**

6
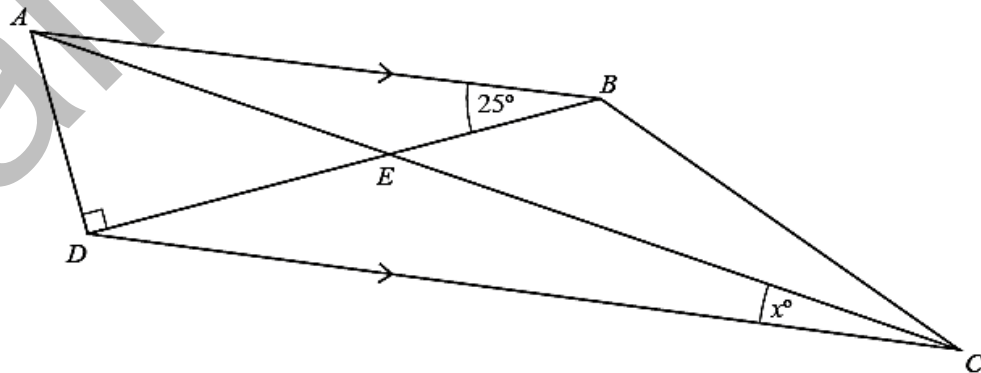
In the diagram AB is parallel to DC.
AC and BD intersect at E.

Triangle ADE is right-angled and isosceles with $AD = DE$.
$A\hat{B}D = 25°$.

Find x.   Answer $x =$ .... [2]

J17/11/Q3

7
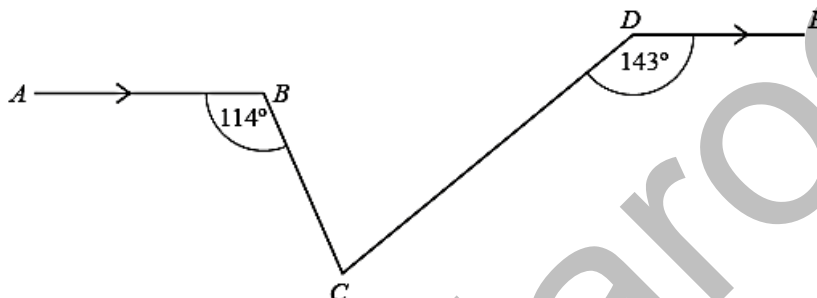
In the diagram AB is parallel to DE.

$A\hat{B}C = 114°$ and $C\hat{D}E = 143°$.

Find $B\hat{C}D$.   Answer $B\hat{C}D =$ .... [2]

J17/11/Q7

8   In the diagram, BCD is a straight line.
$AD = AC$, $D\hat{A}C = 34°$ and $A\hat{B}C = 42°$.

Find

(a) $A\hat{D}C$,

(b) $B\hat{A}C$,

(c) the reflex angle ABC.
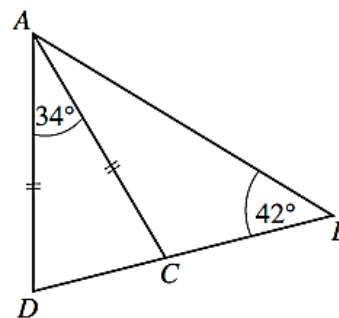
Answer (a) $A\hat{D}C =$ ....[1]

(b) $B\hat{A}C =$ ....[1]

(c) reflex $A\hat{B}C =$ ....[1]

N05/1/Q8

9　**(a)**
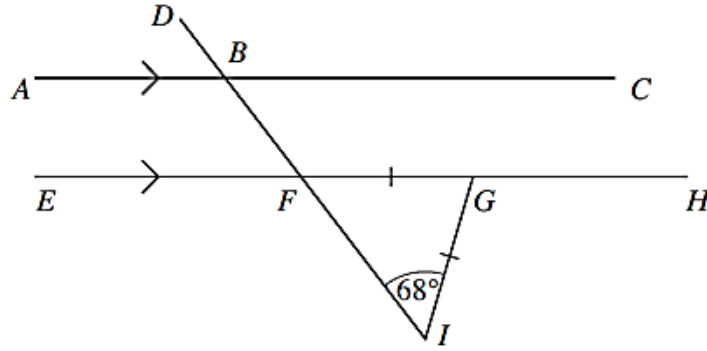
ABC and EFGH are parallel lines.
The line DI intersects AC at B and EH at F.
$F\hat{I}G = 68°$ and $FG = GI$.

Find

(i)　$B\hat{F}G$,

(ii)　$F\hat{G}I$,

(iii)　$D\hat{B}A$.

Answer (a)(i) $B\hat{F}G$ = ...[1]

(ii) $F\hat{G}I$ = ...[1]

(iii) $D\hat{B}A$ = ...[1]

(b)
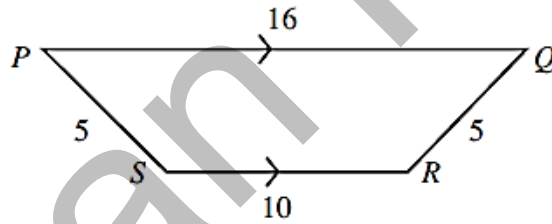
PQRS is a trapezium.
$PS = QR = 5$ cm, $PQ = 16$ cm and $SR = 10$ cm.
Find the area of the trapezium.

Answer (b) .... cm^2 [2]

Answers: **(a)(i)** 112°,　(ii) 44°,　(iii) 68°,　**(b)** 52 cm^2.

10

The diagram shows three parallel lines.

(a) Find x.　Answer (a) x = ...[1]

(b) Find y.　Answer (b) y = ...[1]

(c) Find z.　Answer (c) z = ...[1]
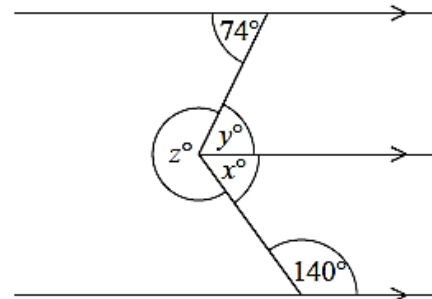
Answers: **(a)** 40 **(b)** 74 **(c)** 246

Lines and Angles paper 2:

1   **(a)** In the diagram, *ABCD* is a parallelogram.
ADE and *BFE* are straight lines.

AF = BF.

$A\hat{B}F = 54°$ and $C\hat{B}F = 57°$.

Find the value of

    (i)   *t*,   [1]

    (ii)   *u*,   [1]

    (iii)   *x*,   [1]
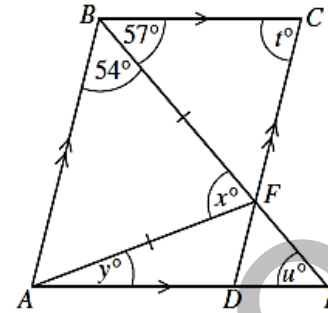
    (iv)   *y*.   [1]

Answers: **(a)** 69°, 57°, 72°, 15°;   **J05/2/Q3**

2   **(a)** Show that each interior angle of a regular octagon is 135°.   [2]

    (b)
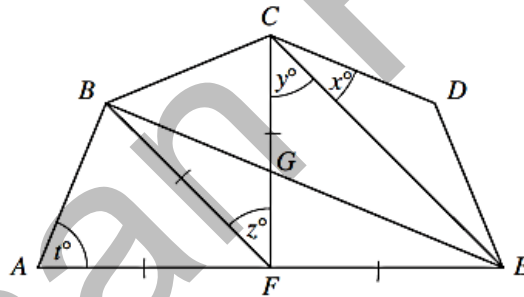
In the diagram, *AB*, *BC*, *CD* and *DE* are four adjacent sides of a regular octagon.
FA = FB = FC = FE.
CF meets *BE* at *G*.

    (i)   Calculate

       (a)   *x*,   [1]

       (b)   *y*,   [1]

       (c)   *z*,   [1]

       (d)   *t*.   [1]

    (ii)   Write down the special name given to the quadrilateral *BCEF*.   [1]

    (iii)   Given that *FC* = 10 cm, calculate *CE*.   [2]

    (iv)   **(a)**   Show that ΔCGE is similar to ΔFGB.   [1]

           (b)   Find $\dfrac{\text{the area of }\Delta CGE}{\text{the area of }\Delta FGB}$.   [1]

3 **(b)**
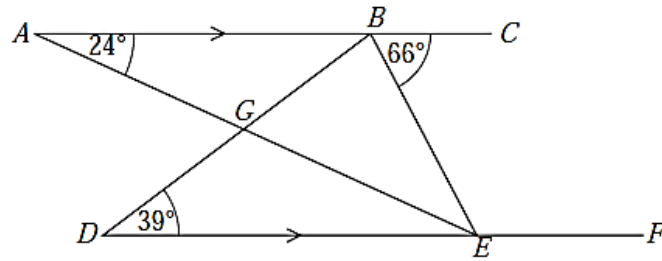
In the diagram, the lines *ABC* and *DEF* are parallel.
AE meets *DB* at *G*.
$B\hat{A}E = 24°$, $C\hat{B}E = 66°$ and $B\hat{D}E = 39°$.

Calculate

(i) $F\hat{E}B$, [1]

(ii) $B\hat{E}A$, [1]

(iii) $A\hat{G}D$. [1]

J09/2/Q4b

(b)(i) 114°, (ii) 42°, (iii) 63°.

4
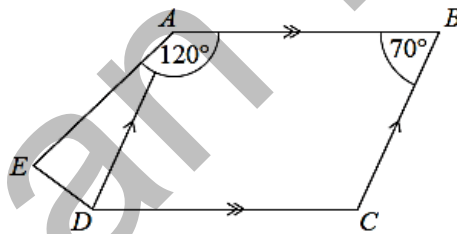
The parallelogram *ABCD* forms part of the pentagon *ABCDE*.
$A\hat{B}C = 70°$ and $B\hat{A}E = 120°$.

(a) Find

(i) $B\hat{C}D$, [1]

(ii) $E\hat{A}D$. [1]

(b) $E\hat{D}C$ is twice $A\hat{E}D$.

Find

(i) $A\hat{E}D$, [3]

(ii) $E\hat{D}A$. [1]

Answers: (a) (i) 110°, (ii) 10°; (b) (i) 80°, (ii) 90°. **J10/22/Q2**

5 (b)
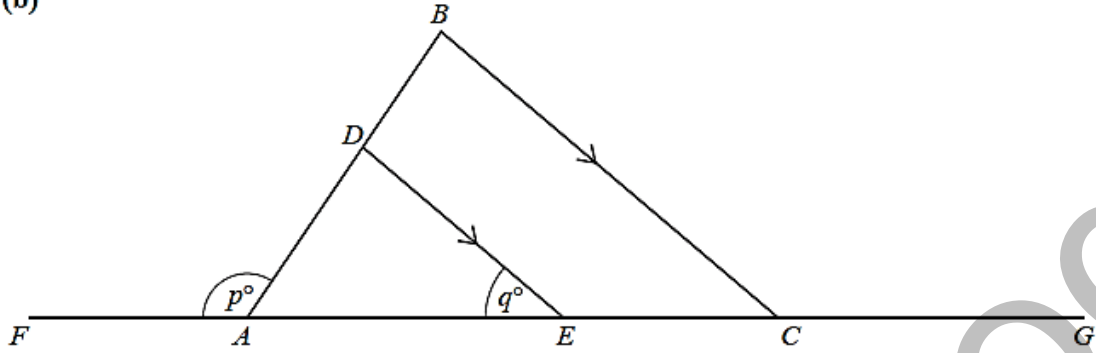
FAECG and ADB are straight lines. DE is parallel to BC.

(i) $F\hat{A}D = p°$ and $A\hat{E}D = q°$.
Find an expression in terms of p and/or q for

(a) $B\hat{C}G$, *Answer* .... [1]

(b) $D\hat{B}C$. *Answer* .... [1]

(ii) $AE = 7\,cm$, $EC = 3\,cm$, $DE = 5.6\,cm$ and $DB = 2.1\,cm$.

(a) Find BC. *Answer* .... cm [1]

(b) Find AD. *Answer* .... cm [1]

(b)(i)(a) $180 - q$ (b) $p - q$ (ii)(a) $8\,cm$ (b) $4.9\,cm$ **J13/21/Q2b**

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6 (a)
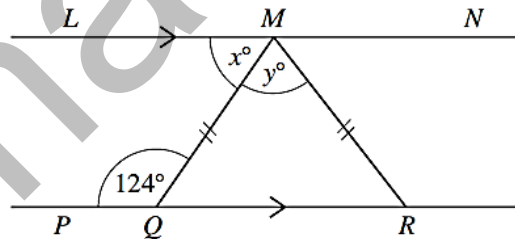
In the diagram, LMN is parallel to PQR.
Angle $PQM = 124°$ and $MQ = MR$.

Find

(i) x, [1]

(ii) y. [1]

Answers: (a)(i) 56°, (ii) 68°, (b)(i) See above (ii) 100 m.   **N07/2/Q3a**

7 **(a)** *AB* and *CD* are parallel.
EGHF is a straight line.
GK bisects *AĜH* and
HK bisects *CĤG*.
EĜB = 40°.
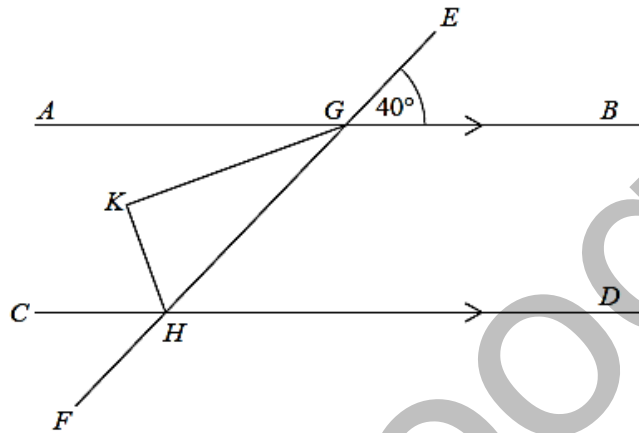
(i) Find *KĜH*.

Answer .... [1]

(ii) Find *GĤK*.

Answer .... [1]

(iii) The bisectors of *HĜB* and *DĤG* intersect at *L*.
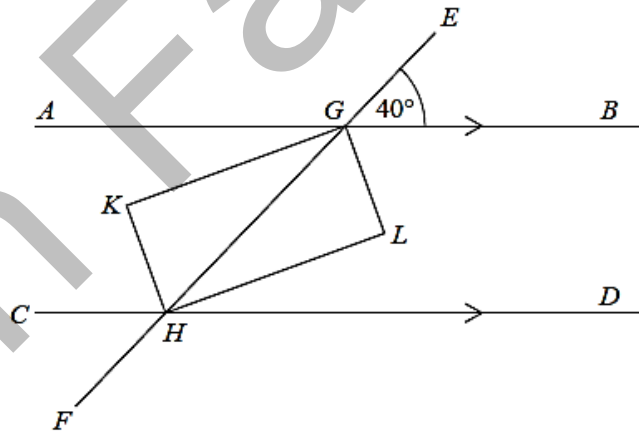
State the name of the special quadrilateral *HKGL* and give your reasons.

Answer *HKGL* is a ....

because ....

....[3]

N12/21/Q4

8
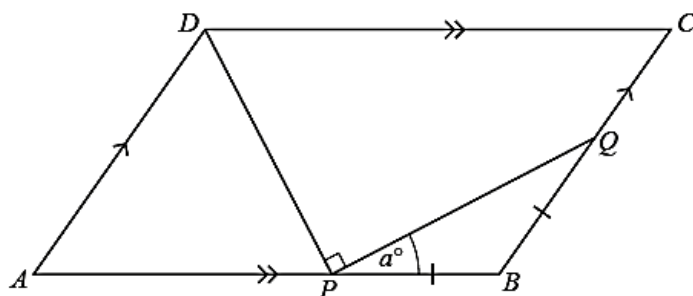
In the diagram, *ABCD* is a parallelogram.
P and *Q* are points on *AB* and *BC* respectively, such that $PB = BQ$ and $D\hat{P}Q = 90°$.
$B\hat{P}Q = a°$.

(a) Find an expression, in terms of *a*, for each of the following angles.
Give each answer in its simplest form.

(i) $P\hat{B}Q$

Answer .... [1]

(ii) $A\hat{P}D$

Answer .... [1]

(iii) $D\hat{A}P$

Answer .... [1]

(iv) $A\hat{D}P$

Answer .... [1]

(b) $AB = 8\,$cm and $AD = 4.7\,$cm.

(i) Find *PB*.

Answer .... cm [1]

(ii) Given also that $D\hat{A}B = 54°$, calculate the area of the parallelogram.

Answer .... cm² [2]

Answers: (a)(i) 180 − 2a (ii) 90 − a (iii) 2a (iv) 90 − a (b)(i) 3.3 (ii) 30.4   **N16/21/Q3**

Angle properties of polygons Paper 1

1

AB and BC are adjacent sides of a regular polygon.
$A\hat{B}C = 140°$.
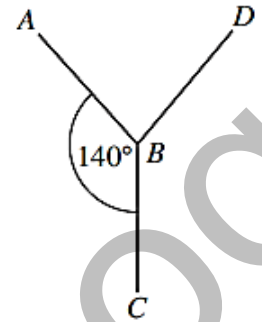
(a) Calculate the number of sides of the polygon.

(b) CB and BD are adjacent sides of a congruent regular polygon.
Calculate $A\hat{B}D$.

Answer (a) ....[2]

(b) $A\hat{B}D = $ ....[1]

Answer: (a)   9   (b)   80°   **J06/1/Q12**

---

2   In the quadrilateral $ABCD$, $\hat{A} = x°$, $\hat{B} = 2x°$, $\hat{C} = 3x°$ and $\hat{D} = 4x°$.

(a) Find x.   *Answer* (a) $x = $...[1]

(b) Explain why AB is parallel to DC.   (b) ....[1]

Answer: (a) 36   **J07/1/Q7**

3

(a) Show that the interior angle of a regular decagon is 144°. [1]

(b) *AB* and *BC* are two sides of a regular decagon.
AB and *BD* are two sides of a regular hexagon.

Work out the value of *x*.
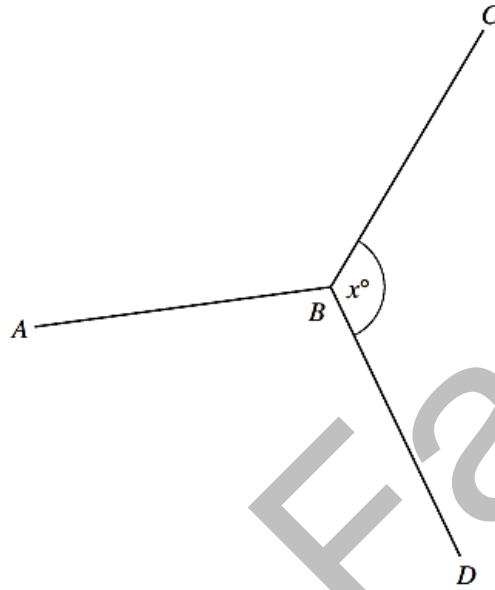
Answer x = ... [2]

4

Each interior angle of a regular polygon is 150°.
Calculate the number of sides of the polygon.

5

The diagram shows part of a regular polygon with *n* sides.
Each interior angle of this polygon is 156°.

Find

(a) the value of *n*,

(b) $A\hat{C}D$,

(c) $A\hat{D}C$.

6

(a) The interior angle of a regular polygon is $160°$.

How many sides does it have? *Answer (a)* .... [2]

(b) *ABCDE......* is part of a regular polygon which has interior angles of $160°$.
CDLM is a square.
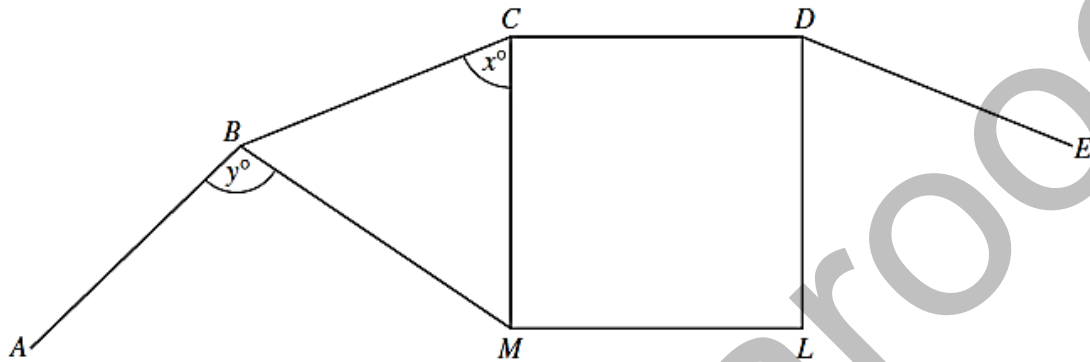
Find
(i) the value of x,

Answer (b) (i) $x =$ .... [1]

(ii) the value of y.

(ii) $y =$ .... [1]

Answers: (a) 18; (b)(i) $x = 70$, (ii) $y = 105$. **N04/1/Q20**

7

(a) Three of the angles of a quadrilateral are each $95°$.
Find the fourth angle. *Answer (a)* .... [1]

(b) Each interior angle of a regular polygon is $165°$.
How many sides has the polygon? *(b)* .... [2]

Answers: (a) 75°; (b) 24. **N05/1/Q10**

8   The 7 sided polygon in the diagram has 6 angles of $x°$ and one of $y°$.
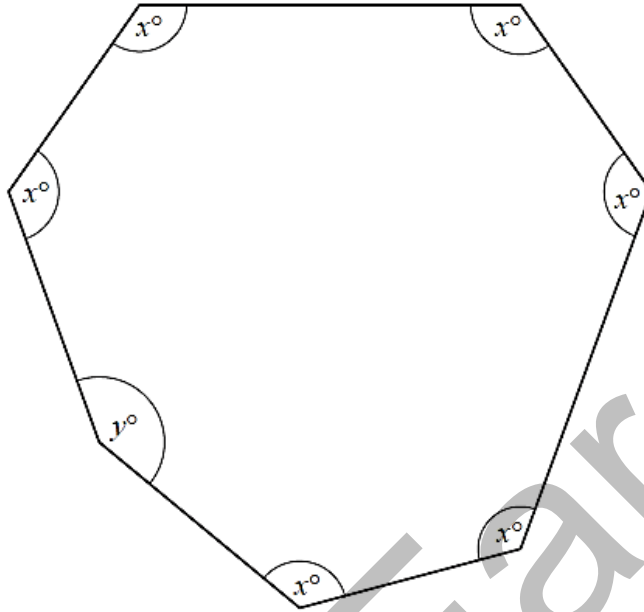
(a) Draw the line of symmetry on the diagram.   [1]

(b) If $y = 126$, calculate the value of x.   *Answer (b) x =* ....[2]

Answer: **(b) 129°.**   **N06/1/Q13**

9
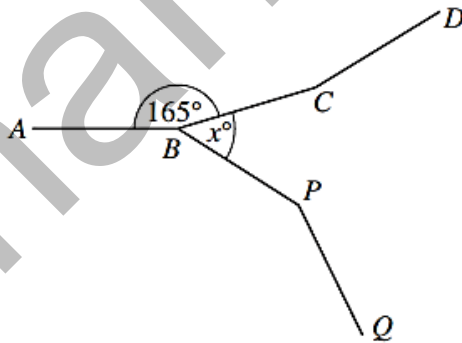
In the diagram, *ABCD* is part of a regular polygon.
Each interior angle is 165°.

(a) How many sides does this polygon have?   *(a)* ....*Answer*[2]

(b) *ABPQ* is part of another regular polygon.
This polygon has 12 sides.
Calculate x.   *(b) x =* ....[2]

Answer: **(a) 24   (b) 45**   **N07/1/Q18**

10 A regular polygon has interior angles of 160°.

(a) Calculate the number of sides of the polygon. *Answer* .... [2]

(b)
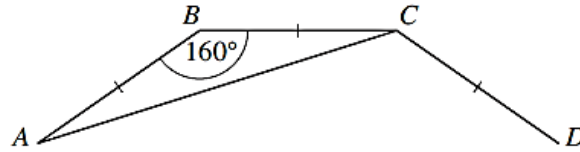
The diagram shows three sides, *AB*, *BC* and *CD*, of this polygon.

(i) Calculate $B\hat{A}C$. *Answer* .... [1]

(ii) Calculate $A\hat{C}D$. *Answer* .... [1]

Answers: (a) 18 (b) (i) 10 (ii) 150

11 Each interior angle of a regular polygon is *p* times each exterior angle.

Find an expression, in terms of *p*, for

(a) an exterior angle, *Answer* .... [1]

(b) the number of sides of the polygon. *Answer* .... [1]

Answer: (a) $\dfrac{180}{p+1}$ (b) $2p + 2$

12 The diagram shows a hexagon.

Find *x*.

ver $x = $ .... [3]

Answer: 76

13 **(a)**
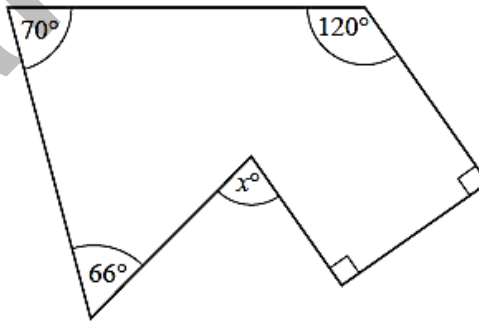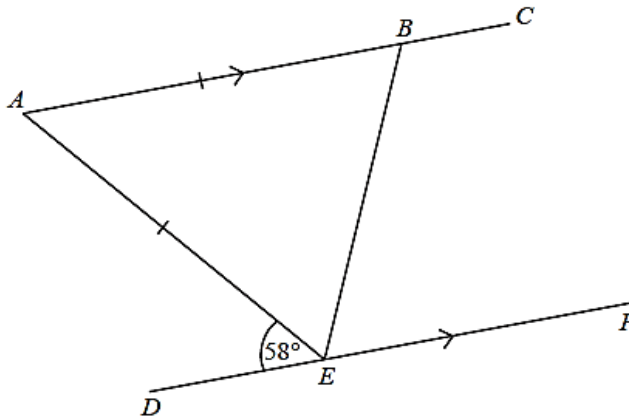
In the diagram the lines ABC and DEF are parallel.
$AB = AE$ and $A\hat{E}D = 58°$.

(i) Complete the statement below.

$E\hat{A}B = 58°$ because ....

.... [1]

(ii) Calculate $E\hat{B}C$.

Answer $E\hat{B}C$ = ... [2]

(b) A pentagon has interior angles of 80°, 95° and 125°.
Each of the remaining angles is equal to $x°$.

Calculate the value of x.

Answer x = ... [2]

Answers: **(a)(i)** ...alternate angles..., **(ii)** 119°; **(b)** 120.

14 Each interior angle of a regular polygon is 171°.

Find the number of sides of the polygon.   *Answer* .... [2]

Answer: 40

Angle Properties of Polygons Paper 2:

1   (a)   Show that the interior angle of a regular hexagon is 120°.   [2]

     (b)   In the diagram, *ABCDEF* is a regular hexagon. *ABPQ* and *FARS* are two squares.
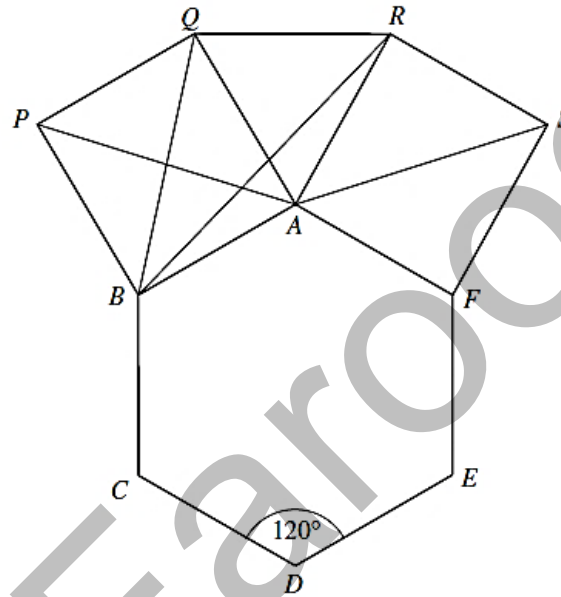
       (i)   Calculate

           (a)   reflex $P\hat{B}C$,   [1]

           (b)   obtuse $P\hat{A}S$,   [2]

           (c)   acute $R\hat{B}A$.   [2]

       (ii)   What is the special name given to triangle *AQR*?   [1]

2   (a)

In the diagram, the 9-sided polygon has 6 angles of $x°$ and 3 angles of $y°$.
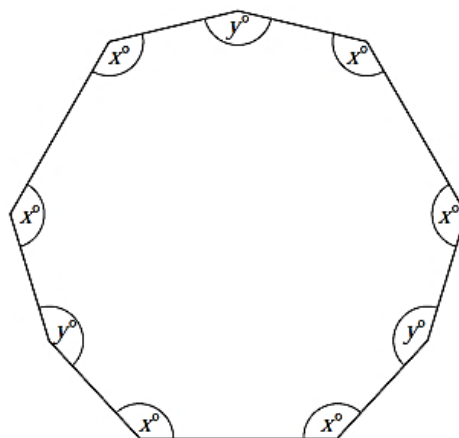
       (i)   For this polygon, state

           (a) the number of lines of symmetry,   [1]

(b) the order of rotational symmetry. [1]

(ii) **(a)** Show that the sum of the interior angles of a 9-sided polygon is 1260°. [1]

(b) Find an expression for y in terms of x. [2]

(c) Given also that $y = 12 + x$, find x. [2]

Answers: **(a)(i)(a)** 3, **(i)(b)** 3, **(ii)(b)** $y = 420 - 2x$, **(ii)(c)** 136; **J09/2/Q4**

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3 **(a)** The interior angle of a regular polygon is 165°.

How many sides has the polygon? *Answer* .... [2]

Answer: **(a)** 24 **J13/21/Q2**

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4 **(b)**
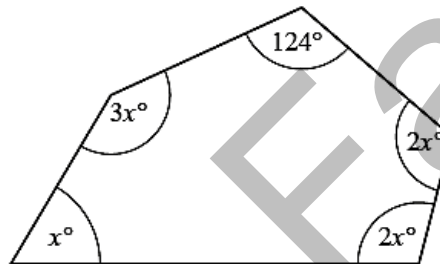
Work out the size of the largest angle in the pentagon.

Answer .... [3]

(c)
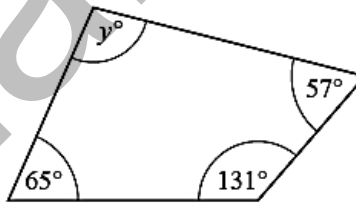
The angles in the quadrilateral are given correct to the nearest degree.

Find the lower bound for the value of y. *Answer* .... [2]

Answers: **(a)(i)** 38° **(ii)** 38°, angles in same segment are equal **(iii)** 112° **(iv)** 106° **(b)** 156° **(c)** 105.5° **N17/21/Q6**

Angle Properties of Circles Paper 1

1 The points A, P, Q, B and S lie on a circle.
AP is produced to R.
$R\hat{P}Q = 69°$, $P\hat{B}A = 24°$ and $Q\hat{S}B = 21°$.
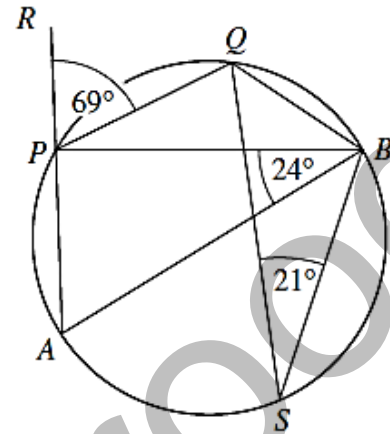
(a) Find $Q\hat{P}B$, giving your reasons.

(b) Given that M is the midpoint of AB, find $P\hat{M}A$.
Give all your reasons.

Answer (a) $QP\hat{\ }B = $ .... because ....

....[1]

(b) $P\hat{M}A = $ .... because ....

....[3]

2
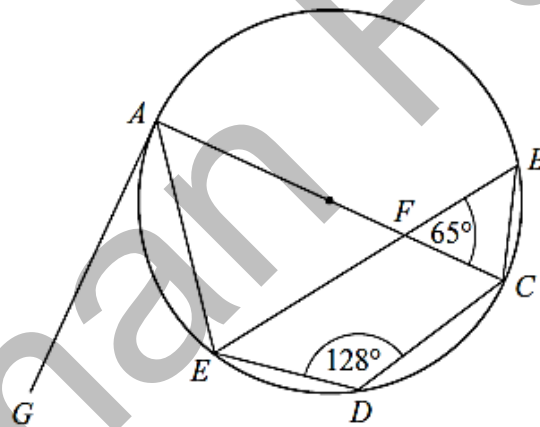
The points A, B, C, D and E lie on the circle with diameter AC.
EB and AC meet at F.
GA is a tangent to the circle at A.
$C\hat{D}E = 128°$ and $B\hat{F}C = 65°$.
Calculate

(a) $G\hat{A}E$,

(b) $A\hat{E}B$.

Answer (a) $G\hat{A}E = $ .... [2]

(b) $A\hat{E}B = $ .... [1]

Answers: **(a)** 38°; **(b)** 63°.

3
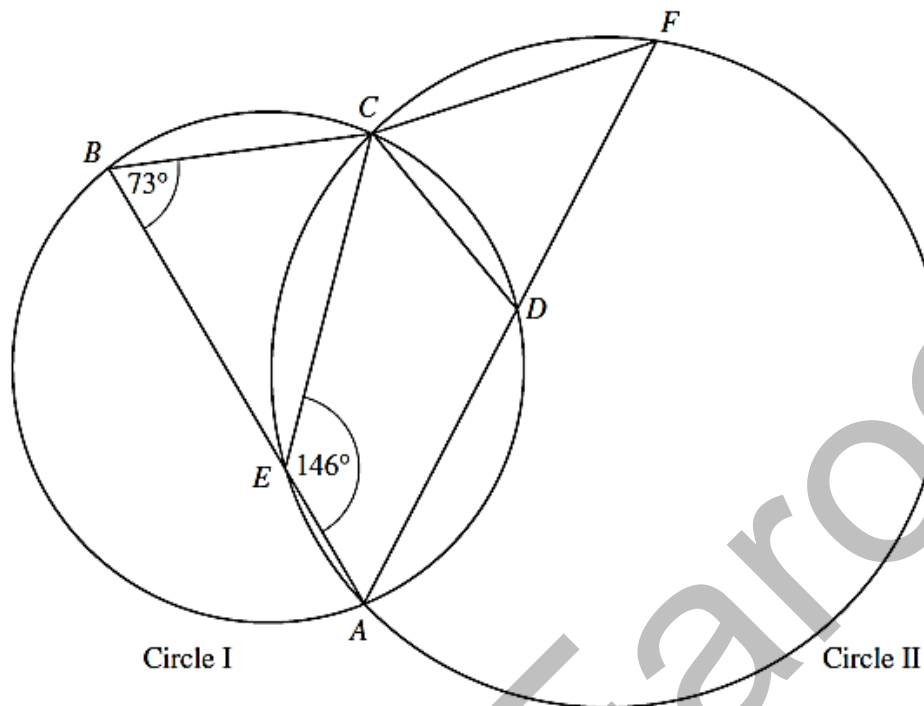
Circle I   Circle II

In the diagram, the points A, B, C and D lie on circle I.
The points A, E, C and F lie on circle II.
AEB and ADF are straight lines.
$E\hat{B}C = 73°$ and $A\hat{E}C = 146°$.

(a)   Calculate

(i) $A\hat{D}C$,

(ii) $C\hat{F}A$.

(b)   Explain why the centre of circle I lies on circle II.

Answer (*a*)(i) .... [1]

(ii) .... [1]

Answers: **(a)(i)** 107°, **(ii)** 34° .   **J04/1/Q22**

212

4 A, B and S are points on a circle, centre O.
TA and TB are tangents.
$A\hat{T}B = 52°$.

Calculate

(a) $A\hat{O}B$,

(b) $O\hat{B}A$,

(c) $A\hat{S}B$.
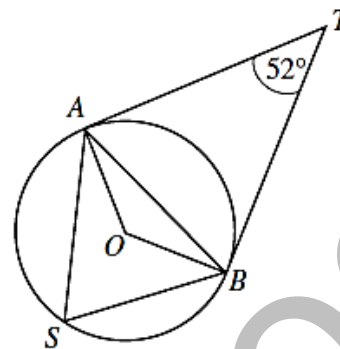
Answer (a) $A\hat{O}B$ = ... [1]

(b) $O\hat{B}A$ = ... [1]

(c) $A\hat{S}B$ = ... [1]

Answers: (a) 128°; (b) 26°; (c) 64°.   **J05/1/Q14**

---

5 A, B, C and D are points on a circle with BD as diameter.
TD is a tangent at D and $T\hat{D}A = 36°$.

Find   Answer (a) $A\hat{D}B$ =..[1]

(a) $A\hat{D}B$,

(b) $A\hat{B}D$,   (b) $A\hat{B}D$ = ... [1]

(c) $A\hat{C}D$.   (c) $A\hat{C}D$ = ... [1]
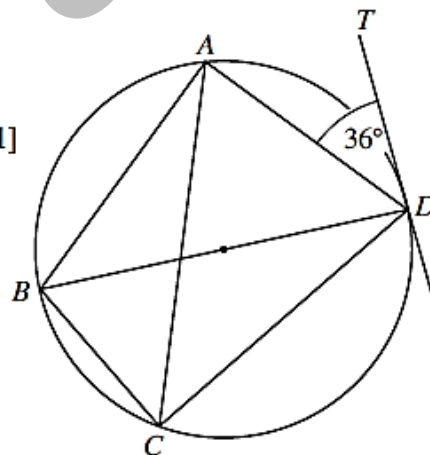
Answer: (a) 54°   (b) 36°   (c) 36°   **J06/1/Q18**

213

6
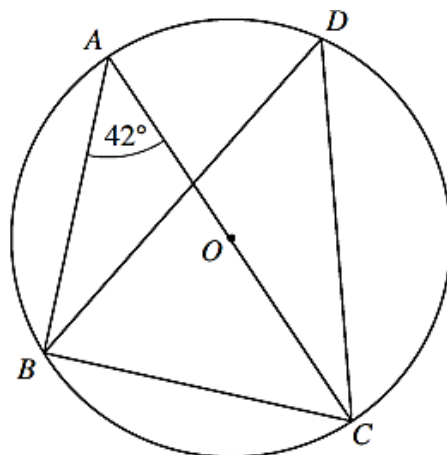
The diagram shows a circle, centre O, passing through A, B, C and D.
AOC is a straight line and $B\hat{A}C = 42°$.

Find

(a) $B\hat{D}C$,

(b) $A\hat{B}C$,

(c) $A\hat{C}B$.

Answer (a) $B\hat{D}C =$ ....[1]

(b) $A\hat{B}C =$ ....[1]

(c) $A\hat{C}B =$ ....[1]

Answers: (a) 42°   (b) 90°   (c) 48°   **J08/1/Q9**

7
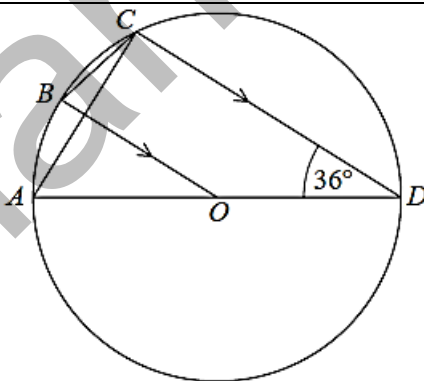
The diagram shows a circle, centre O, passing through A, B, C and D.
AOD is a straight line, BO is parallel to CD and $C\hat{D}A = 36°$.

Find

(a) $B\hat{O}A$,

(b) $B\hat{C}A$,

Answer (a) $B\hat{O}A$ ....[1]

Answer (b) $B\hat{C}A$ ....[1]

(c) $D\hat{C}B$,

(d) $O\hat{B}C$.

Answer: **(a)** 36   **(b)** 18   **(c)** 108   **(d)** 72   **J09/1/Q14**

8
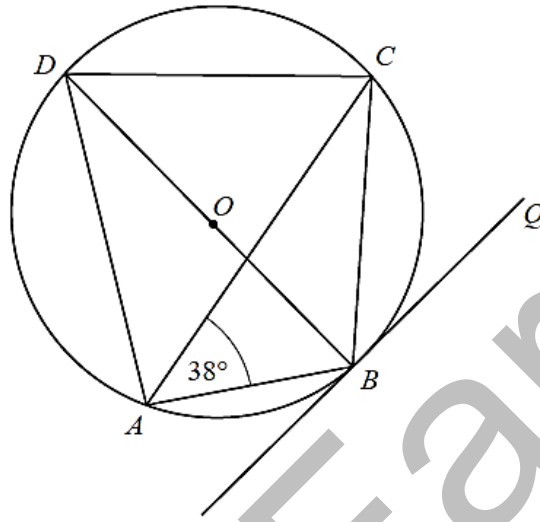
The diagram shows a circle, centre O, passing through A, B, C and D.
BOD is a straight line and $B\hat{A}C = 38°$.
The line BQ is a tangent to the circle at B.

Find

(a) $D\hat{A}C$,

Answer (a) $D\hat{A}C =$ .... [1]

(b) $D\hat{B}C$,

Answer (b) $D\hat{B}C =$ .... [1]

(c) $C\hat{B}Q$.

Answer (c) $C\hat{B}Q =$ .... [1]

Answer: **(a)** 52   **(b)** 52   **(c)** 38   **J10/12/Q15**

9
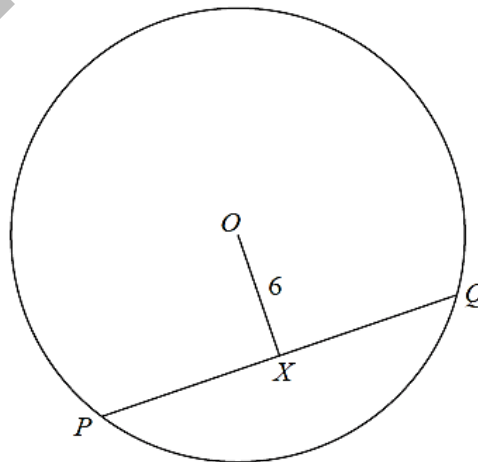
PQ is a chord of the circle, centre O.
X is the midpoint of PQ.
$OX = 6\,\text{cm}$ and the radius of the circle is $10\,\text{cm}$.

Calculate PQ.

Answer .... cm [2]

10
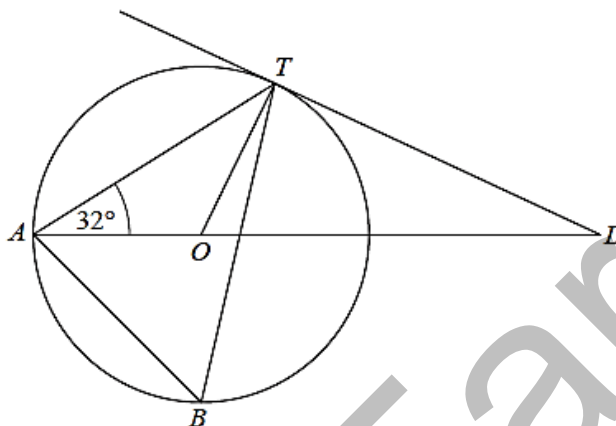
A, B and T are points on a circle, centre O.
AOD is a straight line and DT is a tangent to the circle at T.
$T\hat{A}O = 32°$

Find

(a) $A\hat{T}O$,

Answer $A\hat{T}O$ = ... [1]

(b) $T\hat{D}O$,

Answer $T\hat{D}O$ = ... [1]

(c) $A\hat{B}T$.

Answer $A\hat{B}T$ = ... [1]

11
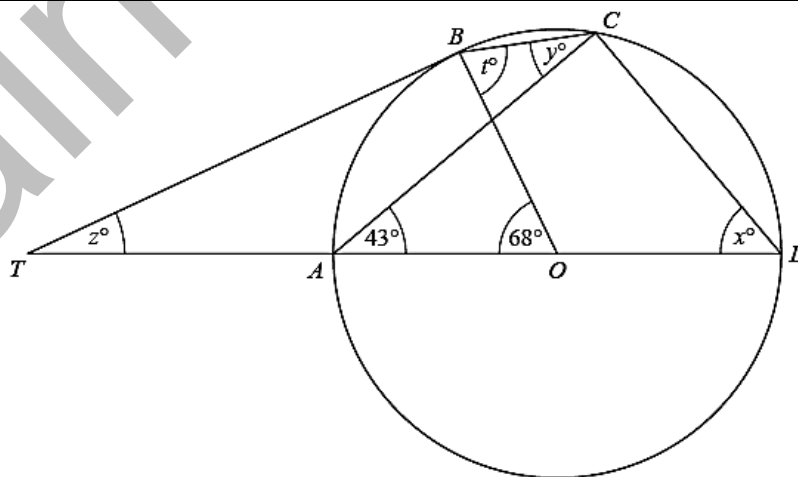
In the diagram, A, B, C and D lie on the circle, centre O.
AD is a diameter.
The tangent to the circle at B meets the line DA produced at T.

$A\hat{O}B = 68°$ and $C\hat{A}O = 43°$.

(a) Find x.

Answer $x =$ .... [1]

(b) Find y.

Answer $y =$ .... [1]

(c) Find z.

Answer $z =$ .... [1]

(d) Find t.

Answer $t =$ .... [1]

Answers: **(a)** 47 **(b)** 34 **(c)** 22 **(d)** 77   **J15/11/Q19**

12
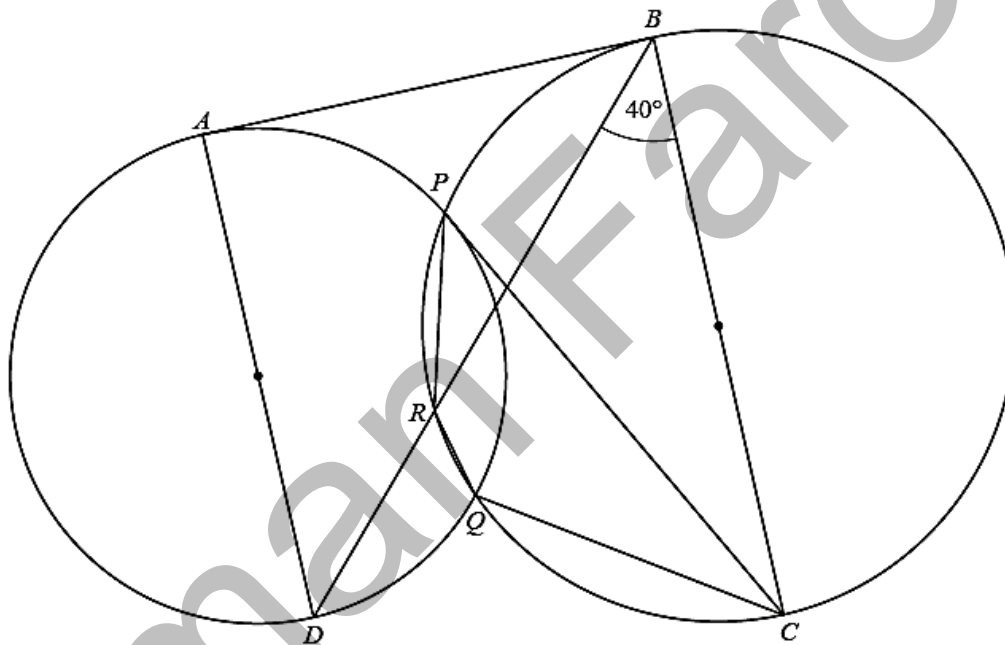
In the diagram, the two circles intersect at P and Q.
The line AB is a tangent to the circles at A and B.
AD and BC are diameters.
BD intersects the larger circle at R.

$D\hat{B}C = 40°$.

(a) Find $C\hat{P}R$.

Answer $C\hat{P}R =$ .... [1]

(b) Find $C\hat{Q}R$.

Answer $C\hat{Q}R =$ .... [1]

(c) Find $A\hat{B}D$.

Answer $A\hat{B}D =$ .... [1]

(d) Find $A\hat{D}B$.

Answer $A\hat{D}B =$ .... [1]

217

13

In the diagram, TB is a tangent to the circle, centre O.

TO meets the circle at C and A. D is another point on the circle. $B\hat{T}C = 22°$.
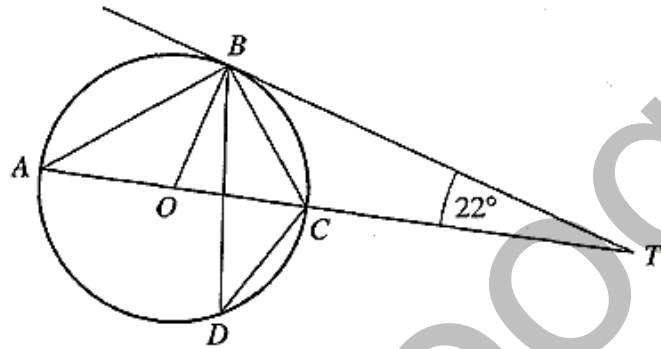
(a) Use the letters in the diagram to name two right angles.

(b) Find

    (i) $O\hat{A}B$,

    (ii) $A\hat{B}T$,

    (iii) $B\hat{D}C$.

14
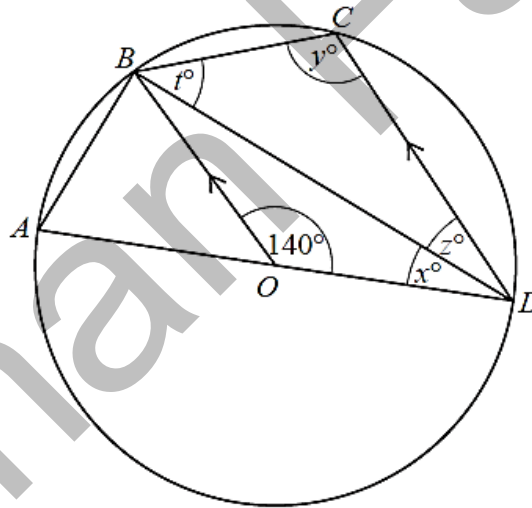
In the diagram, the points A, B, C and D lie on a circle, centre O. AOD is a diameter, OB is parallel to DC and $B\hat{O}D = 140°$.
Find

(a) x,

    Answer (a) $x =$ ....[1]

(b) y,

    (b) $y =$ ....[1]

(c) z,

    (c) $z =$ ....[1]

(d) t.

    (d) $t =$ ....[1]

15
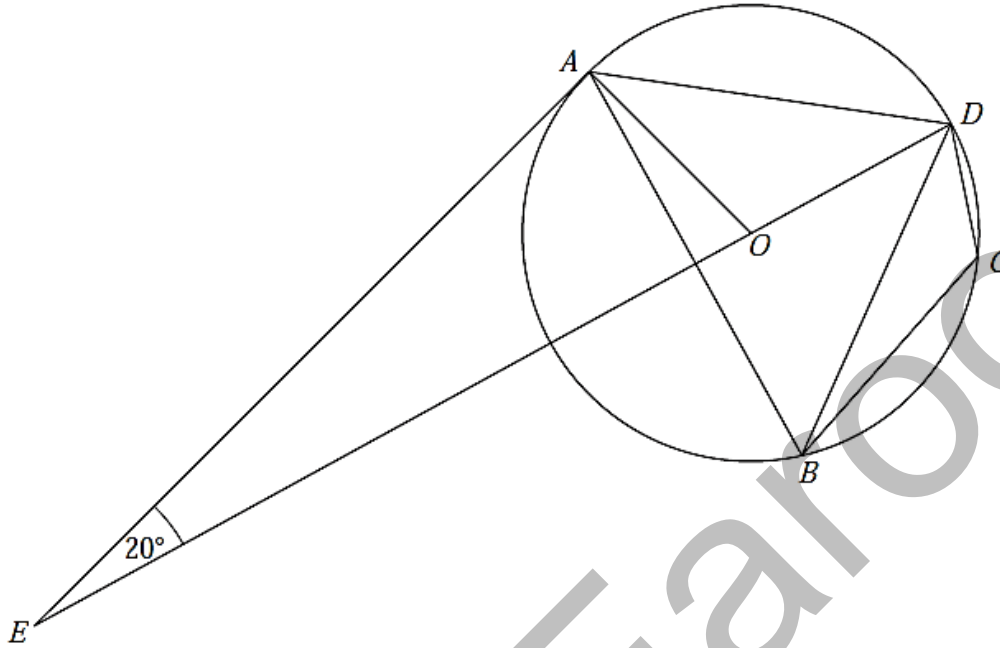
The quadrilateral *ABCD* has its vertices on the circumference of a circle.
AE is a tangent to the circle and $A\hat{E}D = 20°$.
The centre of the circle, *O*, lies on the straight line *DE*.

(a) Find $A\hat{D}O$.

Answer (a) $A\hat{D}O = $ ....[2]

(b) Given that *DE* is the perpendicular bisector of *AB* and $D\hat{B}A = 55°$,

   (i) write down $B\hat{A}D$,

Answer (b)(i) $B\hat{A}D = $ ....[1]

   (ii) find $B\hat{C}D$.

Answer (b)(ii) $B\hat{C}D = $ ....[1]

16 In the diagram, A, B, C, D, E and F
lie on the circle, centre O.
AD and FC are diameters, and $F\hat{C}D = 57°$.
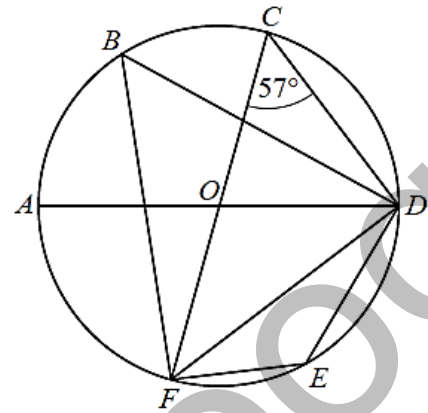
Find

(a) $D\hat{E}F$,   *Answer* *(a)* $D\hat{E}F =$ ....[1]

(b) $F\hat{B}D$,   *Answer* *(b)* $F\hat{B}D =$ ....[1]

(c) $C\hat{F}D$,   *Answer* *(c)* $C\hat{F}D =$ ....[1]

(d) $A\hat{O}F$.   *Answer* *(d)* $A\hat{O}F =$ ....[1]

Answer: **(a)** 123° **(b)** 57° **(c)** 33° **(d)** 66°   **N10/12/Q23**

17

In the diagram, AB touches the
circle, centre O, at T.

OB intersects the circle at C.
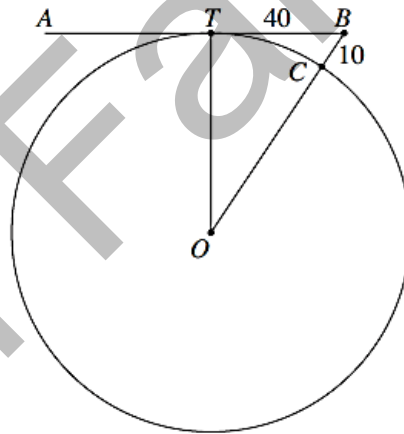
(a) State, with a reason, the value of $B\hat{T}O$.

Answer $B\hat{T}O =$ .... because ....

.... [1]

(b) Given that $TB = 40\,cm$, $CB = 10\,cm$, and the radius of the circle is x centimetres,
form an equation in x, and hence find the radius of the circle.

Answer .... cm [4]

Answers: **(a)** 90° ...tangent- radius property **(b)** 75   **N11/11/Q26**

18

In the diagram, the points A, B, C and D lie on the circle, centre O.

$B\hat{A}D = 47°$.

Find

(a) $B\hat{O}D$, Answer $B\hat{O}D$ = ... [1]

(b) $B\hat{C}D$, Answer $B\hat{C}D$ = ... [1]

(c) $O\hat{B}D$. Answer $O\hat{B}D$ = ... [1]
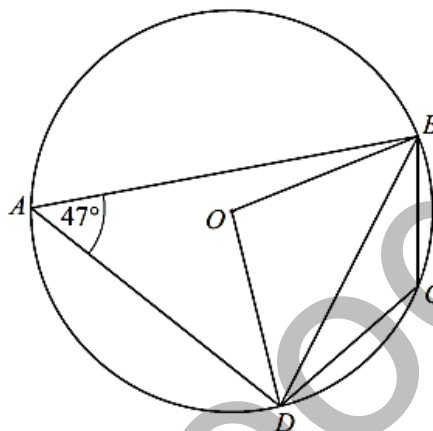
Answer: (a) 94° (b) 133° (c) 43°

19
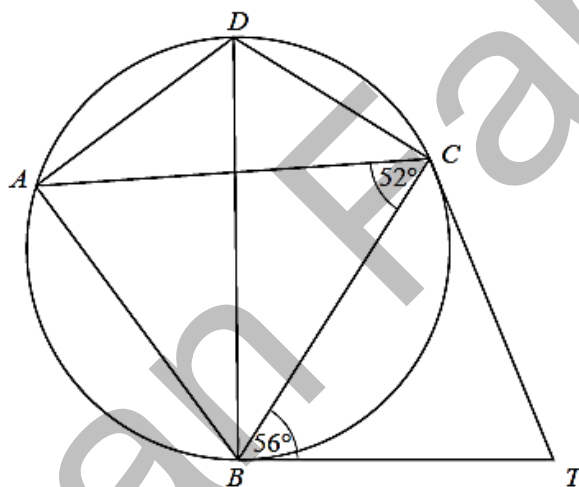
In the diagram, the points A, B, C and D lie on the circle.
BD is a diameter.
The tangents from T touch the circle at B and C.
$A\hat{C}B = 52°$ and $T\hat{B}C = 56°$.

Find

(a) $B\hat{T}C$,   Answer $B\hat{T}C$ = ... [1]

(b) $A\hat{D}B$,   Answer $A\hat{D}B$ = ... [1]

(c) $B\hat{D}C$,   Answer $B\hat{D}C$ = ... [1]

(d) $A\hat{B}C$.   Answer $A\hat{B}C$ = ... [1]

Answers: (a) 68 (b) 52 (c) 56 (d) 72

20
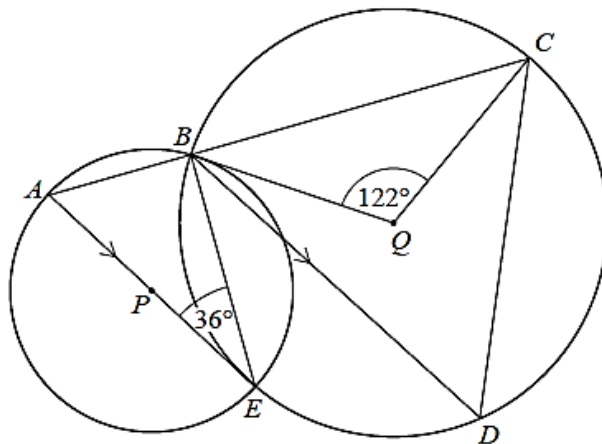
In the diagram, the circles, centres P and Q, intersect at B and E.
ABC and APE are straight lines. BD is parallel to AE.

$B\hat{E}A = 36°$ and $B\hat{Q}C = 122°$.

(a) Find $B\hat{A}E$.

                                     Answer $B\hat{A}E$ =................................[1]

(b) Find $E\hat{B}D$.

                                     Answer $E\hat{B}D$ =................................[1]

(c) Find $B\hat{D}C$.

                                     Answer $B\hat{D}C$ =................................[1]

(d) Find $D\hat{B}Q$.

                                     Answer $D\hat{B}Q$ =................................[1]

Answers: (a) 54° (b) 36° (c) 61° (d) 25°   **N13/11/Q22**

21
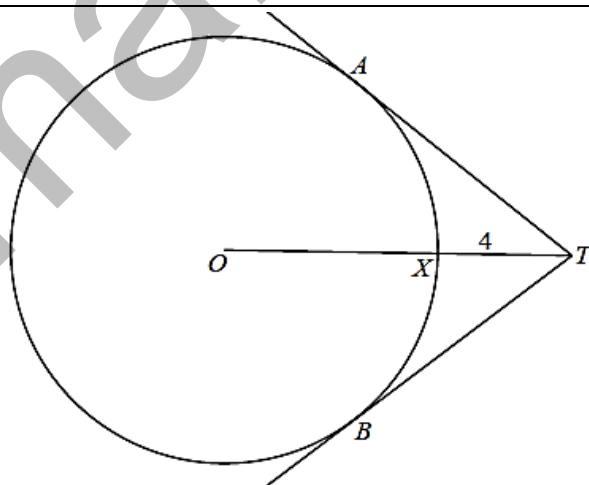
The diagram shows a circle, centre O, with radius 6 cm.
Tangents are drawn from T to touch the circle at A and B.
OXT is a straight line intersecting the circle at X with $XT = 4$ cm.

Calculate *AT*.   *Answer* .... cm [3]

22   **(a)**
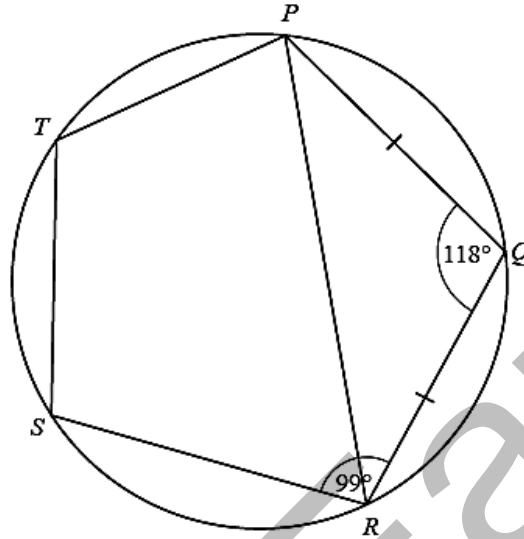
P, *Q*, *R*, *S* and *T* are points on the circumference of a circle.
PQ = *QR*.
PQ̂R = 118° and *QR̂S* = 99°.

Find *PT̂S*.
Show all your working.   *Answer*   *PT̂S* = ... [2]

(b)
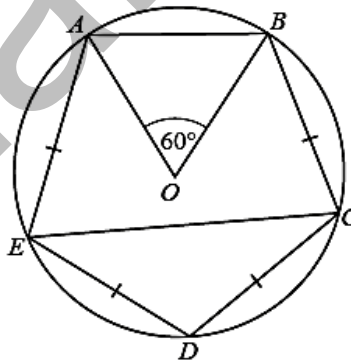
A, *B*, *C*, *D* and *E* are points on the circumference of a circle, centre *O*.
AE = *ED* = *DC* = *CB* and *AÔB* = 60°.

(i)   Find *EĈD*.
          Show all your working.   *Answer*   *EĈD* = ... [2]

223

(ii) The radius of the circle is 12 cm.

Calculate the length of the minor arc *AB*.
Use π = 3.14 .

Answer .... cm [2]

23
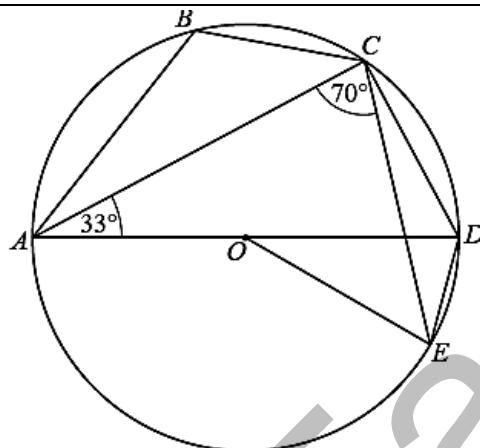
In the diagram, the points *A*, *B*, *C*, *D* and *E* lie on the circle centre *O*.
AD is a diameter.

$D\hat{A}C = 33°$ and $A\hat{C}E = 70°$.

(a) Find $C\hat{D}A$.

Answer $C\hat{D}A =$ .... [1]

(b) Find $D\hat{E}C$.

Answer $D\hat{E}C =$ .... [1]

(c) Find $A\hat{B}C$.

Answer $A\hat{B}C =$ .... [1]

(d) Find reflex $E\hat{O}A$.

Answer reflex $E\hat{O}A =$ .... [1]

Angle Properties of Circles Paper 2

1
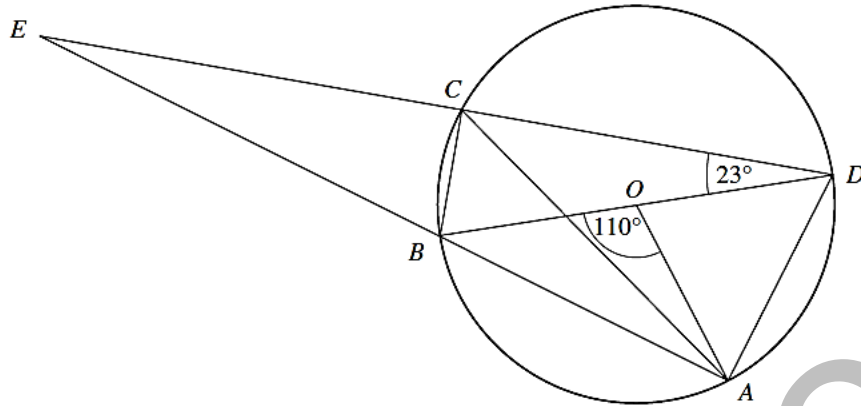
BD is a diameter of the circle, centre *O*.
C and *A* are two points on the circle.
AB and *DC*, when produced, meet at *E*.
$A\hat{O}B = 110°$ and $B\hat{D}C = 23°$.

(a) Find

    (i) $A\hat{D}O$,   [1]

    (ii) $B\hat{A}C$,   [1]

    (iii) $C\hat{B}D$,   [1]

    (iv) $C\hat{E}B$.   [1]

(b) *M* is the midpoint of *CD*.

    (i) Explain why triangle *OMD* is similar to triangle *BCD*.   [2]

    (ii) Write down the value of $\dfrac{\text{Area of } \triangle OMD}{\text{Area of } \triangle BCD}$.   [1]

Answers: **(a)(i)** 55°, **(ii)** 23°, **(iii)** 67°, **(iv)** 12°; **(b)(ii)** $\dfrac{1}{4}$.

J03/2/Q4

---

2 **(a)** In the diagram, the points *A*, *B*, *C* and *D* lie on a circle, centre *O*.

$D\hat{O}B = 124°$ and $C\hat{D}O = 36°$.
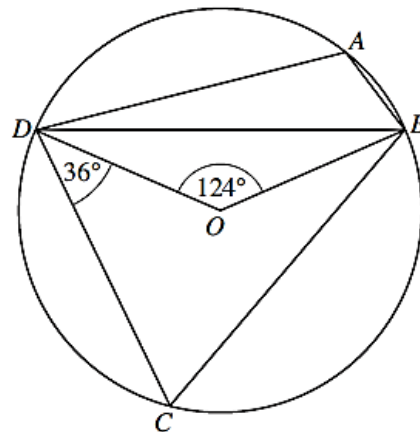
Calculate

    (i) $D\hat{C}B$,   [1]

(ii) $D\hat{A}B$, [1]

(iii) $O\hat{D}B$, [1]

(iv) $C\hat{B}O$. [1]

Answers: (a)(i) 62°, (ii) 118°, (iii) 28°, (iv) 26°;

3

(b) In the diagram, A, B, C and D lie on a circle centre O.

The tangent to the circle at C meets the diameter AD produced, at T.

$D\hat{O}C = 66°$.

Calculate

(i) $D\hat{A}C$, [1]

(ii) $D\hat{T}C$, [1]

(iii) $A\hat{D}C$, [1]

(iv) $A\hat{B}C$. [1]
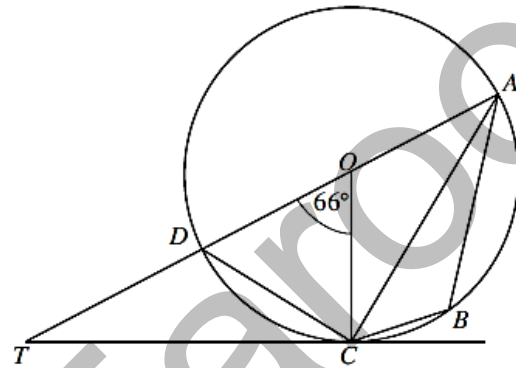
(b) 33°, 24°, 57° and 123°.

4
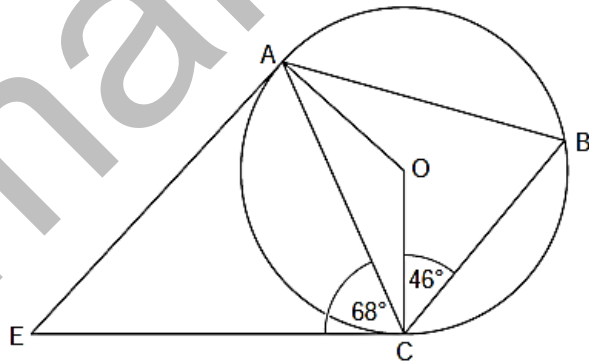
A, B and C are points on the circumference of a circle, centre O.
AE and CE are tangents to the circle.
$A\hat{C}E = 68°$ and $B\hat{C}O = 46°$.

(a) Calculate

(i) $A\hat{O}C$, [2]

(ii) $A\hat{E}C$. [1]

226

(b) Find the three angles of the triangle *ABC* and hence state the name given to this special triangle.

[2]

5
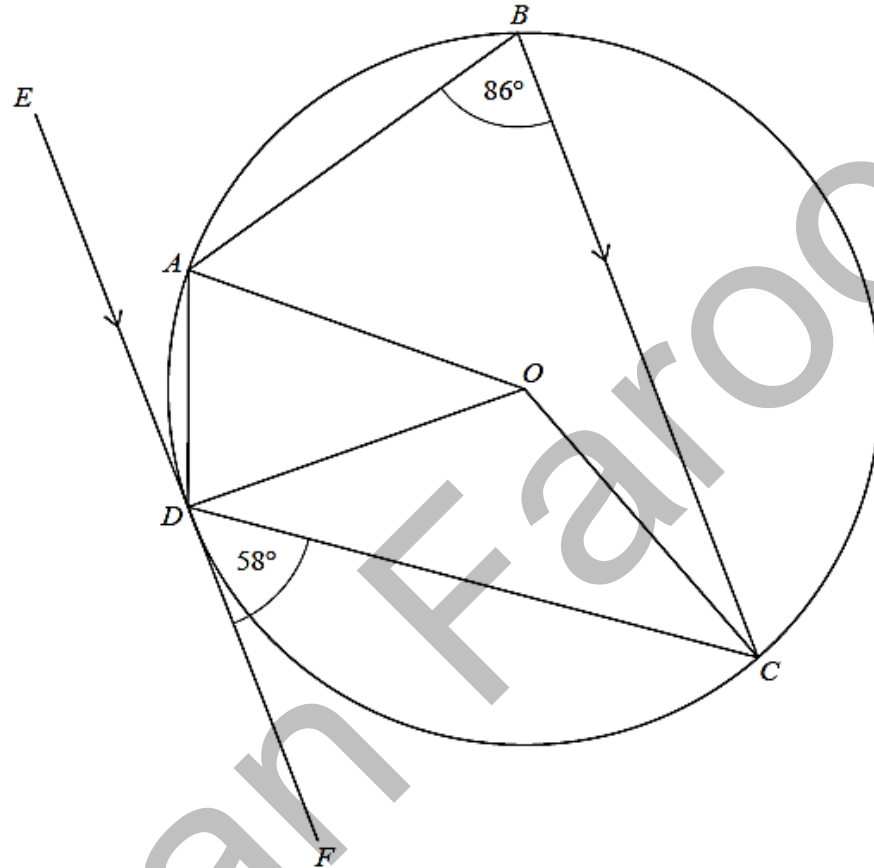
A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
EF is the tangent to the circle at *D* and is parallel to *BC*.
$A\hat{B}C = 86°$ and $C\hat{D}F = 58°$.

(a) Find $O\hat{D}C$.   *Answer* ......[1]

(b) Explain why $O\hat{C}B = 26°$.   *Answer* ....[2]

(c) Find

   (i) $A\hat{D}C$,   *Answer* ....[1]

   (ii) $A\hat{D}E$,   *Answer* ....[1]

   (iii) $A\hat{O}D$,   *Answer* ....[1]

   (iv) $B\hat{A}O$.   *Answer* ....[1]

227

6
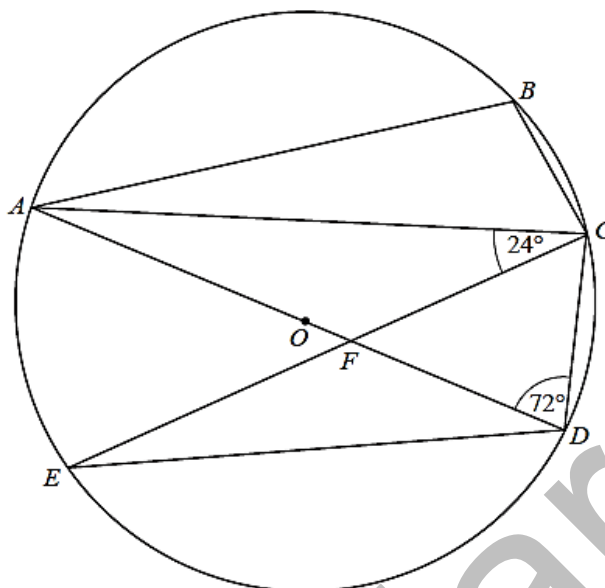
A, B, C, D and E are points on a circle with centre O.
AD is a diameter of the circle and F is the point of intersection of AD and CE.
$A\hat{C}E = 24°$ and $A\hat{D}C = 72°$.

(a) Find

   (i) $A\hat{D}E$,   *Answer* .... [1]

   (ii) $C\hat{E}D$,   *Answer* .... [1]

   (iii)   ,   *Answer* .... [1]

   (iv) $A\hat{B}C$.   *Answer* .... [1]

(b) Given that $DC = 4.5$ cm, calculate

   (i) the diameter of the circle,   *Answer* .... cm [2]

   (ii) DE.   *Answer* .... cm [2]

7
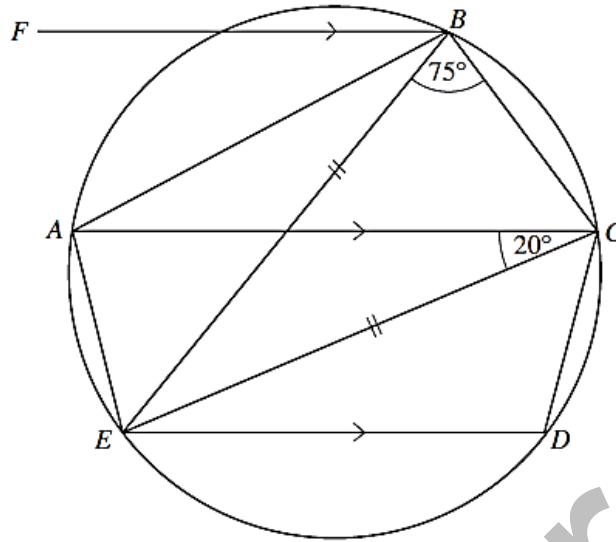
In the diagram, A, B, C, D and E lie on a circle and $EB = EC$.
The lines ED, AC and FB are parallel.
$E\hat{B}C = 75°$ and $A\hat{C}E = 20°$.

Calculate

(a) $C\hat{E}D$, [1]

(b) $C\hat{D}E$, [1]

(c) $E\hat{C}D$, [1]

(d) $A\hat{C}B$, [2]

(e) $A\hat{B}F$. [2]

Answers: **(a)** 20°; **(b)** 105°; **(c)** 55°; **(d)** 55°; **(e)** 30°. **N01/2/Q3**

8
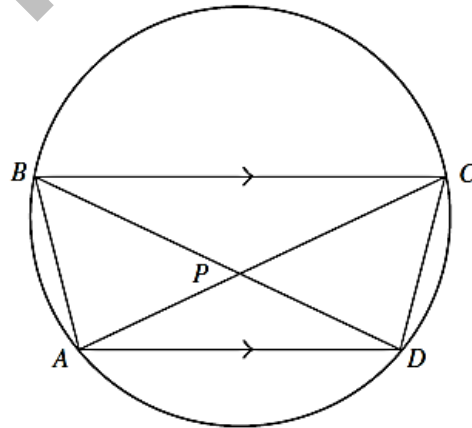
Diagram I

The points A, B, C and D lie on a circle as shown on Diagram I.

AC cuts *BD* at *P*.
AD is parallel to *BC*.

(a) Show that triangle *BPC* is an isosceles triangle. [2]

(b) Given that angle *ACB* = 32° and angle *DAB* = 118°,
calculate angle *ACD*. [2]

(c)
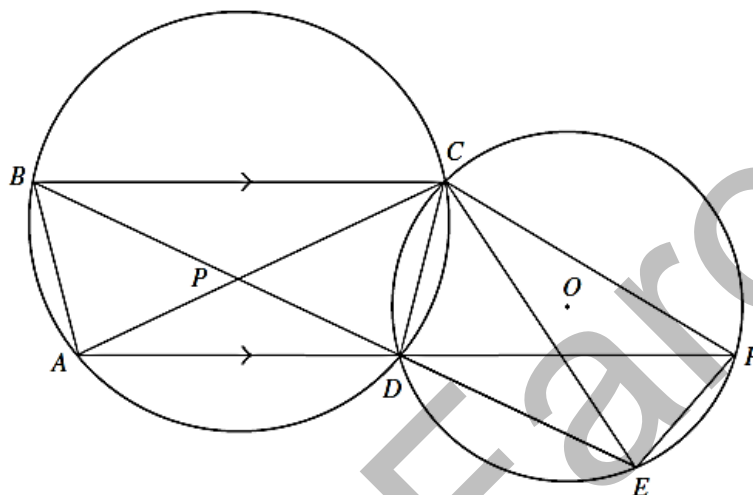
Diagram II

Diagram II shows the circle in Diagram I and a second circle, centre *O*.
The two circles intersect at *C* and *D*.
AD produced cuts the second circle at *F*.
BD produced cuts the second circle at *E*.
Angle *DEF* = 110°.

Calculate

    (i) angle *ACE*, [3]

    (ii) angle *COD*. [2]

Answers: **(b)** 30°; **(c)(i)** 68°, **(ii)** 96°. **N03/2/Q5**

230

9
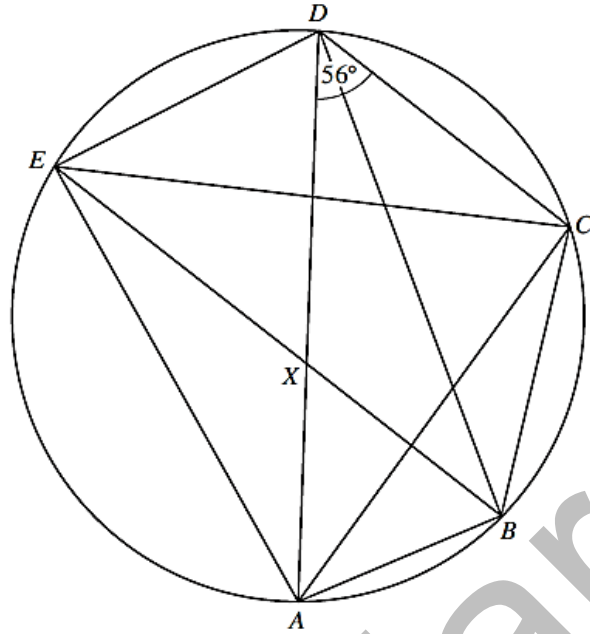
The points A, B, C, D and E lie on a circle.
AD is a diameter of the circle.
DB bisects angle ADC.
Angle $ADC = 56°$.

(a) Giving your reasons, write down

    (i) angle DCA,   [1]

    (ii) angle DAC,   [1]

    (iii) angle CBA,   [1]

    (iv) angle AEB.   [1]

(b) It is given that EB is parallel to DC and that EB cuts AD at X.
    [You **must not** assume that X is the centre of the circle.]

    Show that triangle BDX is isosceles.   [2]

(c) Find angle EBA.   [1]

(d) Hence or otherwise show that X is the centre of the circle.   [1]

Answers: **(a)(i)** 90°, **(ii)** 34°, **(iii)** 124°, **(iv)** 28°; **(c)** 62°.   **N05/2/Q3**

10
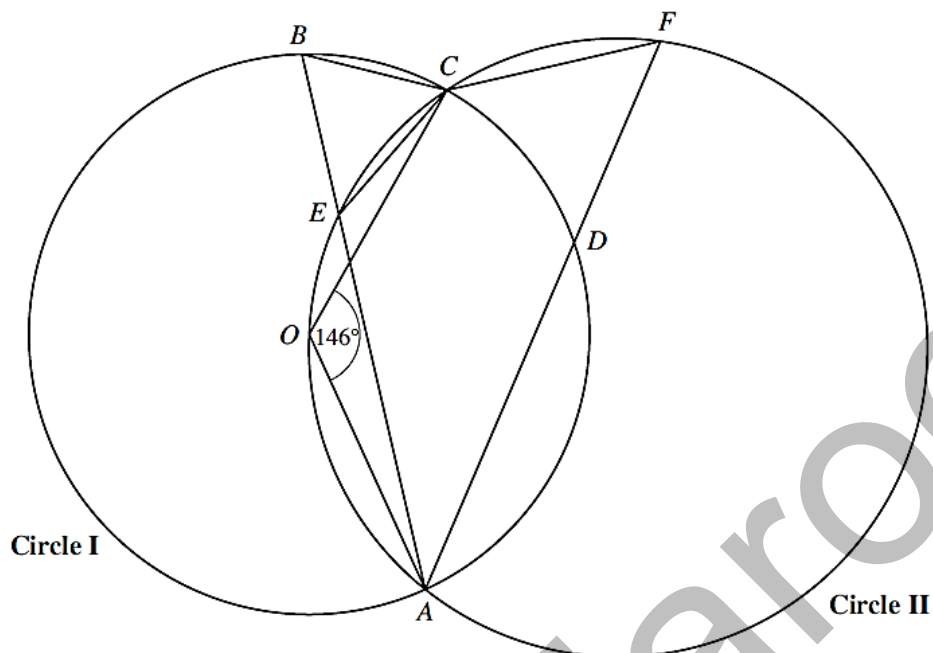
Points A, B, C and D lie on **Circle I**.
O is the centre of **Circle I**.
Points A, O, E, C and F lie on **Circle II**.
AEB and ADF are straight lines.
Angle $AOC = 146°$.

Giving your reasons, write down

   (i) $C\hat{E}A$,   [1]

   (ii) $C\hat{B}A$,   [1]

 (iii) $C\hat{F}A$,   [1]

 (iv) $D\hat{C}F$.   [1]

11 **(a)**
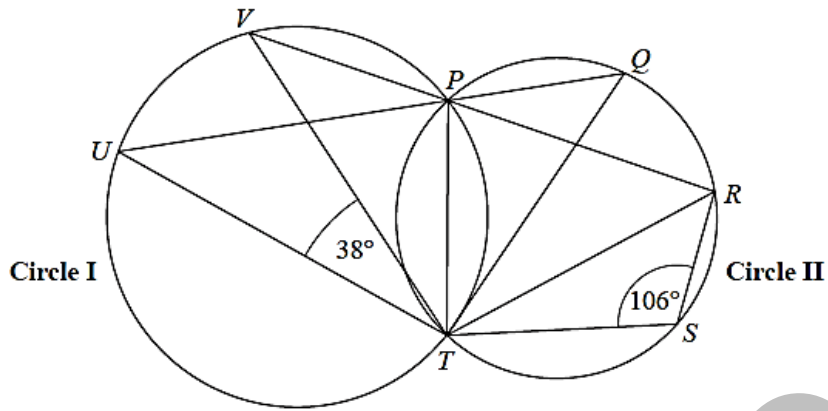
The points P, T, U and V lie on **Circle I**, and the points P, Q, R, S and T lie on **Circle II**.
UPQ and VPR are straight lines.
V T̂U = 38° and T ŜR = 106°.

Find

(i)   V P̂U, [1]

(ii) Q T̂R, [1]

(iii) T P̂R, [1]

(iv) U P̂T. [1]

(b)
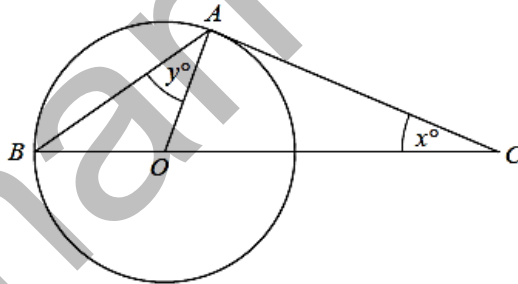
A and B are points on the circle, centre O.
CA is a tangent at A and BOC is a straight line.
A ĈB = x° and B ÂO = y°.

Find an expression for y in terms of x. [3]

Answers: **(a)(i)** 38 **(ii)** 38 **(iii)** 74 **(iv)** 68 **(b)** ½(90 – x)   **N10/21/Q3**

12

(b) In the diagram, $ABCD$ is a square.
The point P lies on the circle through
A, B, C and D.
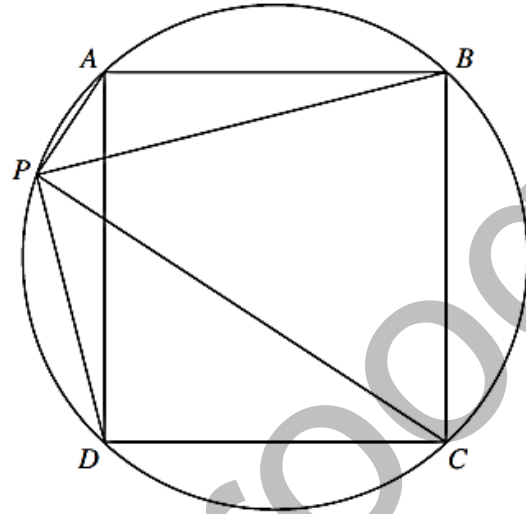
(i) Explain why $A\hat{P}C = 90°$.

Answer....................[1]

(ii) Explain why $A\hat{P}B = B\hat{P}C$.

Answer.......................[2]

(iii) Hence find

(a) $A\hat{P}B$,

Answer ....[1]

(b) $A\hat{P}D$.

Answer ....[1]

(iv) PC and AD intersect at R.

Given that $A\hat{R}C = 127°$, find $P\hat{D}C$.

Answer ....[2]

Answers: **(ii)(a)** $\dfrac{16}{25}$ **(b)** $\dfrac{3}{25}$ **(b)(iii)(a)** 45 **(b)** 135 **(iv)** 98

13   **(a)**
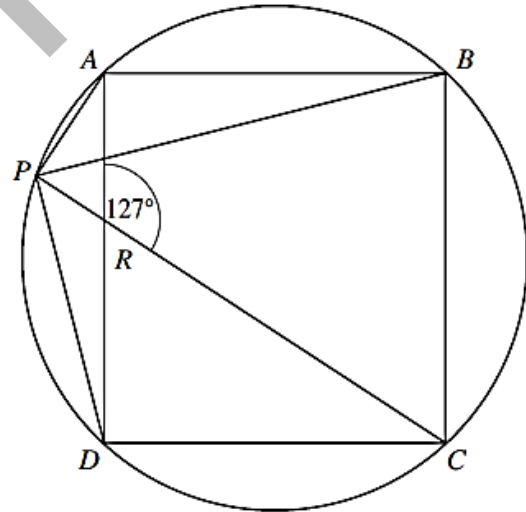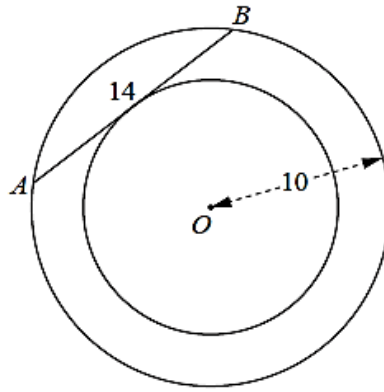
In the diagram, the circles each have centre O.
AB is a chord of the larger circle and also a tangent to the smaller circle.
$AB = 14$ cm and the radius of the larger circle is 10 cm.

Find the radius of the smaller circle.   *Answer* .... cm [3]

(b)
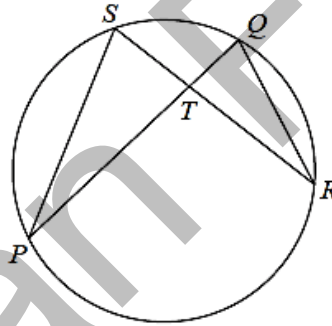
In the diagram, PQ and RS are chords of a circle that intersect at T.

(i) Show that triangles PST and RQT are similar.   [3]

(ii)
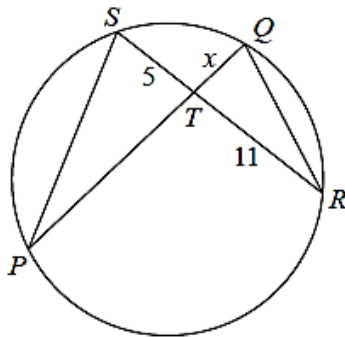
$ST = 5\,cm$, $TR = 11\,cm$ and $TQ = x\,cm$.

Given that $PQ = 18\,cm$, show that x satisfies the equation

$$x^2 - 18x + 55 = 0.$$ [2]

(iii) Solve the equation $x^2 - 18x + 55 = 0$.
Give each solution correct to 1 decimal place.

Answer $x = $ .... or .... [3]

(iv) Find the difference between the lengths of PT and TQ.

Answer .... cm [1]

Answer: (a) 7.14 (iii) 3.9 14.1 (iv) 10.2

14
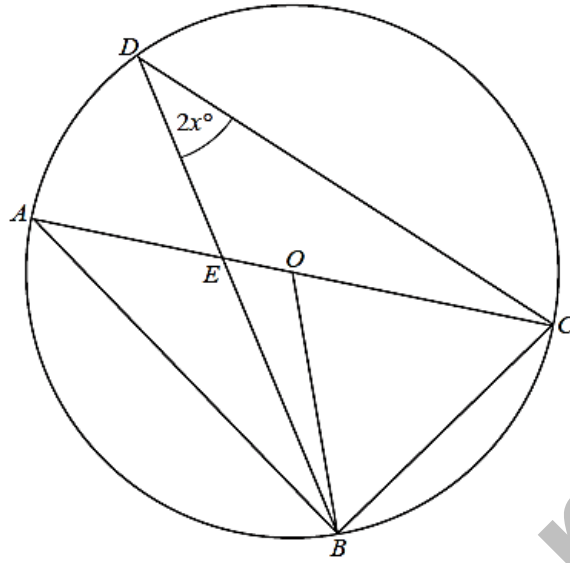
A, B, C and D are points on the circumference of a circle, centre O.
The diameter AC intersects BD at E.
$B\hat{D}C = 2x°$.

(i) Find an expression, in terms of x, for

    (a) $B\hat{A}C$,　　　　　　　　　　　　　*Answer* .... [1]

    (b) $B\hat{O}C$,　　　　　　　　　　　　　*Answer* .... [1]

    (c) $O\hat{C}B$.　　　　　　　　　　　　　*Answer* .... [1]

(ii) Calculate x when $O\hat{B}E = x°$ and $D\hat{E}C = 123°$.

                                                 Answer $x =$ .... [3]

Answers: **(a)(i)(a)** $2x$; **(b)** $4x$; **(c)** $90 - 2x$; **(ii)** 19; **(b)(i)** 22.3; **(ii)** 476.　　　　**N14/21/Q10**

15 **(a)**
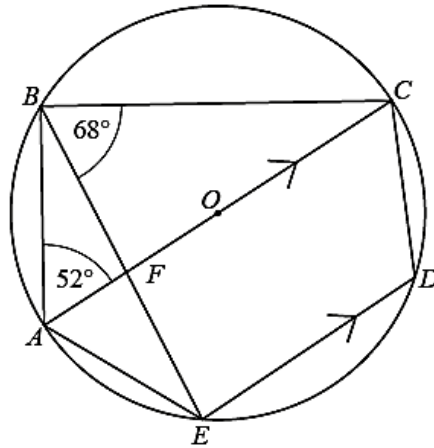
A, B, C, D and E are points on the circumference of the circle, centre O.
AC is a diameter and AC is parallel to ED.
AC and BE intersect at F.
$B\hat{A}C = 52°$ and $C\hat{B}E = 68°$.

(i) Find $A\hat{C}B$.　　　　　　　　　　　　　*Answer* $A\hat{C}B =$ .... [1]

(ii) Find $A\hat{E}F$.
　　　Give a reason for your answer.

　　　Answer $A\hat{E}F =$ .... because .... [1]

(iii) Find $C\hat{D}E$.　　　　　　　　　　　*Answer* $C\hat{D}E =$ .... [1]

(iv) Find $B\hat{C}D$.　　　　　　　　　　　*Answer* $B\hat{C}D =$ .... [2]

Answers: **(a)(i)** 38° **(ii)** 38°, angles in same segment are equal **(iii)** 112° **(iv)** 106° **(b)** 156° **(c)** 105.5°　**N17/21/Q6**

Symmetry, Congruence and Similarity Paper 1

1   **(a)** State the order of rotational symmetry of a regular decagon.

Answer (a) .... [1]

    (b) Write down those letters of the word AMBULANCE which have a vertical axis of symmetry.

Answer (b).. [1]

Answers: **(a)** 10; **(b)** A M and U.

2   The trapezium *PQRS* has one line of symmetry.
$S\hat{P}Q = 120°$.

(a) Explain why $P\hat{S}R = 60°$.

Answer (a) .... [1]

    (b) Three trapezia, each congruent to *PQRS*, are placed together as shown.
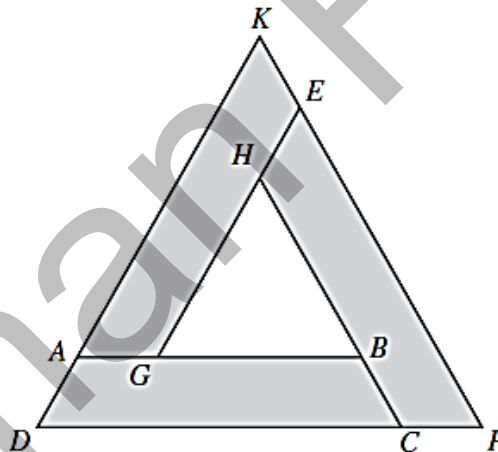
Show that *KDF* is an equilateral triangle.

Answer (b) ....

.... [1]

    (c) Given also that $BC = 1\,m$, $AB = 4\,m$ and $DC = 5\,m$, find

        (i) the length of *GB*,

        (ii) the ratio Area Δ *KDF* : Area Δ *HGB*,

        (iii) the shaded area as a fraction of the area of Δ *KDF*.

Answer (c)(i) ....m [1]

(ii) .... : .... [2]

(iii) .... [1]

3 (a) A prism has a cross-section which is a regular hexagon.
How many planes of symmetry does this prism have?

Answer (a) ....[1]

4 In the diagram, BCDE is a trapezium, and the sides
CD and BE are produced to meet at A.
CB = 12 cm, DE = 9 cm and the perpendicular
distance from D to CB is 4 cm.

Calculate

(a) the area of BCDE,

(b) the perpendicular distance from A to CB.

Answer (a) .... cm² [1]

(b) ....cm [2]

5
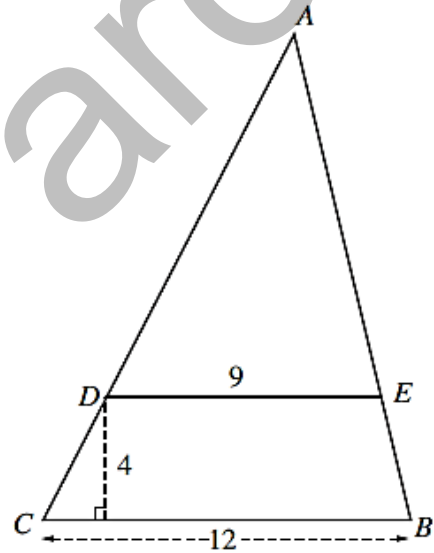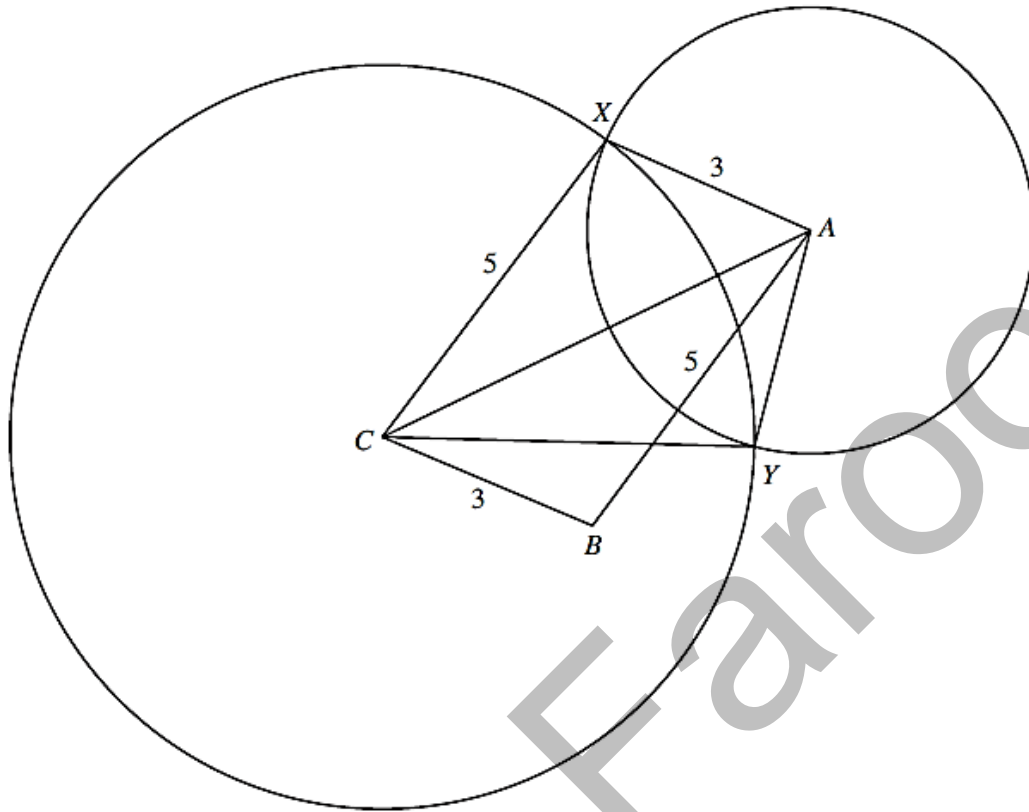
The diagram shows a circle, centre *C*, of radius 5 cm, and a circle, centre *A*, of radius 3 cm.
The circles intersect at *X* and *Y*.
B is a point such that *AB* = 5 cm and *BC* = 3 cm.

(a)   Show that triangles *ABC* and *CYA* are congruent.

Answer (a) ....[2]

(b)   Show that the areas of the quadrilaterals *ABCX* and *AYCX* are equal.

Answer (b) ....[1]

(c)   State the name of the special quadrilateral *AYCX*.

Answer (c) .... [1]

Answer: (c) Kite

J07/1/Q19

6 The following list gives the names of six shapes.

Square   Rectangle   Equilateral triangle

Kite   Trapezium   Parallelogram

From this list, write down the name of the shape which always has

(a) rotational symmetry of order 3,

(b) rotational symmetry of order 2 and exactly 2 lines of symmetry,

(c) one line of symmetry only.

Answers: (a) Equilateral triangle   (b) Rectangle   (c) Kite   **J08/1/Q11**

7 Similar buckets are available in two sizes.
The large bucket has height 30 cm and base diameter 16 cm.
The small bucket has base diameter 8 cm.
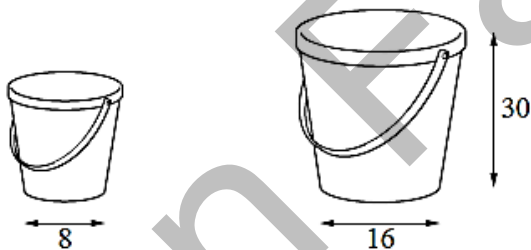
(a) Find the height of the small bucket.   *Answer* (a) .... cm [1]

(b) Given that the small bucket has volume 850 cm³, find the volume of the large bucket.

                                             Answer (b) .... cm³ [2]

Answer: (a) 15   (b) 6 800   **J09/1/Q11**

8
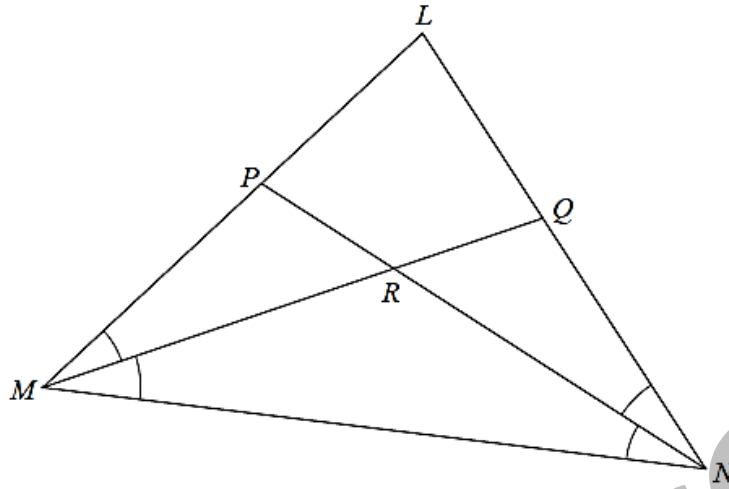
In the diagram, $L\hat{M}Q = Q\hat{M}N = M\hat{N}P = P\hat{N}L$.

(a) Show that triangles LMQ and LNP are congruent.

(b) Show that $M\hat{P}N = M\hat{Q}N$. [3]

[1]

(c) The straight lines MQ and NP intersect at R.

State the name of the special quadrilateral $LPRQ$.   Answer   (c) ....[1]

Answer: **(a)** In triangles LMQ and LNP, LM=LN, $M\hat{L}N$ is common or $L\hat{M}Q = L\hat{N}P$, conclusion that triangles **J09/1/Q19**
    LMQ, LNP are congruent having given all the evidence required.
(b) $L\hat{Q}M = L\hat{P}N$, $M\hat{P}N = 180 - L\hat{P}N$ and $M\hat{Q}N = 180 - L\hat{Q}M$ and conclusion.
(c) Kite

9
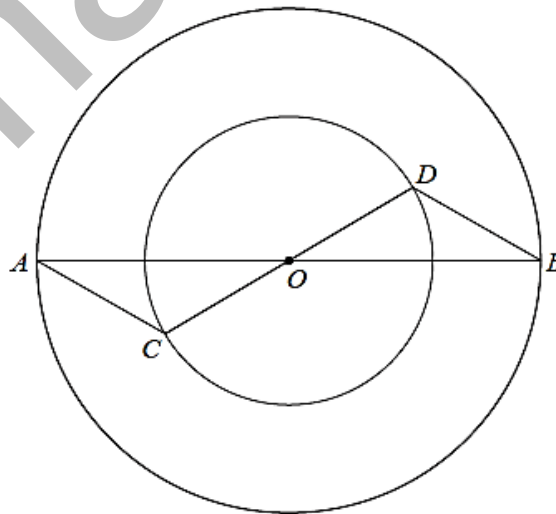
The diagram shows two circles, both with centre O.
CD is a diameter of the small circle and AB is a diameter of the large circle.

Using congruent triangles, show that $BD = AC$.
State your reasons clearly.

In triangles .... and ....

[3]

Answer: congruent triangles established and conclusion

10
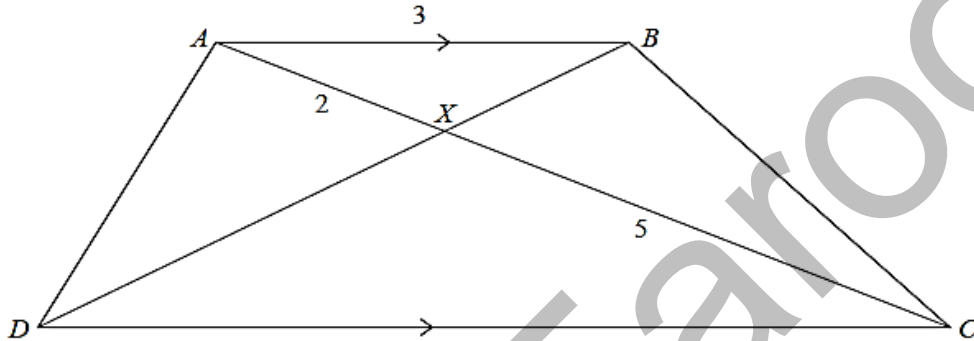
In the diagram, AC and BD intersect at X.
Triangle ABX is similar to triangle CDX.
$AB = 3$ cm, $AX = 2$ cm and $XC = 5$ cm.

(a) Find the ratio of the area of triangle ABX to the area of triangle CDX.

Answer (a)............. : .... [1]

(b) Find the ratio of the area of triangle ABX to the area of triangle BCX.

Answer (b)............. : .... [1]

(c) Calculate CD.

Answer (c) .... cm [2]

Answer: (a) 4 : 25   (b) 2 : 5   (c) 7.5

11   **(a)** On the regular hexagon below, draw all the lines of symmetry.

[1]
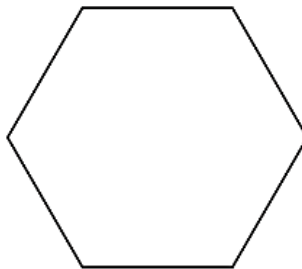
(b) On the grid below, draw a quadrilateral with rotational symmetry of order 2.
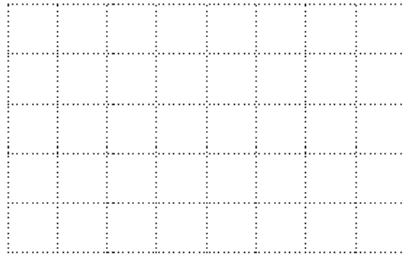
[1]

12 The two triangles below are similar.
The lengths are in centimetres.
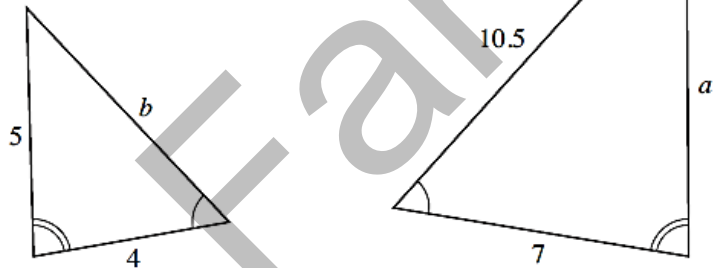
Calculate a and b.

Answer $a = $ ....

$b = $ .... [3]

13 These two cylinders are similar.
The ratio of their volumes is   8 : 27.
The height of cylinder A is 12 cm.

Find the height of cylinder B.

Answer   .... cm [2]

14
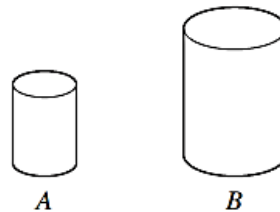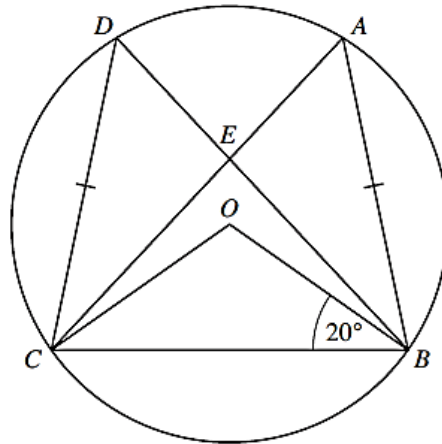
Points A, B, C and D lie on the circumference of a circle, centre O, and $AB = CD$.
AC and BD intersect at E.
$O\hat{B}C = 20°$.

(a) Calculate $B\hat{O}C$.

Answer $B\hat{O}C$ = ... [1]

(b) Calculate $C\hat{A}B$.

Answer $C\hat{A}B$ = ... [1]

(c) Show that triangles AEB and DEC are congruent.

.... [3]

Answer: (a) 140° (b) 70°

15 (a) On the diagram below, shade two more squares to make a pattern that has rotational symmetry of order 2.
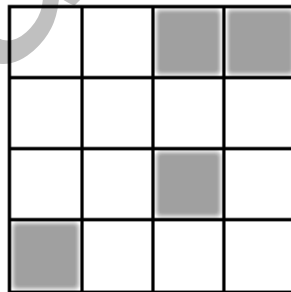
[1]

(b) On the diagram below, shade two more squares to make a pattern that has only one line of symmetry.
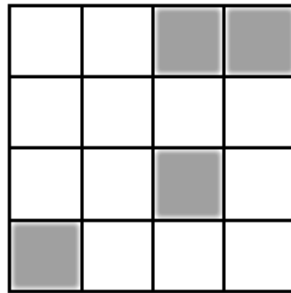
[1]

16 These two triangles are congruent.
The lengths are in centimetres.
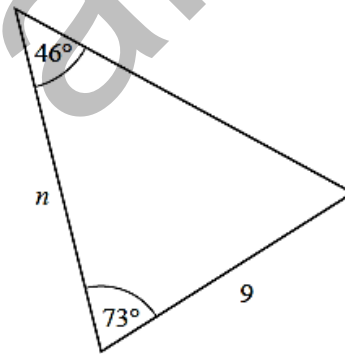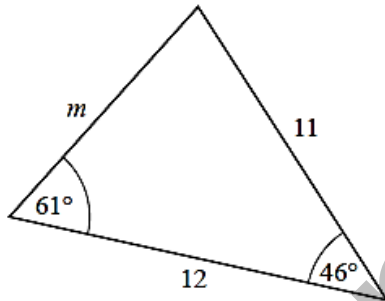
Find m and n.

Answer $m =$ ....

$n =$ ....[2]

Answer: $m = 9$, $n = 11$

17
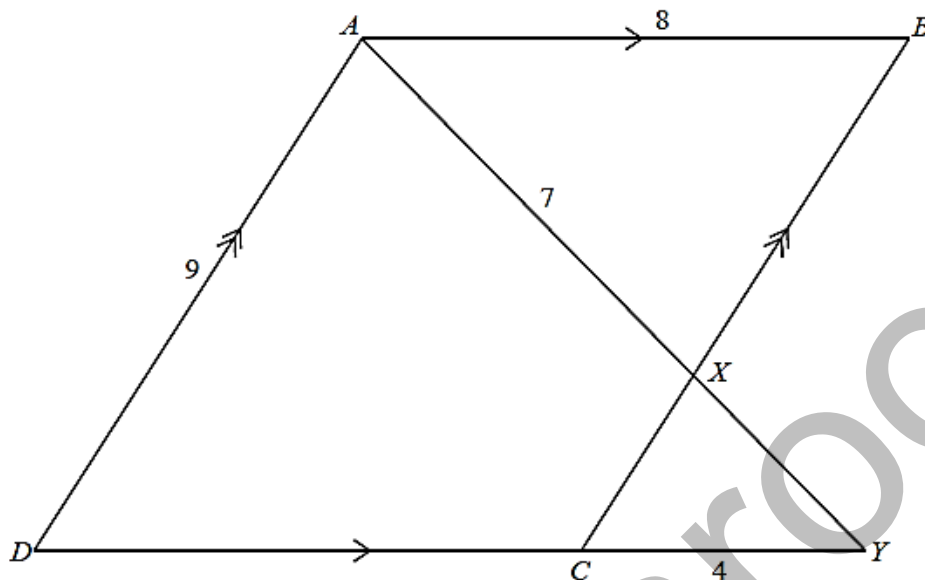
In the diagram, *ABCD* is a parallelogram.
X is a point on *BC*.
AXY and *DCY* are straight lines.
AB = 8 cm, *AX* = 7 cm, *AD* = 9 cm and *CY* = 4 cm.

(a) Show that triangles *ABX* and *YDA* are similar.
Give the reason for each of your statements.

....[2]

(b) Calculate *AY*.   *Answer* .... cm [2]

(c) Calculate *CX*.   *Answer* .... cm [2]

Answer: (b) 10.5 (c) 3   **J12/11/Q24**

18

Maryam makes two geometrically similar cakes.
The heights of the cakes are 6 cm and 9 cm.
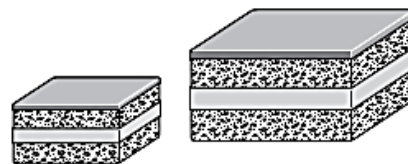
(a) Maryam decorates each cake with a ribbon around the outside.
The length of the ribbon for the larger cake is 66 cm.

Find the length of the ribbon for the smaller cake.   *Answer* .... cm [1]

(b) Maryam uses $1600 \, m^3$ of cake mixture to make the smaller cake.

Find the volume of cake mixture she uses to make the larger cake.

Answer ....cm^3 [2]

19   **(a)** Complete the pattern so that AB is the only line of symmetry.
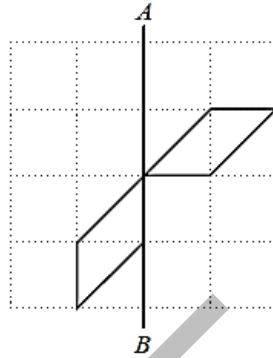
[1]

(b) Shade four more small triangles in the shape below to make a pattern with rotational symmetry of order 3.
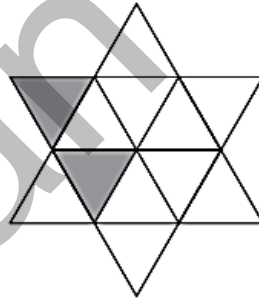
[1]

20   **(a)** In the diagram, six small triangles are shaded.

Shade one more small triangle, so that the diagram will then have one line of symmetry.
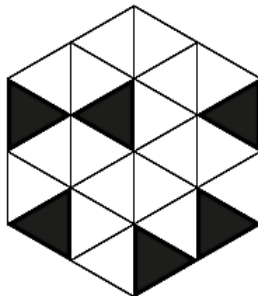
(b) The diagram below has rotational symmetry.

Mark the centre of rotational symmetry with a dot, and write down the order of rotational symmetry.
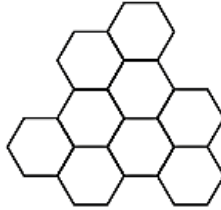
Answer Order of rotational symmetry = ... [1]

21
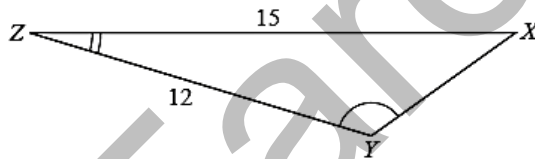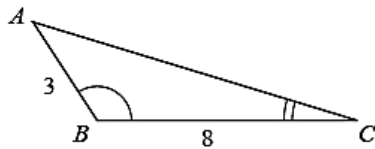
In the diagram, triangle ABC is similar to triangle XYZ.
$AB = 3$ cm, $BC = 8$ cm, $YZ = 12$ cm and $ZX = 15$ cm.

(a) Calculate XY.

Answer .... cm [1]

(b) Given that the area of triangle ABC is 10 cm^2, calculate the area of triangle XYZ.

Answer .... cm^2 [2]

Answers: **(a)** 4.5 **(b)** 22.5

22 **(a)** Complete the diagram to make a quadrilateral $ABCD$ which has AC as its line of symmetry.

[1]

250

(b) Complete the diagram to make a quadrilateral *PQRS* which has rotational symmetry of order 2.

P ———————————— Q

R

[1]
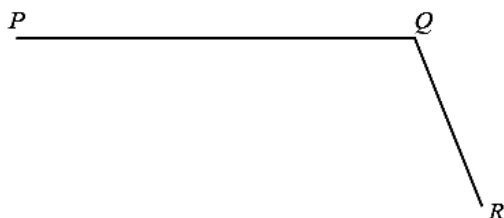
J16/11/Q8

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23   Complete the sentences below which describe two different types of quadrilateral.

   (a)   A .... has two pairs of equal sides and just one line of symmetry.   [1]

   (b)   A ....has two pairs of equal sides, no line of symmetry and

     rotational symmetry of order 2.   [1]

J17/11/Q6

---

24   **(a)**   Complete the diagram to make a quadrilateral *ABCD* which has rotational symmetry of order 2.

     Answer (a)

C

A ———————————— B

[1]

  (b)   Complete the diagram to make a quadrilateral *PQRS* which has *PR* as its line of symmetry.

251

Answer (b)

R

P ————————————————— Q   [1]
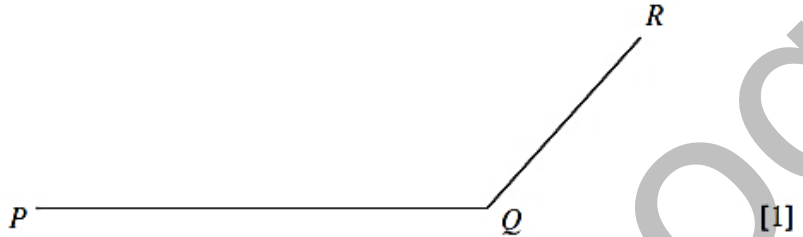
25   The ratio of the areas of the bases of two geometrically similar buckets is 4 : 9.

(a) The area of the top of the smaller bucket is 480 cm^2.
    What is the area of the top of the larger bucket?

(b) **Write down** the ratio of the heights of the two buckets.

(c) Both buckets are filled with sand.
    The mass of sand in the larger bucket is 36 kg.
    Find the mass of sand in the smaller bucket.

Answer (a) .... cm^2 [1]

. [1]
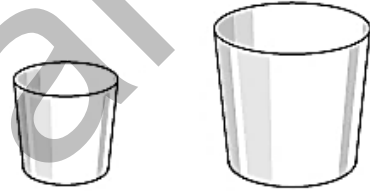
(c) ....kg [2]

26

(a) Shade three more squares so that the completed square grid has rotational symmetry of order 4.

Answer (a)
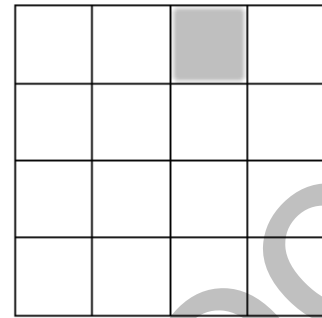
[1]

(b) Shade one more square so that the completed square grid has one line of symmetry.

Answer (b)
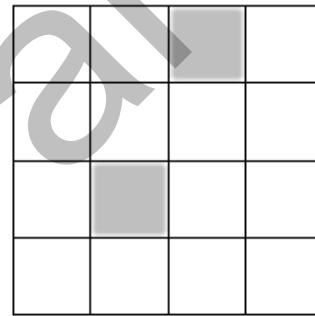
[1]

Answers: **(a)** shading (row1, column 2), (2, 4), (3, 1); **(b)** (3, 4).   **N03/Q7**

27

(a) How many planes of symmetry has a prism with a regular pentagon as its cross-section?
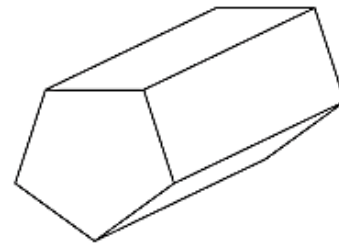
(b) The sides of a rectangular plot of land are measured correct to the nearest metre.

The measurements are given as 20 m by 3 m.

(i) Write down the upper bound of the length of the plot of land.

(ii) Find the least possible perimeter of the plot.

(b)(i) .... m [1]

(ii) .... m [1]

Answers: **(a)** 6; **(b)(i)** 20.5 m, **(ii)** 44 m.   **N03/Q13**

28
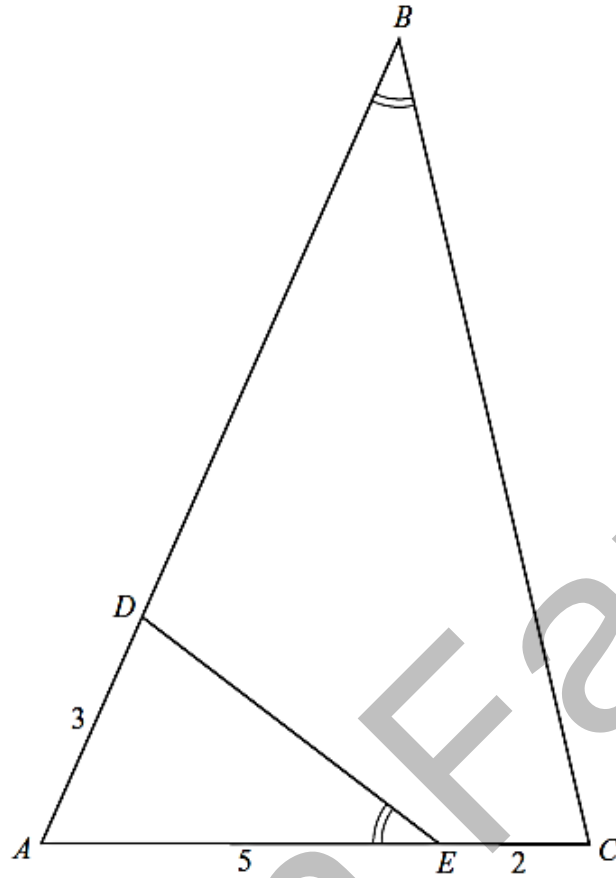
In the diagram, $A\hat{B}C = A\hat{E}D$.

(a) Explain why triangles *ABC* and *AED* are similar.

Answer (a) ....

.... [1]

(b) Given also that *AD* = 3 cm, *AE* = 5 cm and *EC* = 2 cm, calculate

(i) *BD*,
Answer (b)(i) *BD* = ... cm [3]

(ii) $\dfrac{\text{Area of triangle } AED}{\text{Area of triangle } ABC}$.
(ii) .... [1]

Answers: **(a)** $A\hat{B}C = A\hat{E}D$ and $\hat{A}$ is common; **(b)(i)** $8\dfrac{2}{3}$ cm, **(ii)** $\dfrac{9}{49}$.

N03/Q21

254

29
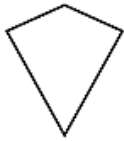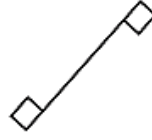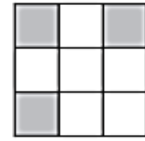
Shape A   B   C   D   E   F

In the answer space, write down the letters of all the shapes which have

(a) exactly 1 line of symmetry,

Answer (a) .... [1]

(b) rotational symmetry of order 2.

(b) .... [1]

    N04/Q2

30

In the diagram, AB is parallel to DC and $A\hat{C}B = C\hat{D}A$.

(a) Explain why triangles ABC and CAD are similar.

(b) Given that $AB = 4\,cm$, $BC = 7\,cm$, $AC = 6\,cm$ and $CD = 9\,cm$, calculate AD.
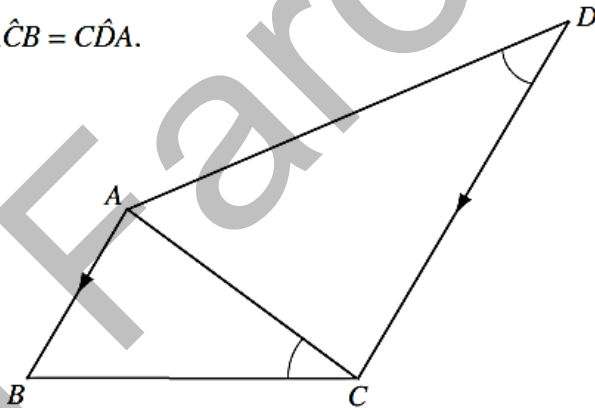
Answer (a) ....

(b) AD = ...cm [2]

    N05/1/Q12

31
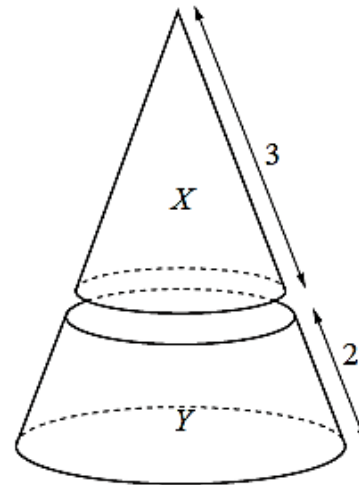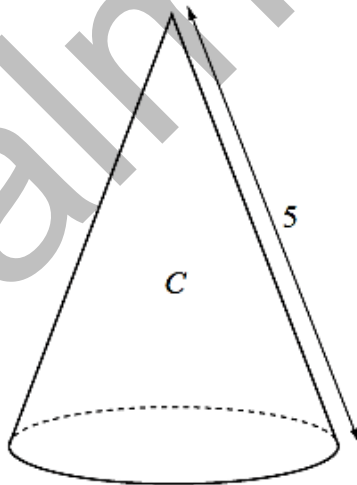
A solid cone, C, is cut into two parts, X and Y, by a plane parallel to the base.
The lengths of the sloping edges of the two parts are 3 cm and 2 cm.
Find the ratio of

(a) the diameters of the bases of X and C, *Answer (a)* .... : ....[1]

(b) the areas of the bases of X and C, (b) .... : ....[1]

(c) the volumes of X and Y. (c) .... : ....[2]

Answer: (a) 3:5; (b) 9:25; (c) 27:98.

---

32 The diagram shows the triangle PQR.
The points S and T lie on the lines PQ and PR respectively.
The line ST is parallel to the line QR.

(a) $S\hat{T}R = 117°$ and $S\hat{Q}R = 48°$.
Find $Q\hat{P}R$.

Answer (a) $Q\hat{P}R =$ ....[1]
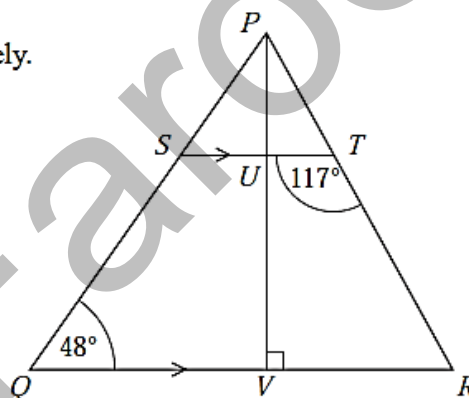
(b) U and V are points on ST and QR respectively.
PUV is a straight line with $2PU = UV$ and $P\hat{V}R = 90°$.

Find *Answer (b)(i)* .... : ....[1]

(i) $PU:PV$,

(ii) the ratio of the area of triangle PQR to the area of the trapezium $STRQ$.

Answer (b)(ii) .... : ....[2]

Answers: (a) 69° (b)(i) 1 : 3 (ii) 9 : 8

33
(a) In the diagram in the answer space, two small squares are shaded.

Shade one more small square, so that the *Answer (a)*
figure will then have one line of symmetry.

[1]

256
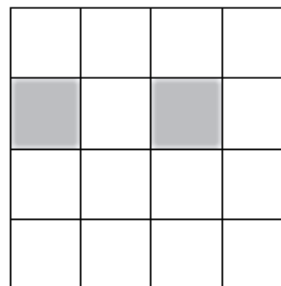
(b) In the diagram in the answer space, two small triangles are shaded.

Shade four more small triangles, so that the figure will then have rotational symmetry of order 3.
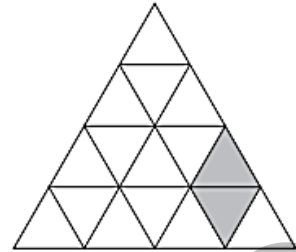
Answer (b)

[1]

Answers: (a) and (b) Correct squares shaded.

34
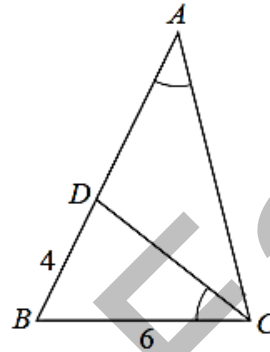
The diagram shows triangle ABC.
D is the point on AB such that $B\hat{C}D = B\hat{A}C$.

(a) Explain why triangle ABC is similar to triangle CBD.

Answer (a) ....

....[1]

(b) Given that $BD = 4$ cm and $BC = 6$ cm, calculate AD.

Answer (b) $AD =$ .... cm [3]

Answers: (b) 5

35

parallelogram		rhombus	
	rectangle		square

Which of these quadrilaterals have

(a) exactly 2 lines of symmetry,

Answer .... [1]

(b) rotational symmetry of order 2,

Answer .... [1]

(c) diagonals that are equal?

Answer .... [1]

N11/11/Q11

36
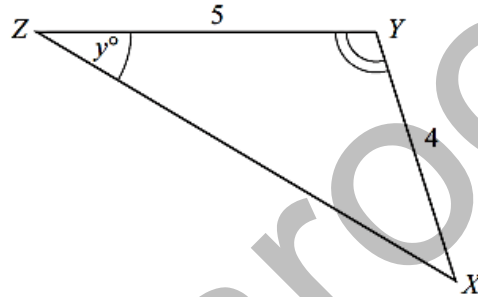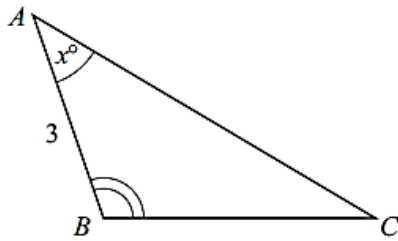
The triangles ABC and XYZ are similar and $A\hat{B}C = X\hat{Y}Z$.

$B\hat{A}C = x°$, $Y\hat{Z}X = y°$ where $x \neq y$.
$AB = 3$ cm, $XY = 4$ cm and $YZ = 5$ cm.

(a) Express $A\hat{B}C$ in terms of x and y.   *Answer* $A\hat{B}C =$ .... [1]

(b) Find BC.   *Answer* $BC =$ .... cm [1]

(c) Write down the value of $\dfrac{\text{area of triangle } ABC}{\text{area of triangle } XYZ}$.   *Answer*   .... [1]

N11/12/Q20

37
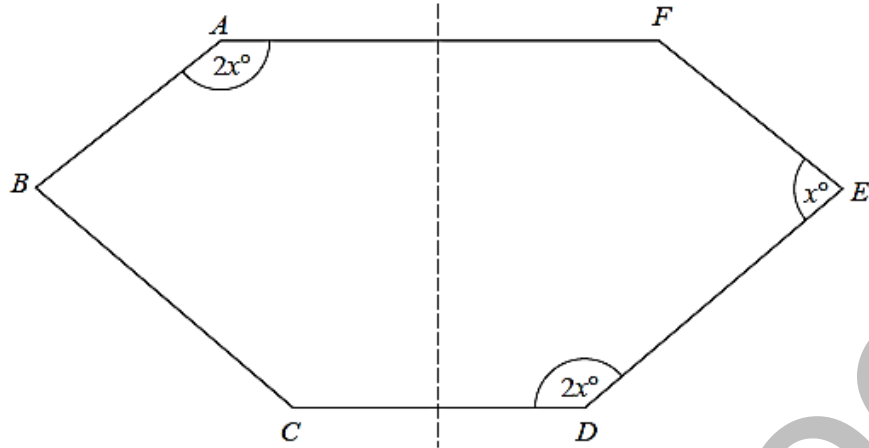
In the diagram, the dashed line is a line of symmetry.
$B\hat{A}F = 2x°$, $F\hat{E}D = x°$ and $C\hat{D}E = 2x°$.

Find the value of x.

Answer $x =$ ....[3]

Answer: 72   **N12/11/Q19**

38

This figure has rotational symmetry of order 3.
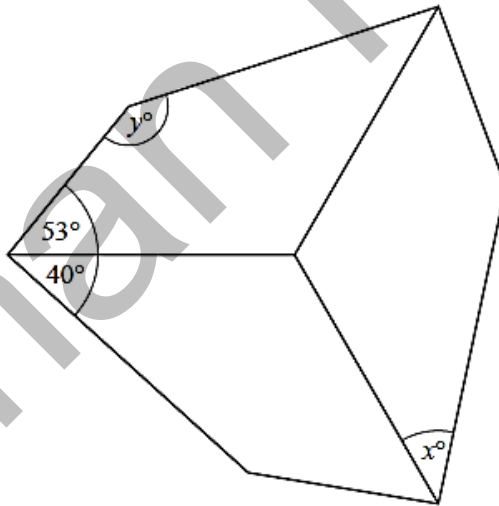
(a) How many lines of symmetry does the figure have?

Answer ....[1]

(b) Find x.

Answer $x =$ ....[1]

(c) Find y.

Answer $y =$..[1]

Answers: (a) 0 (b) 40 (c) 147   **N13/11/Q15**

39
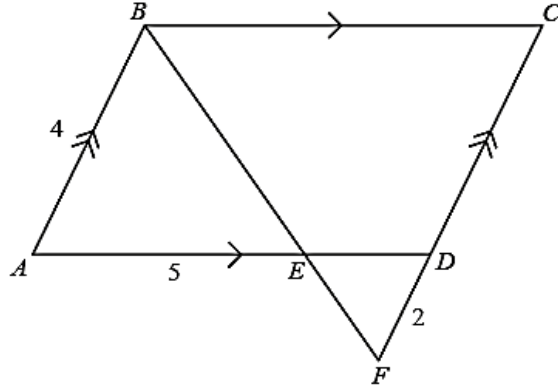
ABCD is a parallelogram.
BEF and *CDF* are straight lines.
AB = 4 cm, *DF* = 2 cm and *AE* = 5 cm.

(a) Show that triangles *ABE* and *CFB* are similar.
Give reasons for each of your statements. [2]

(b) Calculate *BC*. Answer .... cm [2]

(c) Triangle *DFE* is also similar to triangle *ABE*.
Given that the area of triangle *DFE* is *x* cm^2, find the area of *ABCD* in terms of *x*.

Answer .... cm^2 [2]

Answers: (a) Two corresponding pairs of angles equated, with reasons (b) 7.5 (c) 12*x*   **N15/11/Q25**

40
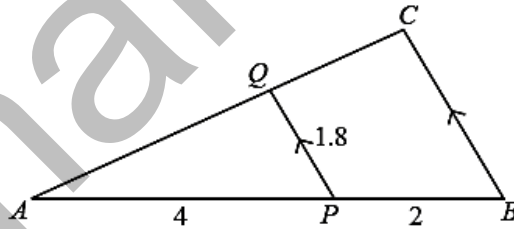
In the diagram, triangles *APQ* and *ABC* are similar.
BC is parallel to *PQ* and *AP* = 4 cm, *PB* = 2 cm and *PQ* = 1.8 cm.

(a) Find *BC*. Answer .... cm [2]

(b) Find $\dfrac{\text{area of triangle } APQ}{\text{area of quadrilateral } PBCQ}$. Answer .... [1]

Answers: (a) 2.7 (b) $\dfrac{4}{5}$ **N16/11/Q14**

260

41　　**(a)**　The diagram shows part of a figure that has *AB* as its line of symmetry.

Complete the figure.
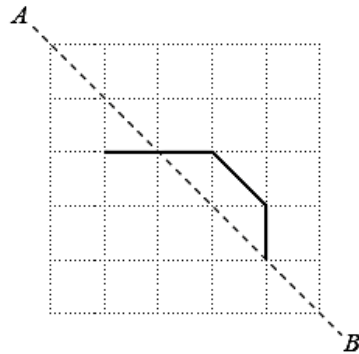
[1]

(b)　In the diagram, six small squares are shaded.

Shade **two** more small squares so that the completed diagram has rotational symmetry of order 4.

[1]
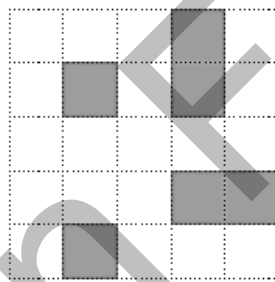
Answers: (a)　　(b)
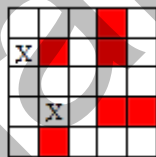
Symmetry, Congruence and Similarity Paper 2

1   (a)   Show that the interior angle of a regular pentagon is 108°.   [2]

(b)
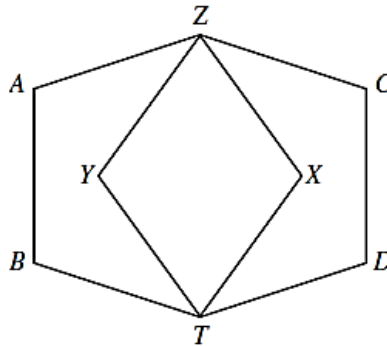
The diagram shows two congruent, regular pentagons, *ZABTX* and *ZCDTY*.

    (i)   Describe fully all the symmetries of this diagram.   [2]

   (ii)   What is the special name given to the quadrilateral *ZXTY*?   [1]

  (iii)   Calculate reflex angle *ZYT*.   [1]

  (iv)   Calculate angle *AZY*.   [1]

Answers: **(b)(i)** 2 lines of symmetry, rotational symmetry of order 2, **(ii)** rhombus, **(iii)** 252°, **(iv)** 36°;   **J04/2/Q4**

2

(b)   This hexagon has rotational symmetry of order 3.

     Calculate the value of *z*.
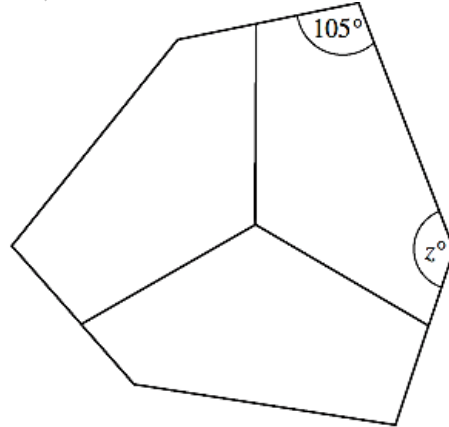
       [2]

(c)   In the diagram, triangle *PQR*
is similar to triangle *PSQ*.
$P\hat{Q}R = P\hat{S}Q$.

     $PQ = 18$ cm, $QR = 14$ cm and $QS = 21$ cm.

     Calculate the length of
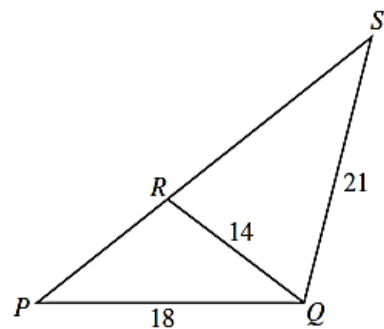
(i) *PR*, [1]

(ii) *RS*. [2]

(b) 135°; **(c)(i)** 12 cm, **(ii)** 15 cm.

---

3 (b) In the diagram, triangle *AQR* is similar to triangle *ABC*.

   $AQ = 8\,cm$, $QB = 6\,cm$ and $AR = 10\,cm$.
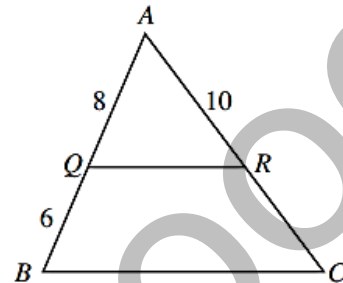
   (i) Calculate the length of *RC*. [2]

   (ii) Given that the area of triangle *AQR* is $32\,cm^2$, calculate the area of triangle *ABC*. [2]

(b) (i) 7.5 cm, (b) 98 cm².

---

4 (b) You may use the grid below to help you answer this question.

   The points $(2, 1), (4, 3), (3, 1)$ and (p, q) form a quadrilateral.
   This quadrilateral has rotational symmetry order 1 and one line of symmetry.

   (i) One possible position of (p, q) is $(2, 2)$.

        Write down the name of this special quadrilateral.

                                    Answer .... [1]

   (ii) Given that *p* and *q* are integers, find two other possible positions of (p, q).

                                    Answer   (........... , ...)

                                                  (........... , ...)   [2]

263
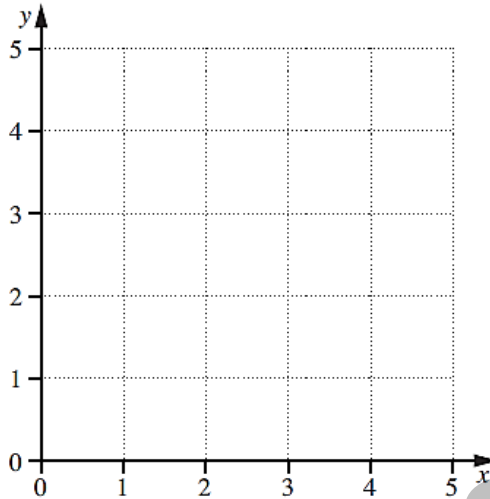
5   **(a)**   **(i)**
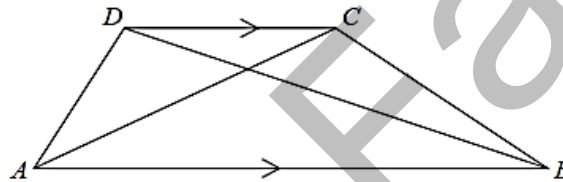
In trapezium *ABCD*, *AB* is parallel to *DC*. *DB* and *AC* are straight lines.

Explain why the area of triangle *ACB* = the area of triangle *ADB*.   [1]

(ii)
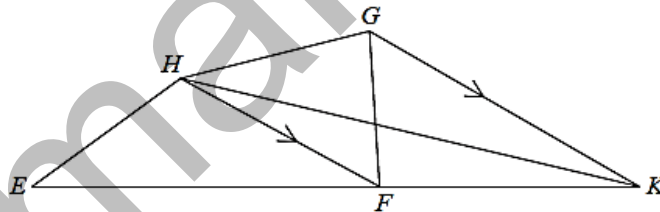
The diagram shows the quadrilateral *EHGK*.
HF is parallel to *GK* and *EFK* is a straight line.

(a) Name a triangle equal in area to triangle *HFK*.   *Answer*   ....[1]

(b) Hence show that the area of triangle *HEK* = the area of quadrilateral *HEFG*.   [1]

(b)
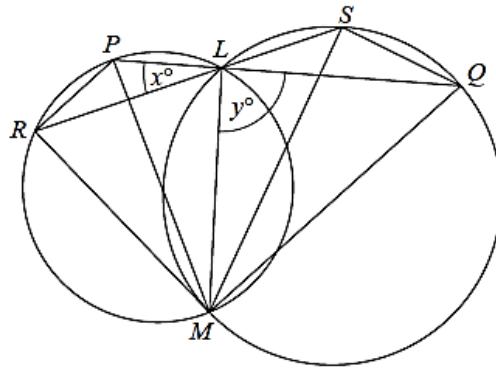
Two circles intersect at L and M.
R and P are on the circumference of one circle. S and Q are on the circumference of the other circle.
PLQ and RLS are straight lines.
$P\hat{L}R = x°$ and $M\hat{L}Q = y°$.

(i) Complete the proof that $S\hat{M}Q = x°$.

Statement	Reason
$x° = P\hat{L}R = S\hat{L}Q$	....
$S\hat{L}Q = S\hat{M}Q = x°$	.... [2]

(ii) Prove that $P\hat{R}M = y°$.

[2]

Statement   Reason

(iii) Complete the following statement, giving your reasons.

The triangles PRM and QSM are ....

Reasons ....

.... [3]

6
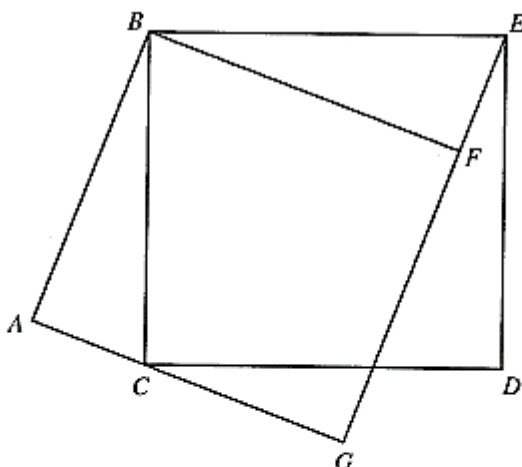
In the diagram, *ABFG* is a rectangle and *BCDE* is a square.

ACG and *GFE* are straight lines.

(a) Show that angle *ABC* = angle *FBE*. [2]

(b) Show that triangle *ABC* is congruent to triangle *FBE*. [3]

(c) Hence show that *ABFG* is a square. [1]

7
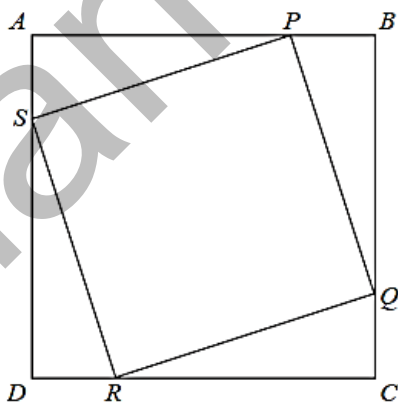
In the diagram, *ABCD* is a square.

Points *P*, *Q*, *R* and *S* lie on *AB*, *BC*, *CD* and *DA* so that *AP = BQ = CR = DS*.

(a) Giving all your reasons, show that

   (i) *PB = QC*, [2]

   (ii) triangle *BPQ* is congruent to triangle *CQR*, [3]

   (iii) *PQR* is a right angle. [2]

(b) Write down two reasons to show that *PQRS* is a square. [2]

8
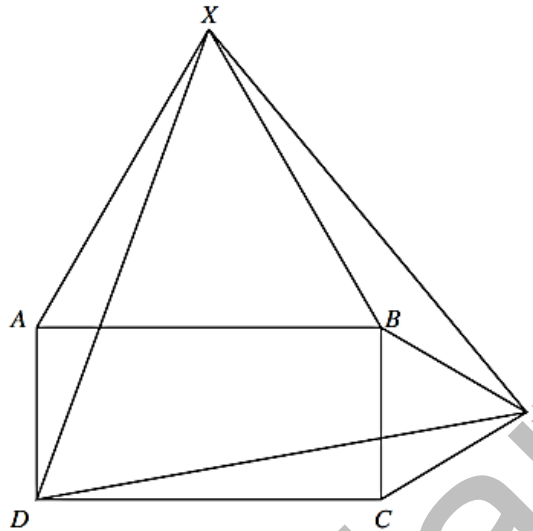
The diagram shows a rectangle *ABCD*.

Triangles *ABX* and *BCY* are equilateral.

(a) Find $X\hat{B}Y$. [1]

(b) Show that triangles *AXD* and *BXY* are congruent. [3]

(c) Show that $D\hat{X}Y = 60°$. [2]

(d) Show that triangle *DXY* is equilateral. [2]

Answer: **(a)** 150°

9 **(b)** In a surveying exercise to find the distance between two points, Y and Z, on opposite banks of a river, angles and lengths were measured.
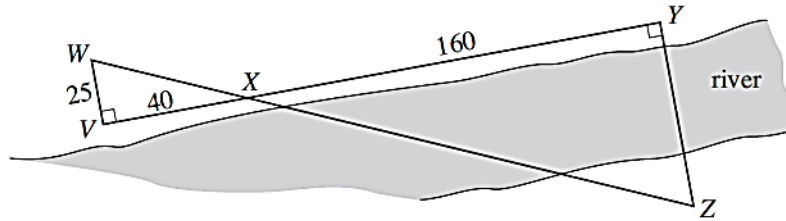
WXZ and VXY are straight lines.
$W\hat{V}X = Z\hat{Y}X = 90°$.

(i) Show that triangles VWX and YZX are similar. [2]

(ii) $VW = 25\,\text{m}$, $VX = 40\,\text{m}$ and $XY = 160\,\text{m}$.

Calculate the distance YZ. [2]

Answers: **(a)(i)** 56°, **(ii)** 68°, **(b)(i)** See above **(ii)** 100 m.
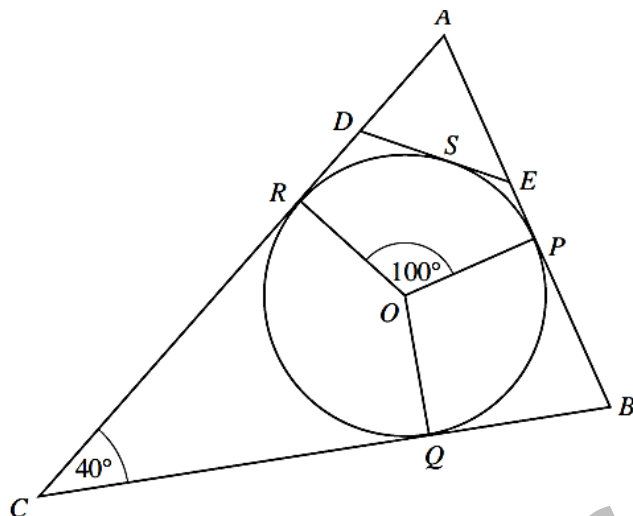
The diagram shows a circle, centre O.
The lines AB, BC and CA touch the circle at P, Q and R respectively.

(a) (i) Explain why $C\hat{Q}O = 90°$. [1]

(ii) Given that $A\hat{C}B = 40°$, find $R\hat{O}Q$. [1]

(b) The line DE touches the circle at S.
The triangles ABC and ADE are similar.

(i) Write down the value of $A\hat{E}D$. [1]

(ii) Given that $R\hat{O}P = 100°$, find $R\hat{O}S$. [2]

(iii)
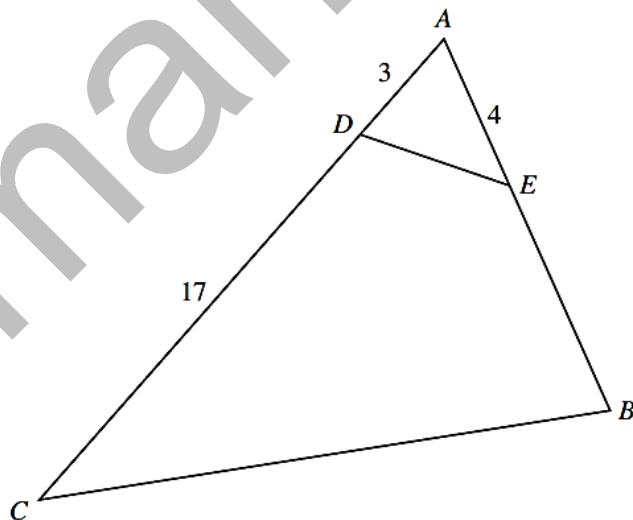
Given also that $AD = 3\,$cm, $CD = 17\,$cm and $AE = 4\,$cm, calculate BE. [2]

Answers: **(a)(i)** An angle between a tangent and a radius **(ii)** 140° **(b)(i)** 40°, **(ii)** 60°, **(iii)** 11.   **N08/2/Q5**

11
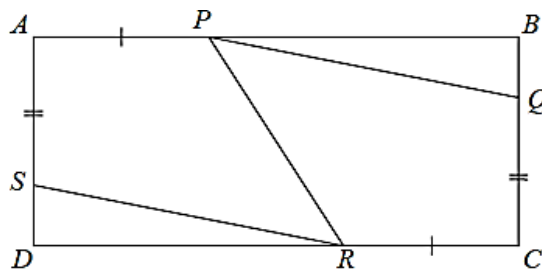
ABCD is a rectangle.
Points P, Q, R and S lie on AB, BC, CD and DA respectively such that AP = CR and QC = SA.

(a) Giving reasons, show that

    (i)   PB = RD,   [1]

    (ii) triangle PBQ is congruent to triangle RDS,   [3]

    (iii) $R\hat{P}Q = P\hat{R}S$.   [3]

(b) State the special name of the quadrilateral PQRS.   [1]

Answers: (a)(i) AB = CD, AP = CR, therefore AB − AP = CD − CR   (ii) PB = RD, BQ = DS, $P\hat{B}Q$ **N09/2/Q2**
= $S\hat{D}R$, therefore triangles PBQ and RDS are congruent   (iii) $B\hat{P}Q = D\hat{R}S$, $R\hat{P}B = P\hat{R}D$,
therefore $R\hat{P}B − B\hat{P}Q = P\hat{R}D − D\hat{R}S$ (b) Parallelogram

---

12   (a) Calculate the interior angle of a regular 10-sided polygon.   [2]

    (b)
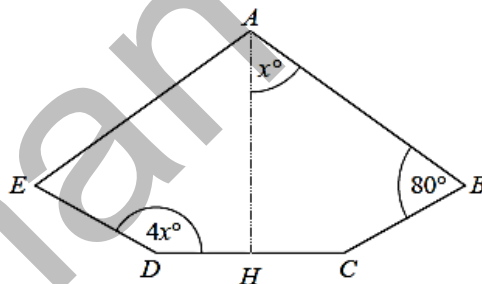
AH is the line of symmetry of the pentagon ABCDE.
$H\hat{A}B = x°$, $A\hat{B}C = 80°$ and $E\hat{D}H = 4x°$.

Find x.   [3]

(c)
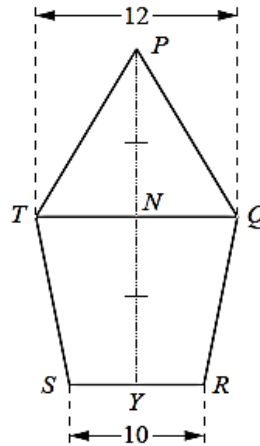
PY is the line of symmetry of the pentagon PQRST.
PY and TQ intersect at N.
PN = NY.
TQ = 12 cm and SR = 10 cm.

(i) Given that PY = 2h centimetres, find an expression, in terms of h, for the area of the trapezium QRST. [2]

(ii) Given that the area of PQRST is 221 cm², calculate h. [2]

Answers: (a) 144° **(b)** 38 **(c)(i)** ½(10 + 12)h, **(ii)** 13 **N10/22/Q3**

13 **(a)**
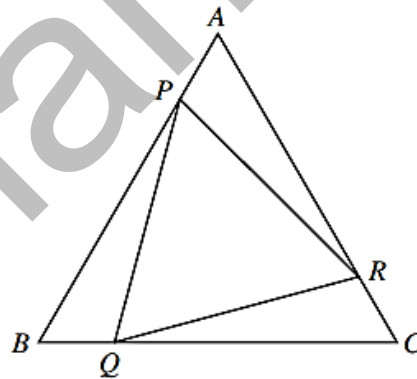
In the diagram, ABC is an equilateral triangle.
The points P, Q and R lie on AB, BC and CA respectively, such that AP = BQ = CR.

(i) Show that triangles APR, BQP and CRQ are congruent. [3]

(ii) It is given that   AB = 5 cm and PQ = 4 cm   .

(a) Find $\dfrac{\text{Area of triangle } PQR}{\text{Area of triangle } ABC}$.   Answer ....[1]

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(b) Find $\dfrac{\text{Area of triangle } APR}{\text{Area of triangle } ABC}$.

Answer ....[1]

Answers: **(ii)(a)** $\dfrac{16}{25}$ **(b)** $\dfrac{3}{25}$ **(b)(iii)(a)** 45 **(b)** 135 **(iv)** 98

14
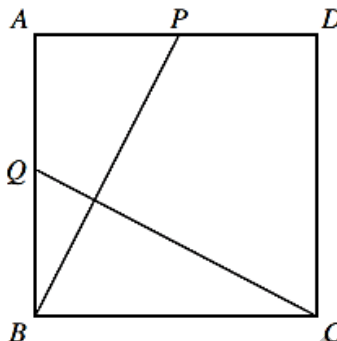
Diagram I

In **Diagram I**, $ABCD$ is a square.
P and Q are the midpoints of AD and AB respectively.

(a) Show that triangles APB and BQC are congruent.

....[3]

(b)

Diagram II
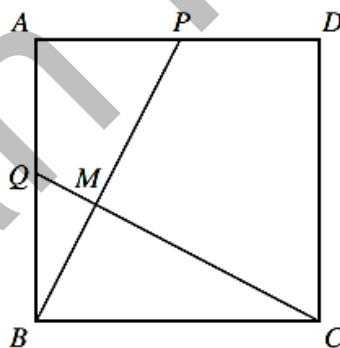
In **Diagram II**, QC and PB intersect at M.

Show that $B\hat{M}C = 90°$.
State your reasons clearly.

....[2]

(c)
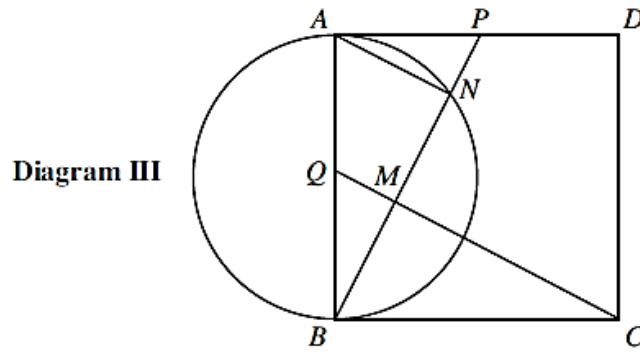
Diagram III

In **Diagram III**, the circle centre Q has diameter AB.
The circle intersects BP at N.

(i) State the reason why $A\hat{N}B = 90°$.

Answer ....[1]

(ii) Triangle BMQ is mapped onto triangle BNA by an enlargement.
Write down the centre and scale factor of the enlargement.

Answer   ....

    .... [1]

(iii) Given that $QM = 3\,\text{cm}$,

  (a) find AN, Answer   .... cm [1]

  (b) show that $MN = 6\,\text{cm}$,

  Answer ....

  ....[1]

  (c) find MC, Answer   .... cm [1]

  (d) find the area of triangle APB.

  Answer   ....cm² [2]

Answers: (c)(i) Angle in a semicircle (ii) Centre B Scale factor 2 (iii)(a) 6 (c) 12 (d) 45   **N11/22/Q9**

273

15 **(b)** *AOB* and *COD* are straight lines.
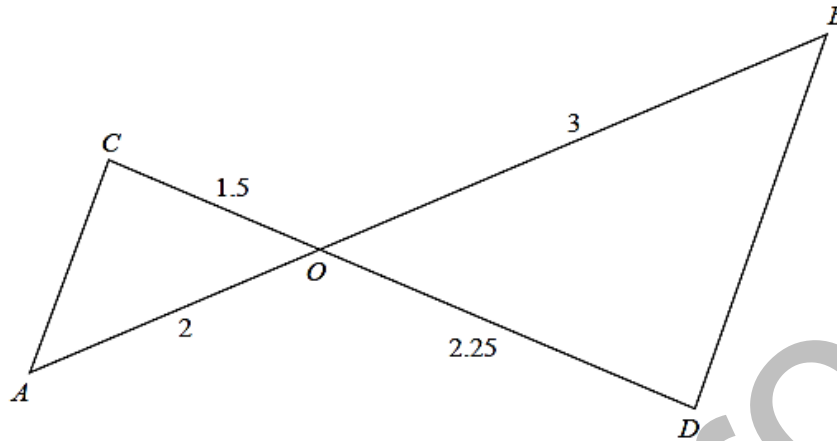
(i) Show that triangles *OCA* and *ODB* are similar.

Answer ....

....[2]

(ii) Given that *BD* = 2.7 cm, find *AC*.

Answer .... cm [1]

Answer: (a)(i) 20° (ii) 70° (iii) Rectangle (b)(ii) 1.8   **N12/21/Q4**

16

(a)
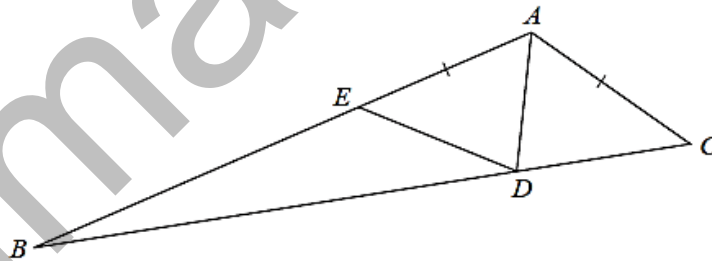
In triangle *ABC*, *D* is the point on *BC* such that *AD* bisects *BÂC* and *E* is the point on *AB* such that *AE = AC*.

(i) Show that triangles *AED* and *ACD* are congruent.   [3]

(ii) Given that *AB̂D* = *x*°, *ED̂B* = *y*° and *AĈB* = *z*°,
find *x* in terms of *y* and *z*.

Answer   *x* = ...[2]

274

(b)
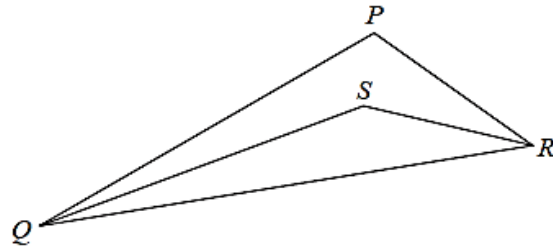
In triangle PQR, QS bisects $P\hat{Q}R$ and RS bisects $P\hat{R}Q$.
$P\hat{Q}R = 42°$ and $P\hat{R}Q = 54°$.

Find reflex angle QSR.   *Answer* ....[2]

Answer: (a)(ii) $z - y$ (b) 228°   **N13/21/Q7**

17   **(a)**
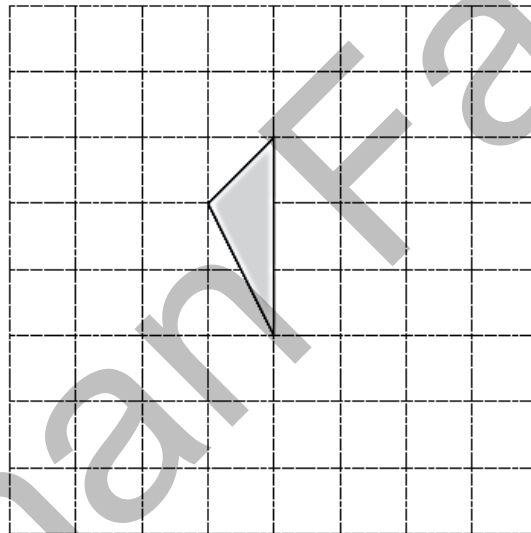
The shaded triangle, drawn on the grid, is part of a quadrilateral with one line of symmetry. The area of the quadrilateral is twice the area of the triangle.

Given that the line of symmetry is **not** vertical, complete the quadrilateral.   [1]

275

(b)
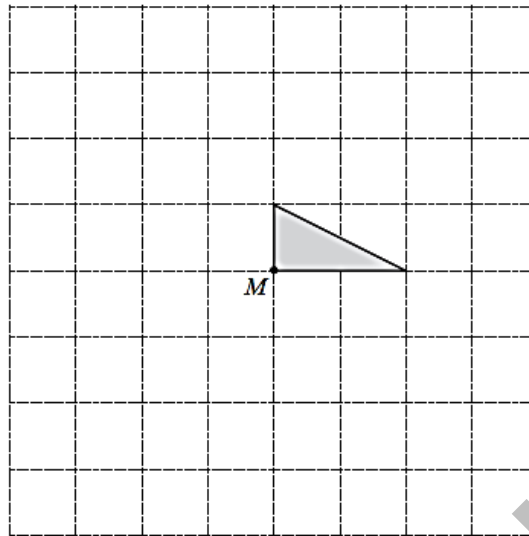
The shaded triangle, drawn on the grid, is part of a shape whose area is 4 times the shaded area and has rotational symmetry of order 4 about M.

Complete the shape. [2]

18   **11**
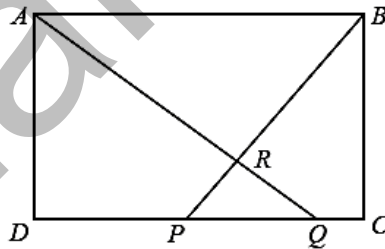
$ABCD$ is a rectangle.
P and Q are points on DC.
AQ and BP intersect at R.

(a) Prove that triangle ARB is similar to triangle QRP.
Give a reason for each statement you make.

.... [3]

Trigonometry Paper 1

1 It is given that $\sin 30° = 0.5$ and $\cos 30° = 0.866$.

    (a) Write down the value of

        (i) $\cos 150°$,   *Answer (a)*(i) $\cos 150° =$ .... [1]

        (ii) $\cos 60°$.

    (b) A triangle has sides of length 6 cm and 5 cm.   (ii) $\cos 60° =$ .... [1]
        The angle between these two sides is 150°.
        Calculate the area of the triangle.   *(b)* .... cm² [2]

Answers: **(a)(i)** – 0.866, **(ii)** 0.5; **(b)** 7.5 cm².

---

2 A man who is 1.8 m tall stands on horizontal ground 50 m from a vertical tree.
The angle of elevation of the top of the tree from his eyes is 30°.
Use as much of the information below as is necessary to calculate an estimate of the height of the tree.
Give the answer to a reasonable degree of accuracy.

    $[\sin 30° = 0.5 \, , \ \cos 30° = 0.866 \, , \ \tan 30° = 0.577]$ *Answer* .... m [4]

Answer: 31 m.

---

3   **(a)**

	sin	cos
30°	0.5	0.87
60°	0.87	0.5

Using as much information in the table as necessary, evaluate 2sin150°.

                        Answer (a)...[1]

  (b)
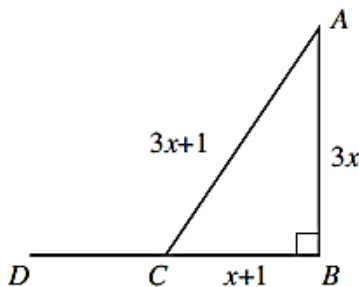
In the triangle ABC, $A\hat{B}C = 90°$, $AB = 3x$ cm, $BC = (x + 1)$ cm and $AC = (3x + 1)$ cm.

(i) Form an equation in x and show that it reduces to $x^2 - 4x = 0$.

Answer (b)(i) ....

....[2]

(ii) Find the value of x.

(iii) Given that BCD is a straight line, state the numerical value of $\cos D\hat{C}A$.

Answer (b) (ii) $x =$ ....[1]

(iii) ....[1]

Answer: (a) 1   (b)(ii) 4   (iii) $-\dfrac{5}{13}$

4
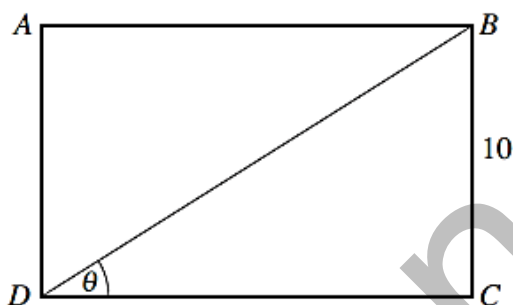
$\sin \theta$	$\dfrac{5}{13}$
$\cos \theta$	$\dfrac{12}{13}$
$\tan \theta$	$\dfrac{5}{12}$

$ABCD$ is a rectangle with $BC = 10$ cm.

Using as much information from the table as is necessary, calculate BD.

Answer $BD =$ .... cm [2]

Answer: 26 cm

5
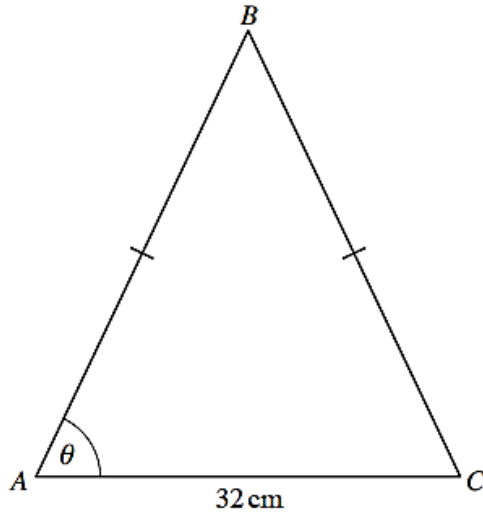
sin θ	$\frac{15}{17}$
cos θ	$\frac{8}{17}$
tan θ	$\frac{15}{8}$

ABC is an isosceles triangle with *AB = BC* and *AC = 32* cm.

Using as much information from the table as is necessary, calculate

(a) *AB*,

Answer (*a*) .... cm [2]

(b) the area of triangle *ABC*.

Answer (*b*) .... cm² [2]

Answer: (a) 34   (b) 480   **J10/12/Q23**

6
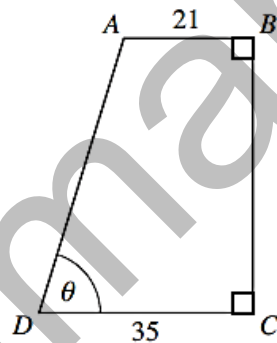
sin θ	$\frac{24}{25}$
cos θ	$\frac{7}{25}$
tan θ	$\frac{24}{7}$

ABCD is a trapezium with *AB = 21* cm and *CD = 35* cm.
ABĈ = BĈD = 90° and *ADĈ = θ*.

Using as much information from the table as is necessary, calculate *AD*.

Answer   .... cm [2]

Answer: 50   **J11/12/Q11**

7 **(a)** One approximate solution of the equation $\sin x° = 0.53$ is $x = 32$.

Use this value of x to find the solution of the equation that lies between $90°$ and $180°$.

Answer ....[1]

(b)
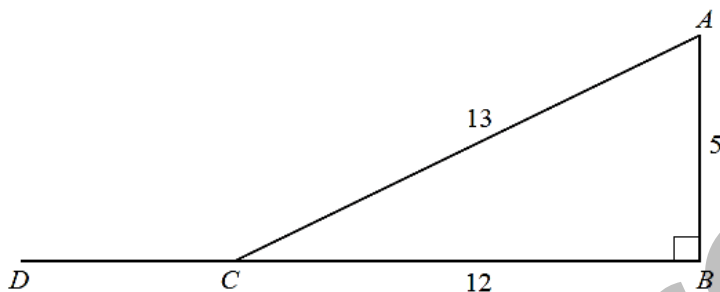
Triangle ABC is right-angled at B and BC is produced to D.
$AB = 5\,\text{cm}$, $BC = 12\,\text{cm}$ and $AC = 13\,\text{cm}$.

Write down the value of $\cos A\hat{C}D$.

Answer $\cos A\hat{C}D = $ ....[1]

8 In the triangle PQR, $PQ = 5\,\text{cm}$, $QR = 7\,\text{cm}$ and $PR = 9\,\text{cm}$. *Answer*   Triangle PQR is ....

Decide whether the triangle is acute angled or obtuse angled.
Show calculations to support your decision.

9 In the diagram, BCD is a straight line,

$BC = 5$ cm, $AB = 12$ cm, $AC = 13$ cm and $A\hat{B}C = 90°$.

Find

(a) tan $B\hat{A}C$,

(b) cos $A\hat{C}D$.

Give both answers as fractions.

Answer (a) tan $B\hat{A}C$ = ... [1]

(b) cos $A\hat{C}D$ = ...[1]

Answers: **(a)** $\dfrac{5}{12}$; **(b)** $-\dfrac{5}{13}$.

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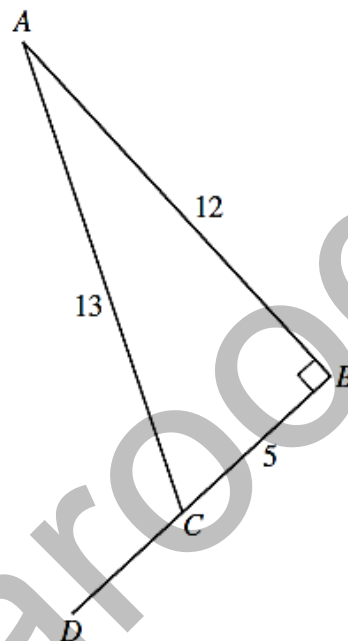
10 In the diagram, the circle, centre O, passes through A, B and C.
AC is a diameter of the circle and the line
TAS is the tangent at A.
$\angle ACB = 34°$, $TA = 3$ cm and $TC = 5$ cm.

(a) Find $\angle BAC$.

(b) Calculate the radius of the circle.

Answer (a) $\angle BAC$ = ...[1]

(b) ....cm [1]

Answer: (a) $56°$ (b) 2 cm

281

11
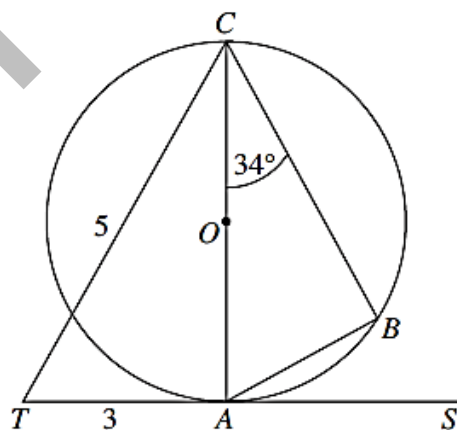
sin $b°$	cos $b°$	tan $b°$
0.85	0.53	1.6
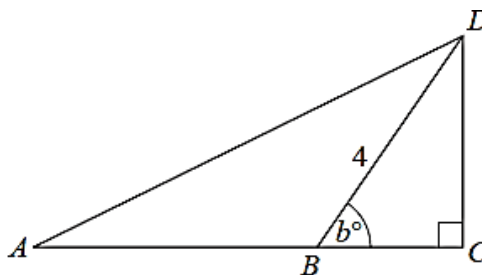
In the diagram, ABC is a straight line.
$BD = 4\,cm$, $B\hat{C}D = 90°$ and $C\hat{B}D = b°$.

Use as much information given in the table as is necessary to answer the following questions.

(a) Calculate the value of $\sin A\hat{B}D + \cos A\hat{B}D$.

Answer (a) ....[2]

(b) Calculate BC.

Answer (b) $BC =$ .... cm [2]

Answers: (a) 0.32 (b) 2.12   **N10/11/Q21**

12

In the diagram, $A\hat{B}C = 90°$,
$BC = 5\,cm$ and $AC = \sqrt{89}\,cm$.

(a) What special kind of number is $\sqrt{89}$?

Answer (a) ....[1]

(b) Calculate AB.

Answer (b) .... cm [2]
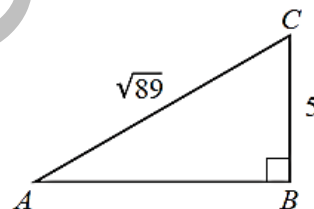
Answer: (a) irrational (b) 8   **N10/12/Q13**

13
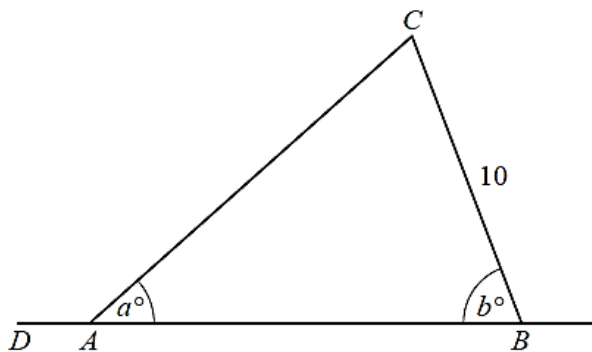
	$x°$	$a°$	$b°$
$\sin x°$	$\dfrac{3}{5}$	$\dfrac{24}{25}$	
$\cos x°$	$\dfrac{4}{5}$	$\dfrac{7}{25}$	
$\tan x°$	$\dfrac{3}{4}$	$\dfrac{24}{7}$	

In the diagram, DAB is a straight line.
$BC = 10\,\text{cm}$, $C\hat{A}B = a°$ and $C\hat{B}A = b°$.

Use as much information given in the table as is necessary to answer the following questions.

(a) Write down the value of $\cos D\hat{A}C$. *Answer* (a) ....[1]

(b) Calculate AC. *Answer* (b) $AC =$ .... cm [3]

Answer: (a) $-\dfrac{4}{5}$ (b) 16

14 **(a)** $\cos y° = -0.54$ where $90 < y < 180$

One solution of the equation $\cos x° = 0.54$ is $x = 57$, correct to the nearest whole number.

Find y correct to the nearest whole number. *Answer* $y =$ .... [1]

(b) Solve $\dfrac{5a - 2}{3} = 11$. *Answer* $a =$ .... [2]

Answers: **(a)** 123 **(b)** 7

15
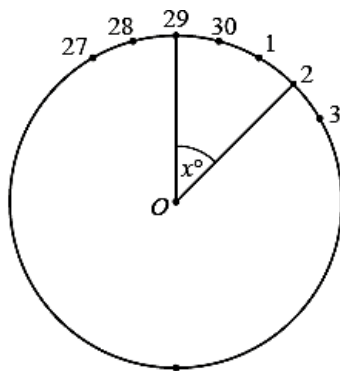
The diagram represents a vertical, circular fairground wheel which turns about its centre O. The wheel has 30 seats, equally spaced around the circumference, numbered consecutively from 1 to 30.

The diagram, which is not drawn to scale, shows seven of the seats, labelled with the seat number.

The seat number 29 is at the top of the wheel.

(a) What is the number of the seat which is at the bottom of the wheel?

Answer .... [1]

(b) Calculate the angle $x°$, as shown on the diagram.   *Answer* .... [1]

(c) Work out the angle of elevation of seat 2 from the bottom of the wheel.

Answer .... [2]

Answers: **(a)** 14 **(b)** 36 **(c)** 72

Trigonometry Paper 2

1 The diagram shows a vertical mast, TB, of height 18 m.
A, B and C are three points on horizontal ground.
TA and TC are two straight wires.
TC has length 27 m and $A\hat{T}B = 31°$.
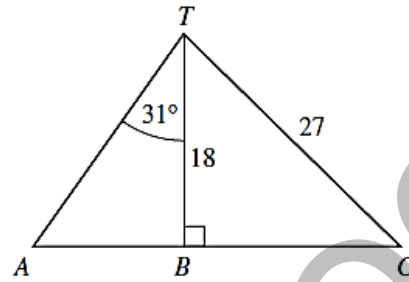
(a) Calculate

    (i) $B\hat{C}T$, [2]

    (ii) the length of TA. [3]

(b) A third straight wire, TD, joins T to a point D on the same horizontal ground.
    Given that $D\hat{T}B = 51°$, find the angle of elevation of T from D. [1]

2
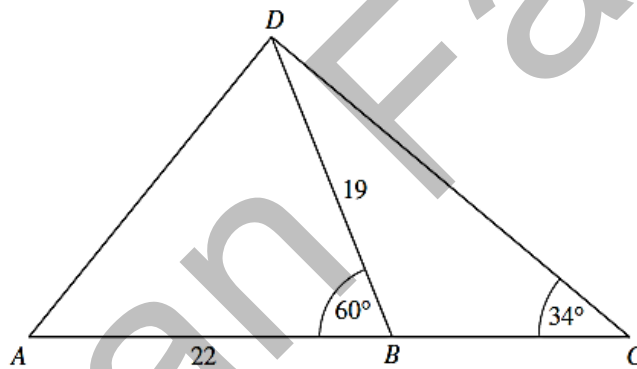
The diagram shows a straight line ABC and a point D.
$AB = 22$ cm, $BD = 19$ cm, $A\hat{B}D = 60°$ and $B\hat{C}D = 34°$.

Calculate

(a) the length of BC, [4]

(b) the length of AD, [4]

(c) the area of triangle ABD, [2]

(d) the shortest distance from B to AD. [2]

3

(a) (i) Evaluate $\dfrac{4.8^2 - 1.7^2}{4.8 \times 1.7}$. [1]

(ii) Find a value of x for which $\sin x° = \tan 12° + \cos 46°$. [1]

(b) The diagram shows a framework $ABCD$.

$AD = 2.2$ m, $BD = 1.9$ m and $B\hat{C}D = 42°$.
$A\hat{B}D = B\hat{D}C = 90°$.
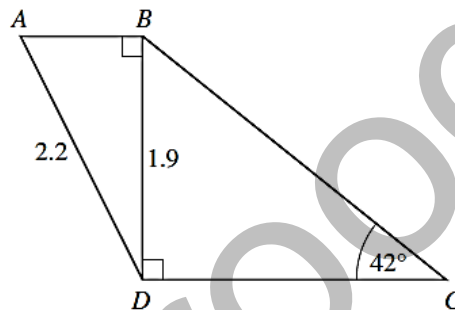
Calculate

(i) $A\hat{D}B$, [2]

(ii) BC. [3]

(c) A vertical flagpole, 18 m high, stands on horizontal ground.

Calculate the angle of elevation of the top of the flagpole from a point, on the ground, 25 m from its base. [2]

Answers: (a)(i) 2.47, (ii) 65.1; (b)(i) 30.3°, (ii) 2.84; (c) 35.8°. **J03/2/Q1**

4

286
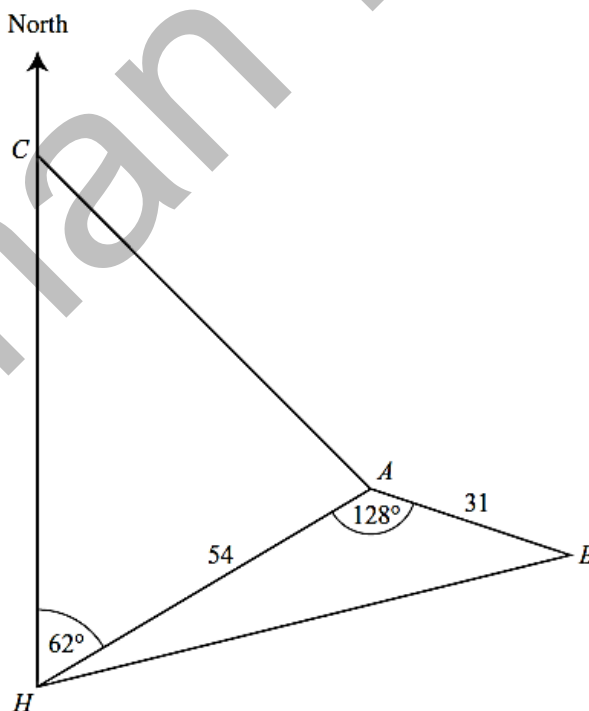
The diagram shows the position of a harbour, H, and three islands A, B and C.

C is due North of H.

The bearing of A from H is $062°$ and $H\hat{A}B = 128°$.

$HA = 54$ km and $AB = 31$ km.

(a) Calculate the distance HB. [4]

(b) Find the bearing of B from A. [1]

(c) The bearing of A from C is $133°$.

Calculate the distance AC. [4]

(d) A lightship, L, is positioned due North of H and equidistant from A and H.

Calculate the distance HL. [3]

Answers: (a) 77.1km; (b) 114°; (c) 65.2km; (d) 57.5 km.

5 (a) The diagram shows a trapezium $ABCD$.

Angle ABC and angle BCD are right angles.

$AB = 9$ cm, $BC = 4$ cm, $CD = 6$ cm and $DA = 5$ cm.

The perpendicular distance from B to AD is h centimetres.
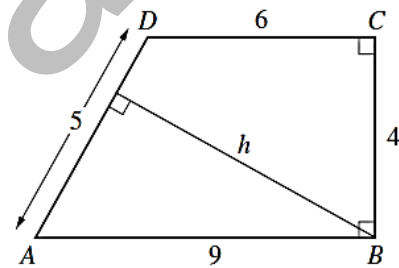
Calculate

(i) the area of the trapezium, [1]

(ii) the value of h, [2]

(iii) angle DAB. [2]

(b) The diagram shows two triangles, PRS and PRQ.

$PR = 8$ cm, $QR = 8.5$ cm, $P\hat{S}R = 90°$, $P\hat{R}S = 51°$ and $R\hat{P}Q = 95°$.
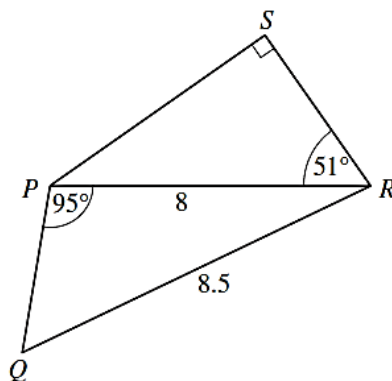
(i) Calculate RS. [2]

(ii) Calculate $P\hat{Q}R$. [3]

6
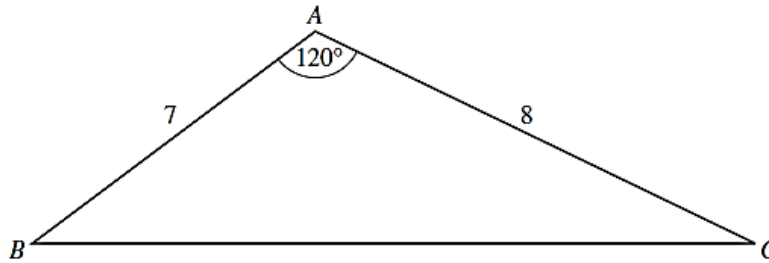
Diagram I

Diagram I shows a triangle *ABC* in which *AB* = 7 cm, *AC* = 8 cm and $B\hat{A}C = 120°$.

(a) Show that *BC* = 13 cm. [2]

(b) Calculate the area of triangle *ABC*. [2]

(c)
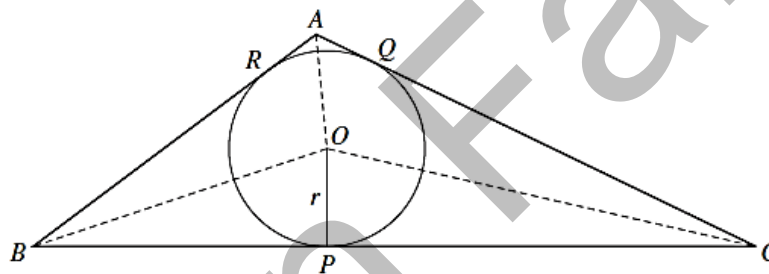
Diagram II

The sides of the triangle *ABC*, shown in Diagram I, are tangents to a circle with centre *O* and radius *r* centimetres.
The circle touches the sides *BC*, *CA* and *AB* at *P*, *Q* and *R* respectively, as shown in Diagram II.

(i) Find an expression, in terms of *r*, for the area of triangle *OBC*. [1]

(ii) By similarly considering the areas of triangles *OAB* and *OAC*, find an expression, in terms of *r*, for the area of triangle *ABC*. [2]

(iii) Hence find the value of *r*. [2]

(d) Calculate the percentage of the area of triangle *ABC* that is **not** occupied by the circle. [3]

Answers: **(b)** 24.2 cm²; **(c)(i)** $\frac{13}{2}r$, **(ii)** 14r, **(iii)** 1.73; **(d)** 61.1%.

7
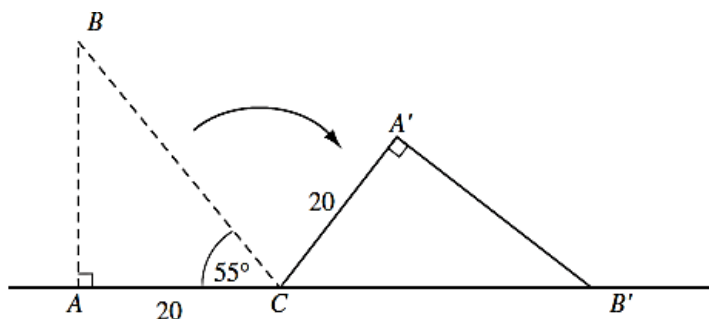
In triangle ABC, $B\hat{A}C = 90°$, $B\hat{C}A = 55°$ and $AC = 20$ cm.
The triangle initially stood with AC on a horizontal surface.
It was then rotated about the point C onto triangle $A'B'C$, where ACB' is a straight line.

(a) Calculate

    (i)   the length of BC,   [2]

    (ii) the distance AB',   [1]

    (iii) the height of A' above CB'.   [2]

(b) Describe **fully** the path which the point A followed under this rotation.   [2]

(c) Calculate the length of the path which the point A followed under this rotation.   [2]

Answers: **(a)(i)** 34.9 cm, **(ii)** 54.9 cm, **(iii)** 16.4 cm; **(b)** 125° arc of a circle, centre C; **(c)** 43.6 cm.   **J05/2/Q4**

8
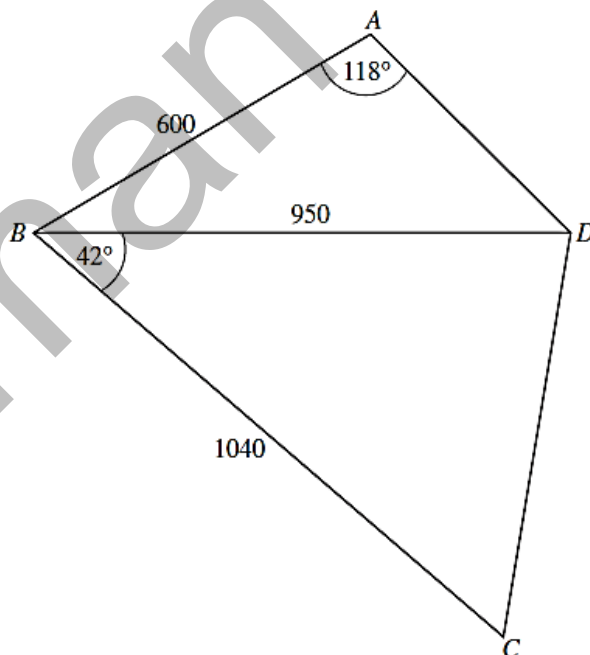
In the diagram, the quadrilateral ABCD represents a level park with a path BD.
AB = 600 m, BC = 1040 m, BD = 950 m, $C\hat{B}D$ = 42° and $B\hat{A}D$ = 118°.

(a) Calculate

    (i)   angle ABD, [4]

    (ii)   the length of CD, [4]

    (iii)   the shortest distance from C to BD. [2]

(b) A helicopter flew directly above the path BD at a constant height of 500 m.

    Calculate the greatest angle of depression of the point C as seen by a passenger on the helicopter. [2]

Answers: **(a)(i)** 28.1°, **(ii)** 718 m, **(iii)** 696 m; **(b)** 35.7°.   **J05/2/Q9**

9   (b) Diagram I represents part of the framework of the ride.

The points A, B, C, D, E and F are on the framework.
The points H, C, G, E and F lie on a horizontal line.
The lines BH and DG are vertical.

BC = 80 m, HC = 60 m, DG = 40 m, GE = 35 m and $D\hat{C}G$ = 32°.
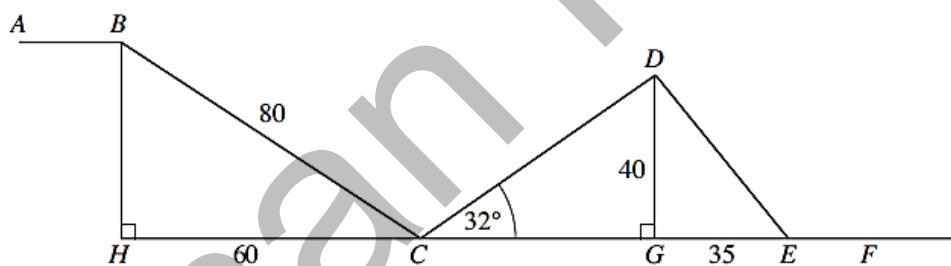
Diagram I

Calculate

    (i) $H\hat{C}B$, [2]

    (ii)   CD, [3]

    (iii)   the angle of depression of E from D. [2]

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(c)
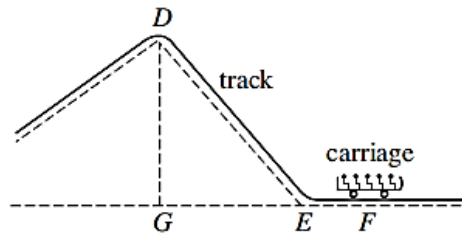
Diagram II

Diagram II shows part of the ride.
The carriage that carried the family was 4.6 m long.
It was travelling at a constant speed of 15 m/s as it passed the point F.

(i) Calculate, correct to the nearest hundredth of a second, the time taken for the carriage to pass the point F. [2]

(ii) Express 15 m/s in kilometres per hour. [1]

(b)(i)41.4°, (ii) 75.5 m, (iii) 48.8°; (c)(i) 0.31 s, (ii) 54 km/h. **J06/2/Q4b**

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10 In the diagram, A and B are two points on a straight coastline.

B is due east of A and $AB = 7$ km.

The position of a boat at different times was noted.
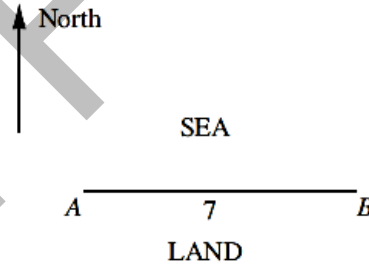
(a) At 8 a.m., the boat was at C, where $A\hat{C}B = 66°$ and $A\hat{B}C = 48°$.

Calculate

(i) the bearing of B from C, [1]

(ii) the distance AC. [3]

291
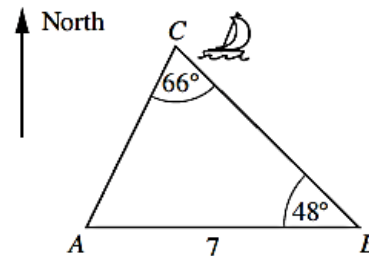
(b) At 9 a.m., the boat was at D, where $AD = 6.3$ km and $D\hat{A}B = 41°$.

North

D

6.3

41°

A   7   B
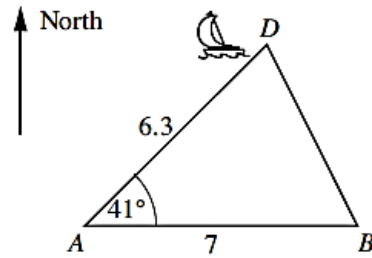
Calculate

(i) the area of triangle ADB, [2]

(ii) the shortest distance from the boat to the coastline. [2]

(c) At 11 a.m., the boat was at E, where $AE = 9$ km and $BE = 5$ km.

North

E

9

5

A   7   B
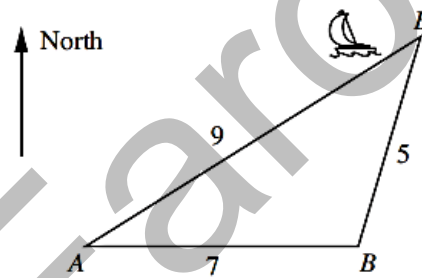
Calculate the bearing of E from A. [4]

Answers: (a) (i) 138°, (ii) 5.69 km (b)(i) 14.5 km², (ii) 4.13 km; (c) 56.4°. **J06/2/Q9**

11

The diagram represents a framework.

$BC = 1.3$ m, $BD = 1.9$ m and $BE = 1.5$ m.
$B\hat{C}D = 76°$, $B\hat{A}E = 68°$ and $B\hat{E}D = 90°$.

1.3

76° C

B

1.9

1.5

68°

A   E   D
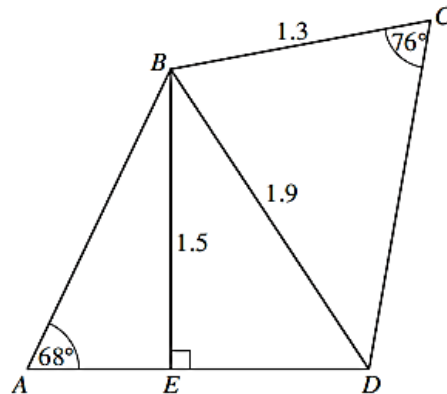
Calculate

(a) $D\hat{B}E$, [2]

(b) AE, [2]

(c) $B\hat{D}C$. [3]

Answer: (a) 37.9°;   (b) 0.606 m;   (c) 41.6° **J07/2/Q2**

12
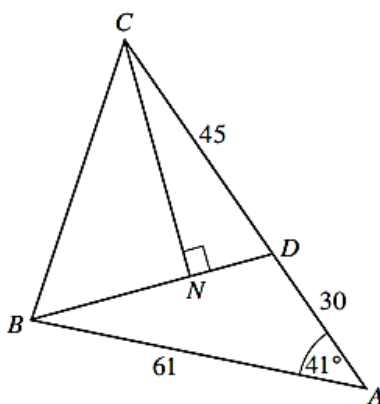
Diagram I

In Diagram I, the point D lies on AC and N is the foot of the perpendicular from C to BD.
$AB = 61$ m, $AD = 30$ m and $DC = 45$ m.
Angle $BAC = 41°$.

(a) Calculate BD. [4]

(b) Show that, correct to the nearest square metre, the area of triangle BDA is 600 m^2. [2]

(c) Explain why $\dfrac{\text{area of } \triangle BCD}{\text{area of } \triangle BDA} = \dfrac{3}{2}$. [1]

(d) Calculate the area of triangle BCD. [1]

(e) Hence calculate CN. [2]

(f)

Diagram II
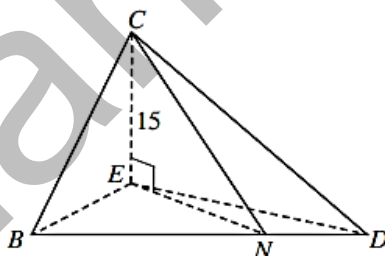
The same points B, C, D and N lie on a sloping plane.
The point E is 15 m vertically below C.
The points B, E, D and N lie on a horizontal plane.
Diagram II represents this information.
Calculate the angle of elevation of C from N. [2]

Answer: **(a)** 43.1 m; **(c)** same height; **(d)** 900 m^2; **(e)** 41.8 m; **(f)** 21.1°.   **J07/2/Q9**

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13 **(a)** A flagpole is a cylinder of length 15 m and diameter 14 cm.

Calculate the volume of the flagpole.
Give your answer in **cubic metres**. [3]

(b) The flagpole, represented by *TP* in the diagrams below, is hinged at the point *P*.
It is raised by using two ropes.
Each rope is fastened to the top of the flagpole and the ropes are held at *A* and *B*.
The points *A*, *P*, *B* and *T* are in a vertical plane with *A*, *P* and *B* on horizontal ground.
TP = 15 m, *AP* = 23 m and *BP* = 12 m.

(i) When $A\hat{T}P = 90°$, calculate $T\hat{P}A$.

[2]
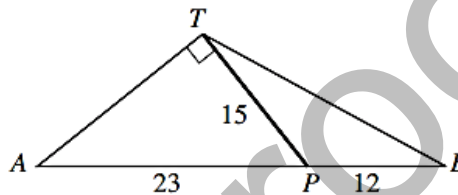
(ii) When $T\hat{B}P = 37°$, calculate $B\hat{P}T$.
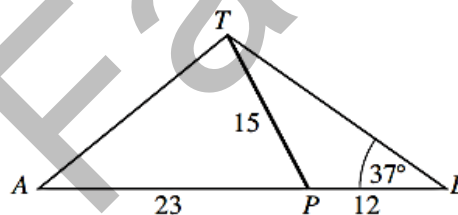
[3]

(iii) When the flagpole is vertical, calculate the angle of elevation of the top of the flagpole from *A*.
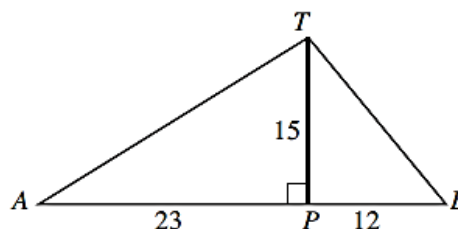
[2]

Answers: **(a)** 0.231, **(b) (i)** 49.3,° **(iii)** 114,° **(iv)** 33.1.° **J08/2/Q1**

294

14
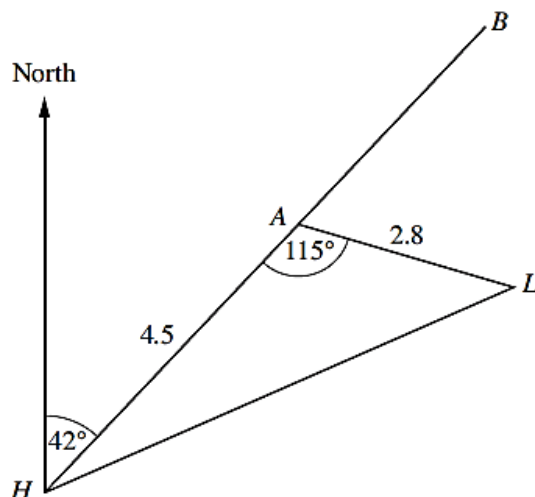
The diagram shows the positions of a harbour, *H*, a lighthouse, *L*, and two buoys *A* and *B*.
HAB is a straight line.
The bearing of *A* from *H* is 042°.
HA = 4.5 km, *AL* = 2.8 km and $H\hat{A}L$ = 115°.

(a) Find the bearing of

    (i) *H* from *A*,   [1]

    (ii) *L* from *A*.   [1]

(b) Calculate

    (i) *HL*,   [4]

    (ii) the area of triangle *HAL*.   [2]

(c) A boat sailed from the harbour along the line *HAB*.

    (i) Calculate the shortest distance between the boat and the lighthouse.   [2]

    (ii) The boat sailed at a constant speed of 3 m/s.

        Given that the boat reached *A* at 07 15, find at what time it left the harbour.   [2]

Answers: (a)(i) 222°,   (ii) 107°;   (b)(i) 6.22 km, 5.71 km²;   (c) (i) 2.54 km,   (ii) 06 50.   **J08/2/Q9**

15 (a) A heavy ball hangs from a point *P*,
11 m above horizontal ground, by means
of a thin wire.

The point *D* is on the ground vertically below *P*.
The point *B* is on the ground 4 m from *D*.
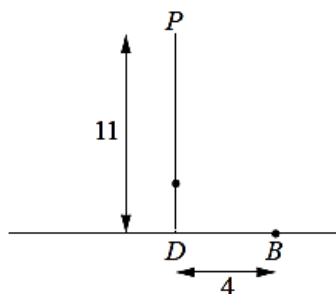
(i) Calculate the angle of elevation of P from B. [2]

(ii) The ball swings, with the wire straight, in the vertical plane PDB.

When the ball is at X, directly above B, $D\hat{P}X = 28°$.

Calculate

(a) PX, [2]
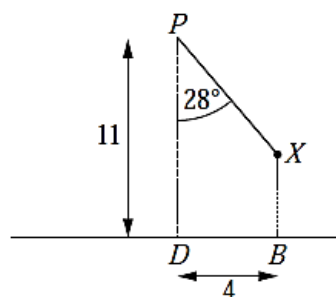
(b) XB. [3]

16   **(a)**
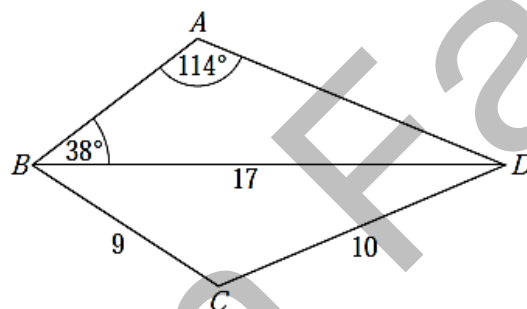
In the diagram, $BD = 17\,\text{cm}$, $CD = 10\,\text{cm}$, $BC = 9\,\text{cm}$, $B\hat{A}D = 114°$ and $A\hat{B}D = 38°$.

Calculate

(i) AD, [3]

(ii) $B\hat{C}D$. [3]

17   **(a)**
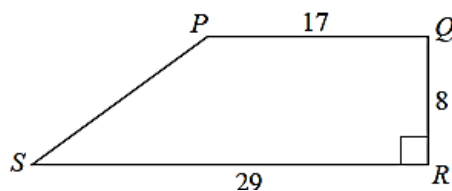
PQRS is a trapezium.
PQ = 17 cm, *QR* = 8 cm, *SR* = 29 cm and *SR̂Q* = 90°.

Calculate

(i) the area of *PQRS*, [1]

(ii) *PŜR*. [2]

(b)
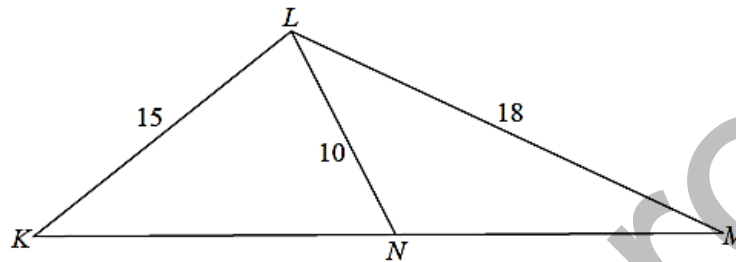
In the diagram, triangle *KLM* is similar to triangle *LNM*.
KL = 15 cm, *LM* = 18 cm and *LN* = 10 cm.

(i) Find *KM*. [2]

(ii) Find *KN*. [2]

(iii) *P* is the point on *LM* such that *PN* is parallel to *LK*.

Find $\dfrac{\text{the area of triangle } NPM}{\text{the area of trapezium } KLPN}$.

Give your answer as a fraction in its simplest form. [2]

Answers: **(a)(i)** 184 cm², **(ii)** 33.7°; **(b)(i)** 27 cm, **(ii)** 15 cm, **(iii)** $\dfrac{16}{65}$. **J10/22/Q7**

18
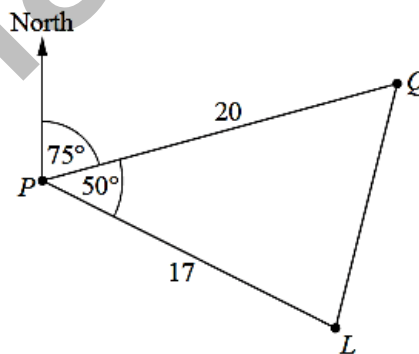
The diagram shows two ports, *P* and *Q*, and a lighthouse *L*.
PQ = 20 km, *PL* = 17 km, *QP̂L* = 50° and the bearing of *Q* from *P* is 075°.

297

(a) Find the bearing of P from L. [1]

(b) Calculate QL. [4]

(c) **(i)** Calculate $P\hat{L}Q$. [3]

(ii) Hence find the bearing of Q from L. [1]

(d) A boat leaves P and sails in a straight line to Q.

(i) It takes 4 hours and 53 minutes to sail from P to Q.
It arrives at Q at 02 23.

At what time does it leave P? [1]

(ii) Calculate the shortest distance between the boat and the lighthouse. [2]

Answers: **(a)** 305° **(b)** 15.9 km **(c)(i)** 74.9°, **(ii)** 019.9°; **(d)(i)** 2130, **(ii)** 13.0 km. **J10/22/Q9**

19
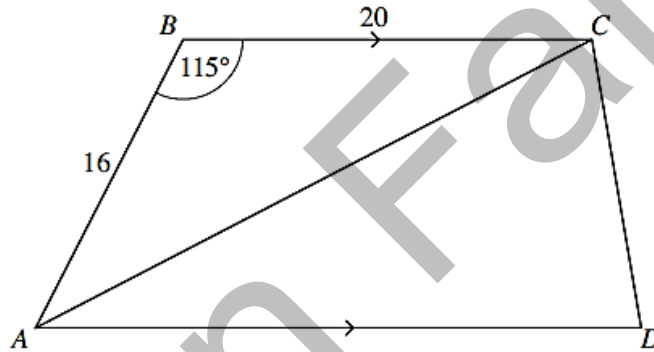
$ABCD$ is a trapezium with AD parallel to BC.
$AB = 16$ cm, $BC = 20$ cm and $A\hat{B}C = 115°$.

(a) Find AC.

Answer .... cm [4]

(b) Show that the perpendicular distance between BC and AD is 14.5 cm.

(c) The area of the trapezium $ABCD$ is 348 cm².

(i) Find AD. *Answer* .... cm [2]

(ii) Show that the area of triangle ACD is 203 cm². [1

(iii) Hence, or otherwise, find $C\hat{A}D$. *Answer* .... [3]

Answers: **(a)** 30.4 **(c)(i)** 28 **(iii)** 28.4° to 28.5° **J11/21/Q7**

20

(c)
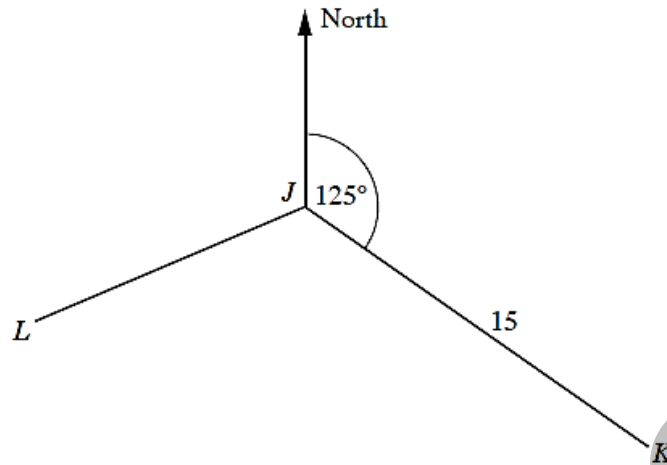
The scale drawing shows a map of three towns, J, K and L.
The distance of K from J is 15 km and the bearing of K from J is 125°.

(i) M is due south of J and due west of K.

Calculate the distance, in kilometres, of M from K.
Give your answer correct to 2 decimal places.

Answer .... km [2]

(ii)   Using measurements from the diagram, find

(a) the bearing of L from J,   *Answer* .... [1]

(b) the actual distance, in kilometres, of L from J.   *Answer* .... km [1]

(c)(i) 12.29 (ii)(a) 247° (b) 10.2 to 10.7   J11/21/Q11c

21
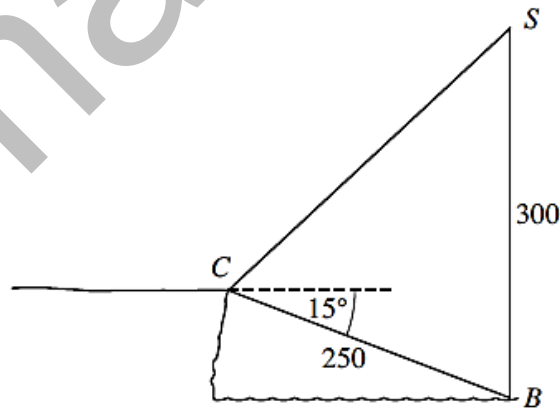
The angle of depression of a buoy, B, from a point, C, on a cliff is 15°.
The distance BC is 250 m.
A seagull, S, hovers so that it is vertically above B and $SB = 300$ m.

(a) **(i)** Find $S\hat{B}C$.

Answer ....[1]

(ii) Find SC.

Answer ....m [3]

(iii) Find the angle of elevation of S from C.

Answer .... [3]

(b)
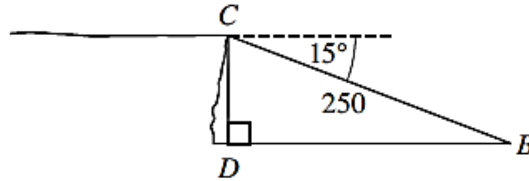
D is a marker at sea level vertically below C and due west of B.

(i) Find DB.

Answer ....m [2]

(ii) M is a marker at sea level 200 m from B and $D\hat{B}M = 30°$.

Find the area of triangle DBM.

Answer .... m² [2]

(iii) N is a marker at sea level due south of B and $DN = 450$ m.
A boat sails on a circular course through D, B and N.

Write down the radius of the circle.

Answers: **(a)(i)** 75° **(ii)** 337 **(iii)** 44.3° **(b)(i)** 241 **(ii)** 12100 **(iii)** 225   **J11/22/Q10**

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22 **(a)** Three towns, A, B and C, are located such that $AB = 90$ km, $BC = 100$ km and $AC = 85$ km.
The bearing of B from A is 127°.

(i) Write down the bearing of A from B.

Answer ....[1]

(ii) Of the three towns, A is the furthest north.

Using a scale of 1 cm to 10 km, construct a scale drawing to show the positions of the three towns.

300

A •
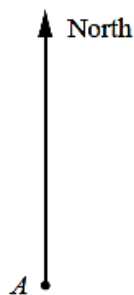
[3]

(iii) Measure the bearing of B from C.   *Answer* ....[1]

Answers: (a)(i) 307°   (iii) 074°   **J12/21/Q8**

23
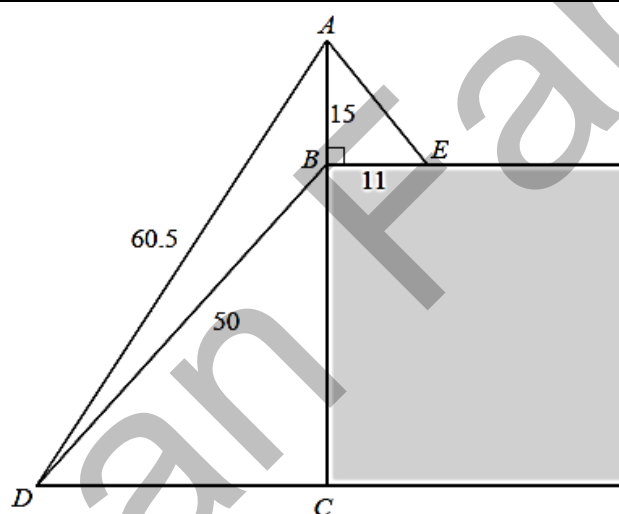
A vertical mast, AB, is 15 m tall and is attached to the top of a building at B.
The top of the mast is attached to the roof of the building at E using a wire.
$A\hat{B}E = 90°$ and $BE = 11$ m.

(a) (i) Calculate AE.   *Answer* ....m [2]

(ii) D is a point on the ground such that $AD = 60.5$ m and $BD = 50$ m.

Calculate $A\hat{D}B$.   *Answer* ....[4]

(b)
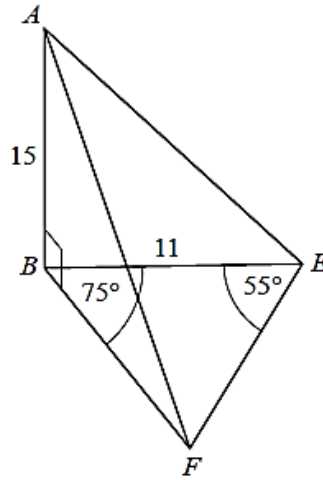
The top of the mast is also attached to the roof of the building at *F* using a wire.
FB̂E = 75°, *BÊF* = 55° and *AB̂F* = 90°.

(i) Calculate *BF̂E*.

Answer ....[1]

(ii) Calculate *FB*.

Answer ....m [3]

(iii) Calculate the angle of depression of *F* from the top of the mast.

Answer ....[2]

Answers: (a)(i) 18.6   (ii) 11.2   (b)(i) 50°   (ii) 11.8   (iii) 51.9   **J12/21/Q11**

24   (a) *ABCD* is a trapezium with *BC* parallel to *AD*.
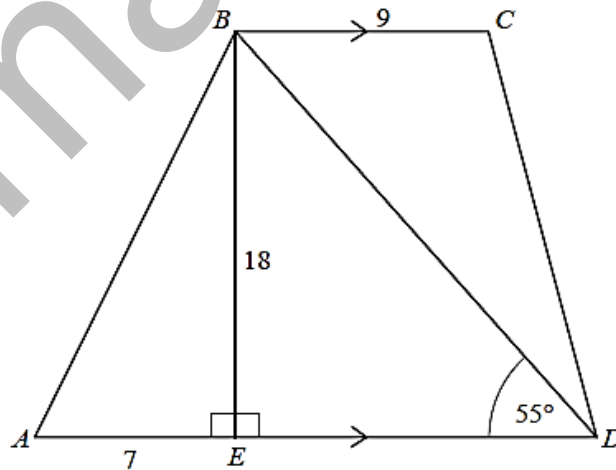
E is the point on AD such that BE is perpendicular to AD.
$B\hat{D}A = 55°$, $AE = 7\,cm$, $BE = 18\,cm$ and $BC = 9\,cm$.

Calculate

(i) $B\hat{A}E$,   *Answer* .... [2]

(ii) the area of the trapezium $ABCD$.   *Answer* ....cm² [4]

(b)
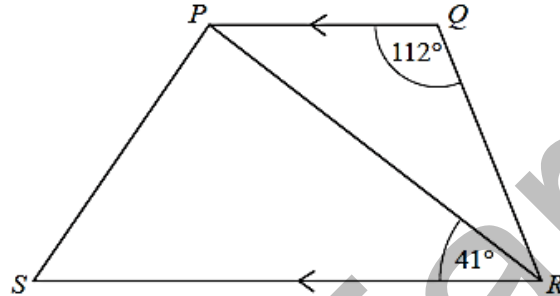
$PQRS$ is another trapezium.
$P\hat{Q}R = 112°$ and $P\hat{R}S = 41°$, each measured correct to the nearest degree.

Find the smallest possible value of $Q\hat{R}P$.

                                                    Answer .... [2]

Answer: (a)(i) 68.7° (ii) 257 to 257.5 (b) 26°   **J13/21/Q6**

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25   (b) The diagram shows the positions, P, Q and R, of three buoys.
The bearing of Q from P is 054°, $PQ = 250\,m$, $QR = 340\,m$ and $PR = 160\,m$.
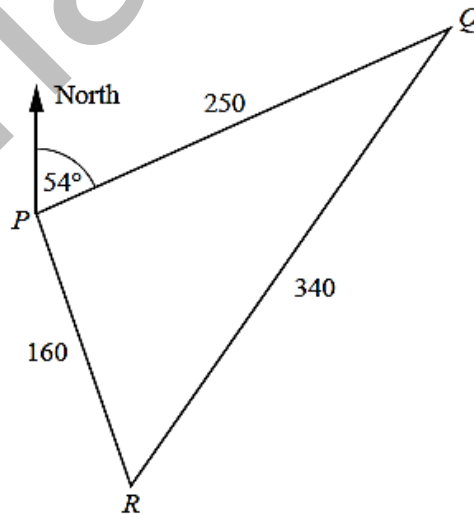
(i) **Calculate** the bearing of R from P.   *Answer* .... [4]

303

(ii) Calculate the area of triangle *PQR*.   *Answer* .... m² [2]

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26 The diagram shows a field on horizontal ground.
The side *AB* is next to a straight road *XY*.
AB = 130 m, *BC* = 164 m and *A B̂ C* = 115°.
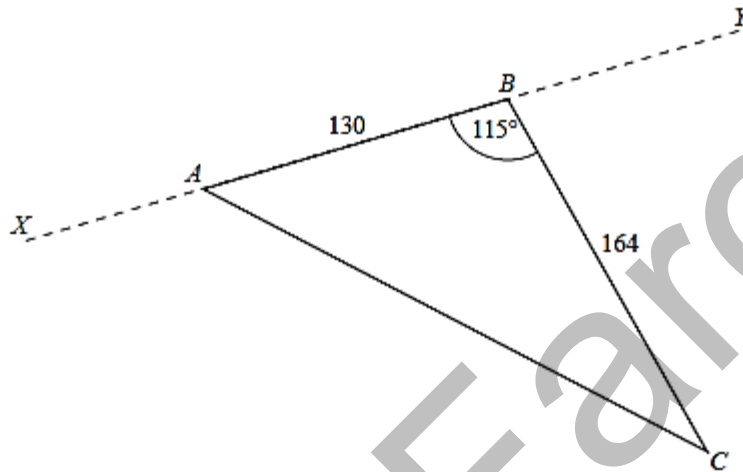
(a) Calculate *AC*.   *Answer* .... m [3]

(b) Work out the area of the field.   *Answer* .... m² [2]

(c) The field is to be sown with grass seed.
Each square metre of the field is sown with 3.25 g of seed.
The seed is only sold in 5 kg bags.

How many bags of grass seed must be bought?   *Answer* .... [2]

(d) A bird is hovering directly above *B*.
The angle of elevation of the bird from *A* is 18.5°.

Calculate the height of the bird above *B*.   *Answer* .... m [2]

(e) Calculate the shortest distance from *C* to the road *XY*.   *Answer* .... m [3]

27
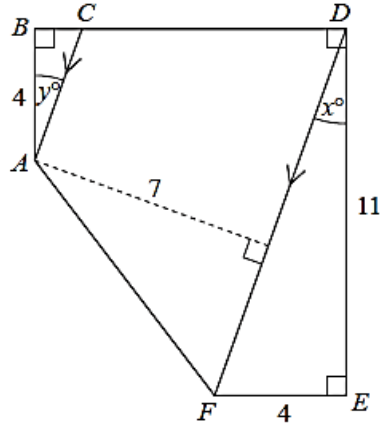
In the framework *ABCDEF*, *BCD* is a straight line, and *CA* is parallel to *DF*.
$A\hat{B}D$, $B\hat{D}E$ and $D\hat{E}F$ are right angles.
$AB = 4\,\text{m}$, $DE = 11\,\text{m}$ and $EF = 4\,\text{m}$.

(a) $F\hat{D}E = x°$.

    Show that $x = 20.0$ correct to 3 significant figures. [2]

(b) $B\hat{A}C = y°$.

    Stating your reasons, explain why $y = x$. [1]

(c) Calculate *AC*.   *Answer* .... m [3]

(d) The perpendicular distance between the parallel lines *CA* and *DF* is 7 m.

    Calculate the area of *ACDF*.   *Answer* .... m² [4]

    J15/21/Q5

28
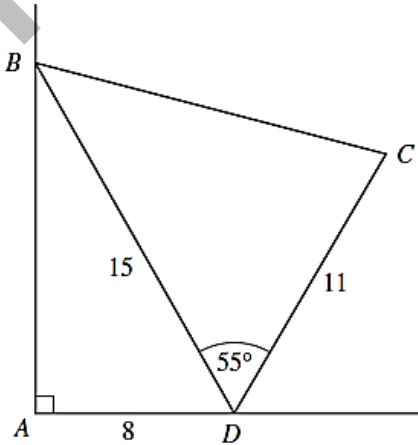
In the diagram, AB is a vertical wall.

A beam, CD, of length 11 metres, rests with one end, D, on horizontal ground.

It is held in place by two cables, BC and BD.

Given that AD = 8 metres, BD = 15 metres and angle BDC = 55°, calculate

(a)   the length AB,   [2]

(b)   the length of the cable BC,   [4]

(c)   the angle between the beam CD and the ground.   [3]

29
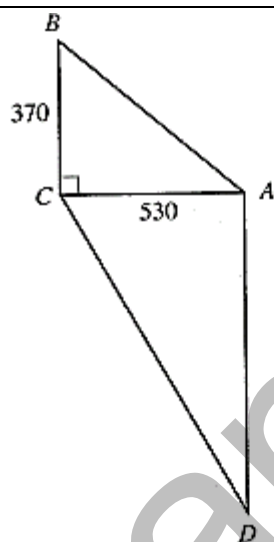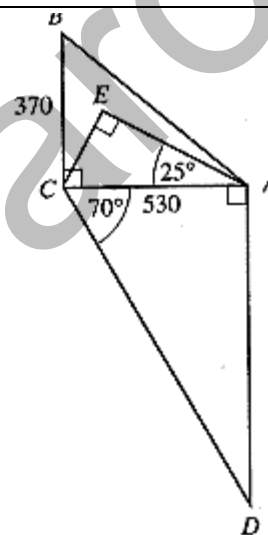
Diagram I   Diagram II

Diagram I shows a path, AC, in a park ABCD.
It is given that AC = 530 m, BC = 370 m and that AC is perpendicular to BC.

(a) Calculate angle ABC.   [2]

(b) Diagram II shows two other paths, AE and CE, in the park.

     Given that angle CAE = 25° and angle AEC = 90°, calculate the length of AE.   [2]

(c) Given also that angle ACD = 70° and angle CAD = 90°, calculate

     (i)   the length of CD,   [2]

     (ii) the area of the park ABCD.   [3]

30
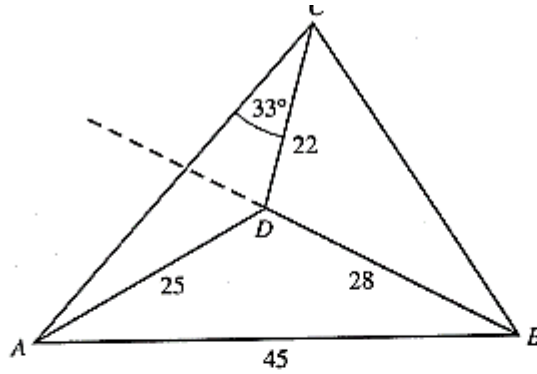
The diagram shows four points, A, B, C and D, on a piece of horizontal land.
It is given that AB = 45 metres, AD = 25 metres and BD = 28 metres.

(a) Calculate angle ADB. [4]

(b) Given also that CD = 22 metres and that angle ACD = 33°, calculate angle ADC. [3]

(c) The line BD is produced beyond D.

Calculate the shortest distance from C to this extended line. [2]

(d) D is the foot of a vertical mast, DT.
The angle of elevation of the top of the mast, T, from A is 40°.

Calculate the angle of elevation of T from B. [3]

Answers: (a) 116.1°; (b) 118.4°; (c) 17.9 m; (d) 36.8°.　　　　　　　　N02/2/Q8

31
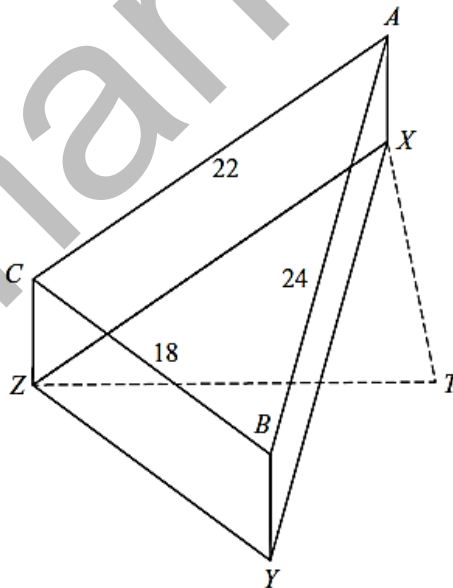
An aircraft waiting to land is flying around a triangular circuit ABC.
A, B and C are vertically above three beacons, X, Y and Z.
T is the control tower at the airport, and T, X, Y and Z lie in a horizontal plane.

BC = 18 km, CA = 22 km and AB = 24 km.

307

(a) (i) The plane is flying at 200 km/h.

Calculate the time, in minutes and seconds, that the aircraft takes to complete one circuit. [2]

(ii) Calculate the largest angle of triangle *ABC*. [4]

(b) *Z* is due West of *T*.
The bearing of *X* from *Z* is 042° and the bearing of *X* from *T* is 338°.

(i) Find the angles of triangle *TXZ*. [2]

(ii) Calculate *TX*. [2]

(c) The aircraft is flying at a constant height of 2600 metres.

Calculate the angle of depression of the tower, *T*, from the aircraft when it is at *A*. [2]

Answers: **(a)(i)** 19 minutes 12 seconds, **(ii)** 73.0°; **(b)(i)** 68°, 64°, 48°, **(ii)** 17.6 km; **(c)** 8.4°.   **N03/2/Q10**

32
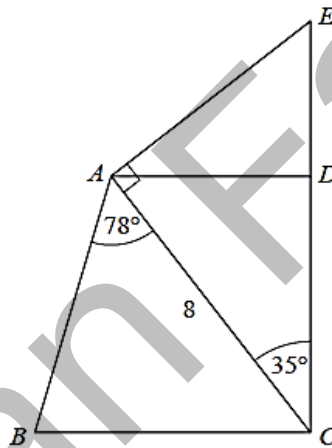
The diagram represents some beams which support part of a roof.

AD and *BC* are horizontal and *CDE* is vertical.

AC = 8 metres, $B\hat{A}C$ = 78°, $A\hat{C}D$ = 35° and $C\hat{A}E$ = 90°.

Calculate the length of the beam

(a) *AD*, [2]

(b) *CE*, [2]

(c) *AB*. [3]

Answers: **(a)** 4.59 m; **(b)** 9.77 m; **(c)** 8.96 m.   **N04/2/Q1**

33 **(a)**
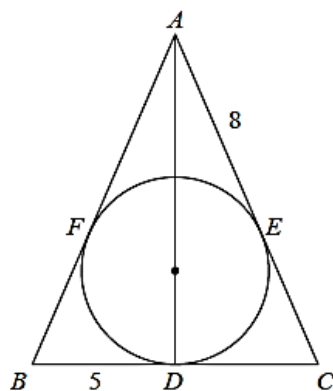
The diagram shows a circle which passes through D, E and F.
AFB, BDC and CEA are tangents to the circle.
D is the midpoint of BC.

Given that $BD = 5\,\text{cm}$ and $AE = 8\,\text{cm}$, find

(i) EC, [1]

(ii) $C\hat{A}D$. [2]

(b)
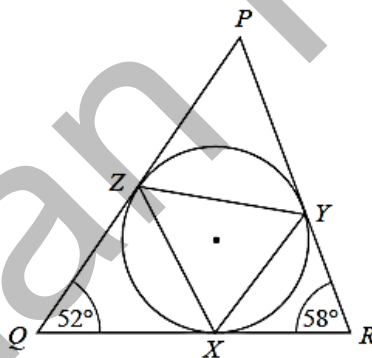
The diagram shows a circle which passes through X, Y and Z.
PZQ, QXR and RYP are tangents to the circle.

Given that $P\hat{Q}R = 52°$ and $Q\hat{R}P = 58°$, calculate

(i) $Q\hat{P}R$, [1]

(ii) $Q\hat{Z}X$, [2]

(iii) $Z\hat{X}Y$. [2]

Answers: **(a)(i)** 5 cm, **(ii)** 22.6°; **(b)(i)** 70°, **(ii)** 64°, **(iii)** 55°.   **N04/2/Q6**

309

34
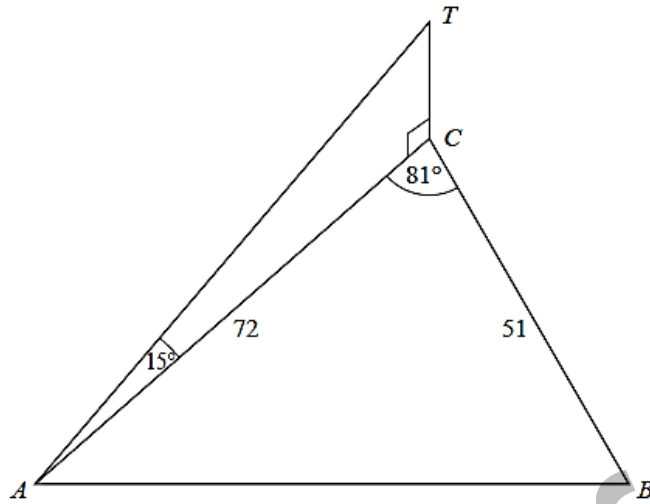
Three paths, *AB*, *BC* and *CA*, run along the edges of a horizontal triangular field *ABC*.
BC = 51 m, *AC* = 72 m and angle *ACB* = 81°.

(a) Calculate the length of *AB*. [4]

(b) Calculate the area of the field *ABC*. [2]

(c) Calculate the shortest distance from *C* to *AB*. [2]

(d) A vertical tree, *CT*, has its base at *C*.
The angle of elevation of the top of the tree from *A* is 15°.

Calculate the height of the tree. [2]

(e) John measured the largest angle of elevation of the top of the tree as seen from the path *AB*.

Calculate this angle of elevation. [2]

Answers: **(a)** 81.5 m; **(b)** 1810 m²; **(c)** 44.5 m; **(d)** 19.3 m; **(e)** 23.4°.   **N04/2/Q7**

35
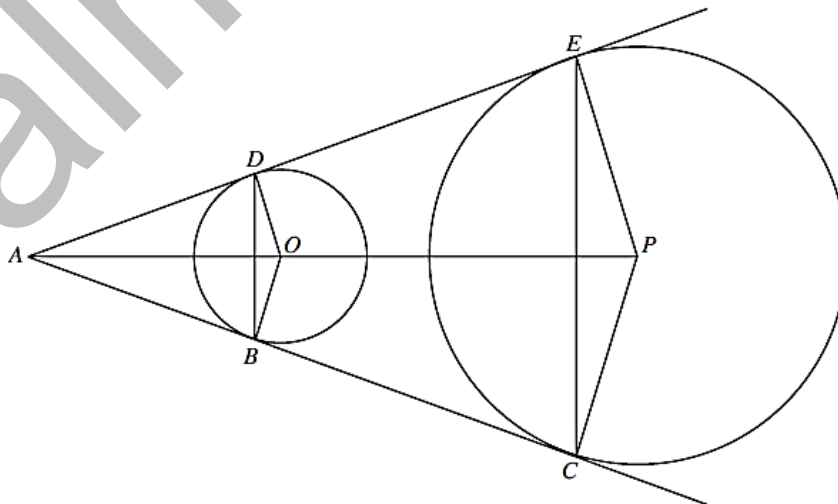
The diagram shows two circles with centres *O* and *P*.
ABC and *ADE* are tangents to the circles at *B*, *C*, *D* and *E* as shown.
AOP is a straight line.

(a) Giving a reason for your answer, write down angle *ABO*.   [1]

(b) It is given that *OB* = 6 cm, *AO* = 13 cm and *PC* = 15 cm.

   (i)   Show that angle *OAB* = 27.5°, correct to one decimal place.   [1]

   (ii) Calculate *AC*.   [2]

   (iii) Calculate *CE*.   [3]

Answers: **(a)** 90°; **(b)(ii)** 28.8 cm, **(iii)** 26.6 cm.   N05/2/Q1

36
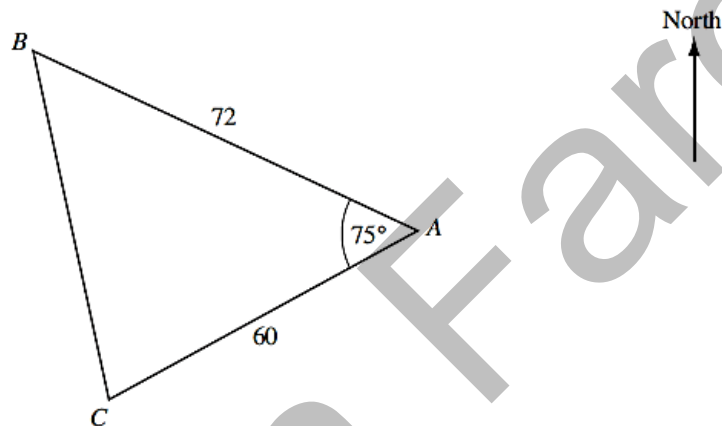
Three points, *A*, *B* and *C*, lie on a horizontal field.
Angle *BAC* = 75° and the bearing of *C* from *A* is 217°.
AB = 72 m and *AC* = 60 m.

(a) Calculate

   (i)   the bearing of *B* from *A*,   [1]

   (ii) *BC*,   [4]

   (iii) angle *ABC*,   [3]

   (iv) the bearing of *C* from *B*.   [1]

(b) A girl standing at *B* is flying a kite.
     The kite, *K*, is vertically above *A*.
     The string, *BK*, attached to the kite is at 24° to the horizontal.

     Calculate the angle of elevation of the kite when viewed from *C*.   [3]

Answers: **(a)(i)** 292°, **(ii)** 80.9 m, **(iii)** 45.7°, **(iv)** 157.7°; **(b)** 28.1°.   N05/2/Q8

37
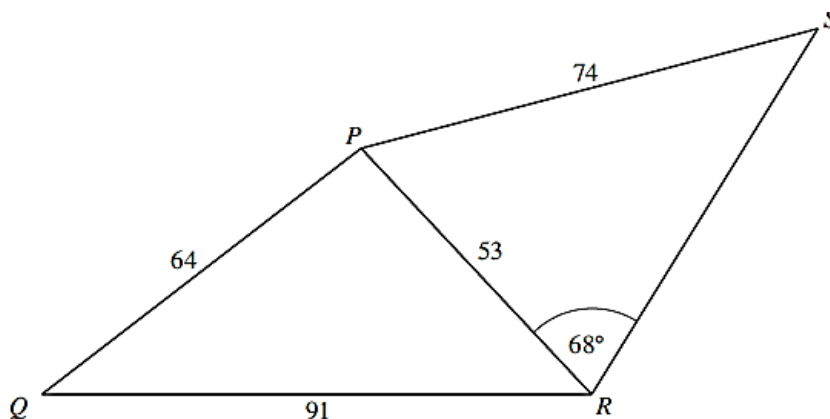
The diagram shows a footpath PR across a park PQRS.
PQ = 64 m, PR = 53 m, PS = 74 m and QR = 91 m.
Angle PRS = 68°.

Calculate

(a) $Q\hat{P}R$,   [3]

(b) $R\hat{P}S$,   [3]

(c) the area of triangle PRS.   [2]

Answer. (a) 101.7°   (b) 70.4°   (c) 1850 m²

38

A vertical flagpole, BF, stands at the top of a hill.
AB is the steepest path up the hill.
N lies vertically below B and $A\hat{N}B = 90°$.

AN = 100 m and AB = 104 m.
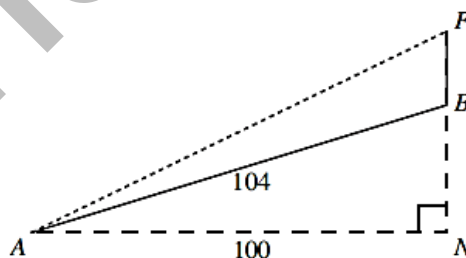
(a) Show that BN = 28.6 m.   [1]

(b) It is given that $F\hat{A}N = 25°$.

  (i)   Write down the size of the angle of depression of A from F.   [1]

  (ii) Calculate the height, BF, of the flagpole.   [3]

312
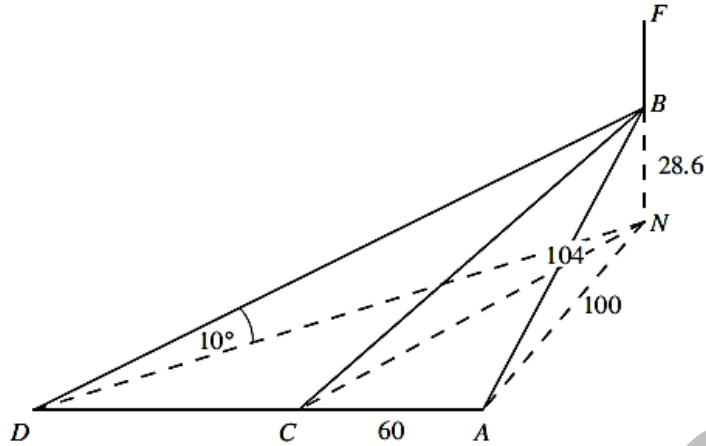
The diagram shows three other straight paths (*CB*, *DB* and *ACD*) on the hill.

The path *ACD* is horizontal and $B\hat{A}C = N\hat{A}C = 90°$.

CN and *DN* are horizontal lines.

(i)   Given that *AC* = 60 m, calculate $B\hat{C}N$. [4]

(ii) Given that $B\hat{D}N = 10°$, calculate $D\hat{B}A$. [3]

Answer: **(b)(i)** 25° **(ii)** 18.0 or 18.1 m **(c)(i)** 13.7° or 13.8° **(ii)** 50.8°

39
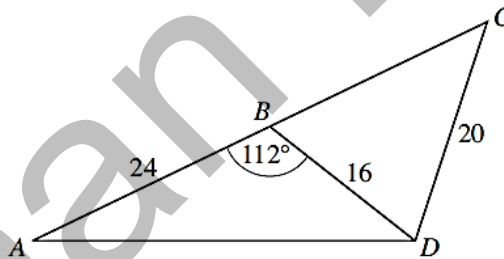
The points *A*, *B*, *C* and *D* represent four towns on a map.
ABC is a straight line.
AB = 24 cm, *BD* = 16 cm and *CD* = 20 cm.
Angle *ABD* = 112°.

(a) Calculate

(i)   *AD*, [4]

(ii) angle *BCD*, [3]

(iii) the area of triangle *ABD*. [2]

(b) The scale of the map is 1 : 250000.

Calculate the actual distance, in kilometres, from town *A* to town *B*. [1]

Answers: **(a)(i)** 33.5 cm, **(ii)** 47.9°, **(iii)** 178 cm², **(b)** 60.

40 Eight straight paths in a level garden form this shape with rotational symmetry of order four.
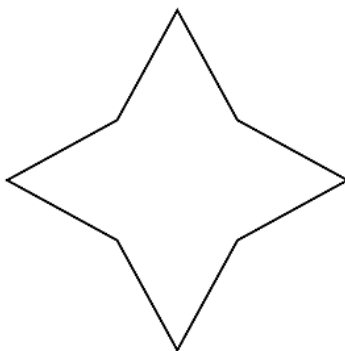
(a)
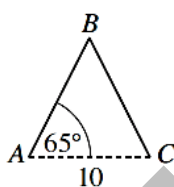
The two paths shown, AB and BC, form part of the isosceles triangle ABC.
$AC = 10$ m and angle $BAC = 65°$.

Calculate

(i) the length of the path AB, [2]

(ii) the area of triangle ABC, [2]

(iii) the area of garden enclosed by all 8 paths. [2]

(b)
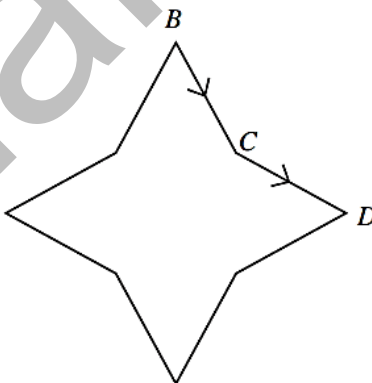
Ada walked along the paths BC and CD.

(i) Calculate $B\hat{C}D$. [2]

(ii) After walking in the direction BC, Ada turned to walk in the direction CD.

State the value of the angle through which she turned. [1]

Answers: **(a)(i)** 11.8, **(ii)** 53.3 to 53.7, **(iii)** 313.2 to 314.5, **(b)(i)** 140°, **(ii)** 40°. **N08/2/Q2**

41
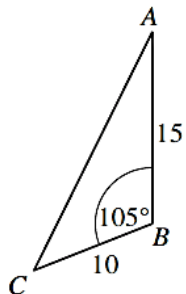
C is a point on a slope with highest point B, where $CB = 10$ m.
A vertical mast, AB, of height 15 m stands at B and $A\hat{B}C = 105°$.
AC is one of the wires that supports the mast.

(a) **(i)** Write down the angle between CB and the horizontal. [1]

(ii) Calculate the length of the wire AC. [4]

(b)
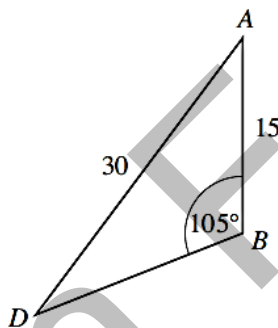
D is another point further down the slope.
AD is another supporting wire.

Given that $AD = 30$ m, calculate $A\hat{D}B$. [3]

(c)
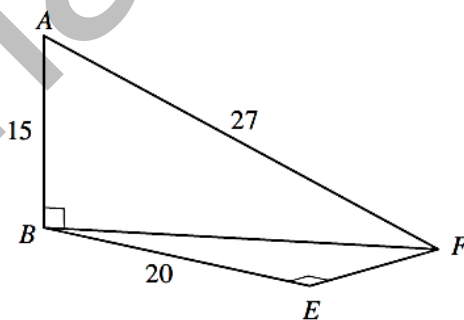
The points B, E and F are on horizontal ground at the top of the slope.
The mast is supported by another wire AF.
$B\hat{E}F = 90°$, $AF = 27$ m and $BE = 20$ m.

Calculate

(i) EF, [2]

(ii) the angle of elevation of the top of the mast from F. [2]

42 A small submarine dived in a straight line from a point A on the surface to examine an object at the point W on the seabed.

(a)
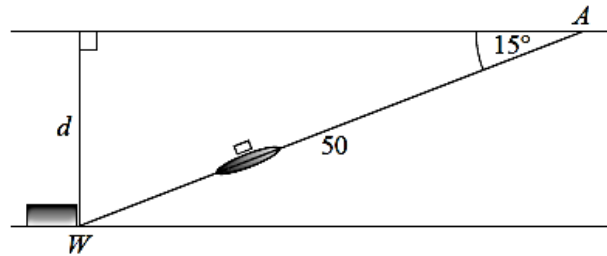
It dived at an angle of 15° to the horizontal and reached W after travelling 50 m.

Calculate the depth, d metres, of the seabed at W. [2]

(b)
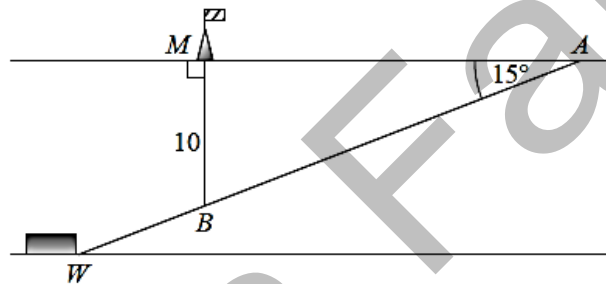
A marker is at the point M on the surface.
When at B, the submarine was 10 m vertically below M.

Calculate AB. [3]

(c)
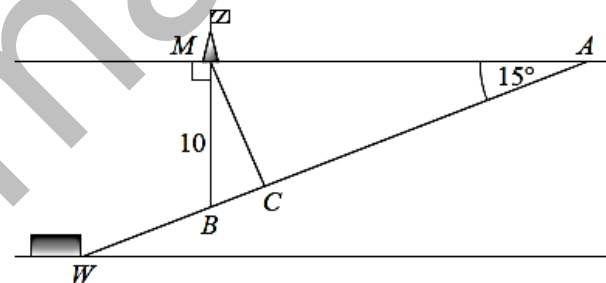
When at C, the submarine was at its nearest point to M.

(i) Find $B\hat{M}C$. [1]

(ii) Calculate CM. [2]

43 **(a)** The diagram shows the side *ABCDE* of a building.
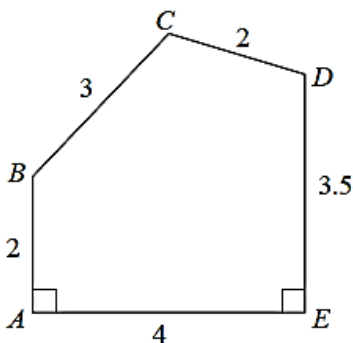
$AB = 2\,\text{m}$, $BC = 3\,\text{m}$, $CD = 2\,\text{m}$, $DE = 3.5\,\text{m}$ and $EA = 4\,\text{m}$.
AB and DE are vertical.
AE is horizontal.

   (i) Using a scale of 2 cm to 1 m, construct an accurate scale drawing of *ABCDE*.   [3]

   (ii) Measure and write down $A\hat{B}C$.   [1]

(b) The diagram shows the pentagon *PQRST*.
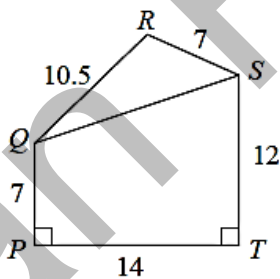
$PQ = 7\,\text{cm}$, $QR = 10.5\,\text{cm}$, $RS = 7\,\text{cm}$, $ST = 12\,\text{cm}$ and $TP = 14\,\text{cm}$.
$Q\hat{P}T = P\hat{T}S = 90°$.

   (i) Explain why the shapes *ABCDE* and *PQRST* are not geometrically similar.   [1]

   (ii) Show that $QS^2 = 221$.   [2]

   (iii) Calculate $Q\hat{R}S$.   [3]

   (iv) Calculate $R\hat{Q}S$.   [2]

Answers: **(a)(i)** Accurate scale drawing of *ABCDE* of the correct size   **(ii)** 135'   **(b)(i)** $DE : ST \neq 1 : 3.5$ **N09/2/Q9**
      (ii) $QS^2 = (12 - 7)^2 + 14^2$ **(iii)** 115'   **(iv)** 25.1 to 25.5'

44 **(a)**
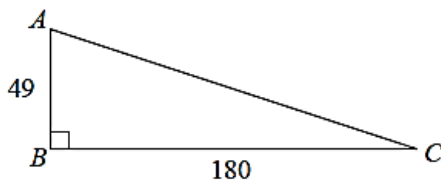
AB is vertical and *BC* is horizontal.
AB = 49 m and *BC* = 180 m.

    (i) Calculate $B\hat{A}C$.   [2]

    (ii) State the value of the angle of depression of *C* from *A*.   [1]

(b) *S* and *P* are two positions at sea and *L* is a Lighthouse.
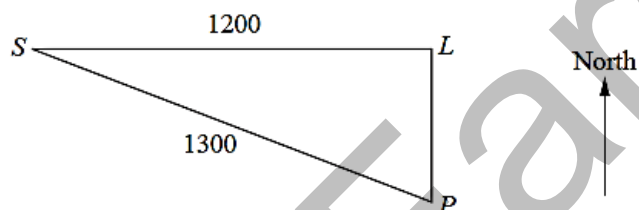
S is due West of *L* and *P* is due South of *L*.
SL = 1200 m and *SP* = 1300 m.

    (i) Calculate *LP*.   [2]

    (ii) Calculate the bearing of *S* from *P*.   [3]

    (iii) A boat sailed from *S* to *P*.
        It left *S* at 15 56 and reached *P* at 16 04.

        Calculate its speed in kilometres per hour.   [2]

Answers: **(a)(i)** 74.8 **(ii)** 15.2 **(b)(i)** 500 **(ii)** 293 **(iii)** 9.75   **N10/21/Q2**

45 **(a)**
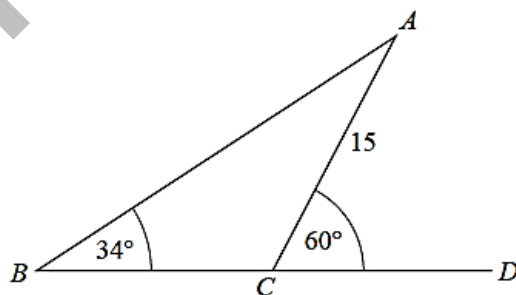
ABC is a triangle in which $A\hat{B}C = 34°$ and *AC* = 15 cm.
BCD is a straight line and $A\hat{C}D = 60°$.

Find

318

(i) $B\hat{A}C$, [1]

(ii) BC. [2]

(b)
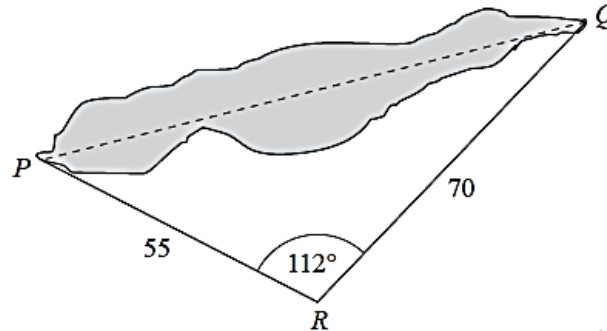
The shaded part of the diagram shows a lake. P and Q are points at each end of the lake.
R is a point on the same level ground as P and Q.
$PR = 55\,$m, $QR = 70\,$m and $P\hat{R}Q = 112°$.

(i) Calculate PQ. [4]

(ii) A plan is drawn using a scale of 1 centimetre to 5 metres.

    (a) State the lengths of PR and QR on the plan. [1]

    (b) Find the area of triangle PQR on the plan. [2]

    (c) The area of the lake on the plan is $32.4\,$cm^2.

        Calculate the actual area, in square metres, of the lake. [2]

Answers: **(a)(i)** 26 **(ii)** 11.8 **(b)(i)** 104 **(ii)(a)** 11 and 14 **(b)** 71.4 **(c)** 810   **N10/21/Q9**

46
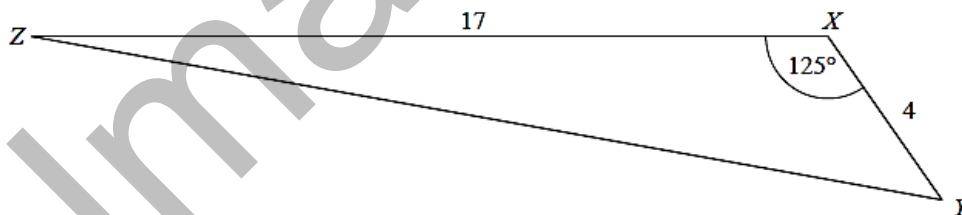
The points X, Y and Z are on horizontal ground.
$XY = 4\,$km, $XZ = 17\,$km and $Y\hat{X}Z = 125°$.

(a) Calculate YZ. [4]

(b)
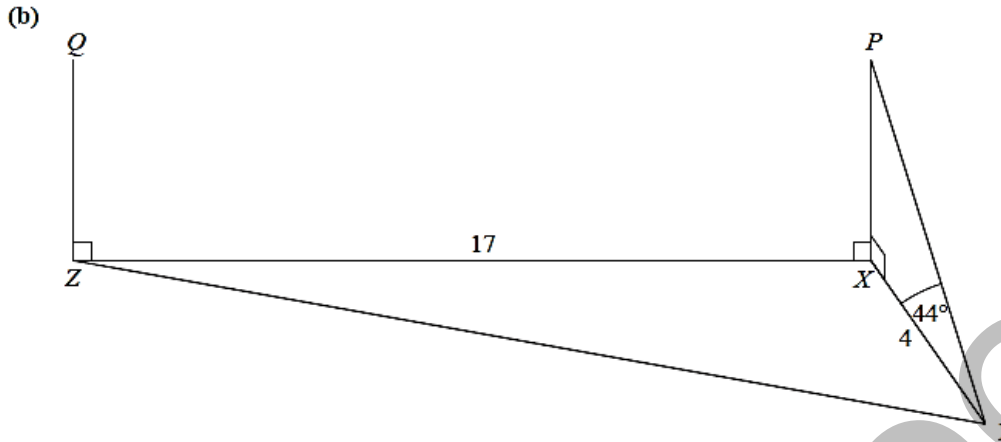
The points P and Q are the same height vertically above X and Z respectively.

(i) When an aircraft was at P, its angle of elevation from Y was 44°.

Calculate PX.
Give your answer in metres, correct to the nearest 100 metres. [3]

(ii) The aircraft took 2 minutes 54 seconds to fly from P to Q.

(a) The aircraft reached its destination 39 minutes 6 seconds after flying over Q.
The flight ended at 15 03.

At what time did the aircraft fly over P? [2]

(b) Calculate the average speed of the aircraft as it flew from P to Q.
Give your answer in kilometres per hour. [3]

Answers: (a) 19.6 km (b)(i) 3 900 m (ii)(a) 14 21 (b) 352 km/h

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47 ABC is a triangle in which $A\hat{B}C = 90°$, $B\hat{A}C = 40°$
and $BC = 10$ cm.
P is the point on AB such that $P\hat{C}B = 20°$.

Calculate

(a) PB, *Answer* .... cm [2]

(b) AP, *Answer* .... cm [2]

(c) the perimeter of triangle PBC. *Answer* .... cm [3]
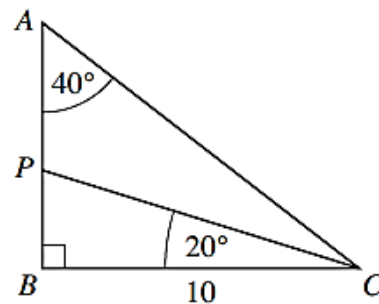
Answers: (a) 3.64 (b) 8.24 – 8.28 (c) 24.2, 24.3

48 P, Q, R and S are four points on level ground.
PQ is parallel to RS and $Q\hat{P}S = 60°$.
PS and RQ intersect at T.
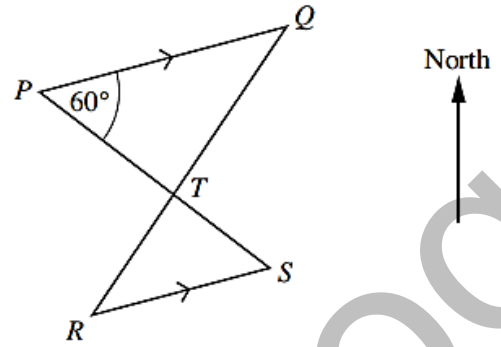
(a) Write down the value of $P\hat{S}R$. Give a reason for your answer.

Answer $P\hat{S}R$ = ... because ....

....[1]

(b) The bearing of Q from P is $070°$.
Find the bearing of

(i) S from P,   Answer .... [1]

(ii) P from S,   Answer .... [1]

(iii) R from S.   Answer .... [1]

(c) (i) Explain why triangles PQT and SRT are similar.   ....[1]

(ii)   Given that $PT = 54\,\text{m}$, $TS = 36\,\text{m}$ and $RQ = 85\,\text{m}$, find TQ.

Answer .... m [3]

Answers: (a) 60° alternate angles (b)(i) 130° (ii) 310° (iii) 250° (c)(i) triangles equiangular (ii) 51   **N11/21/Q3**

49   (a)
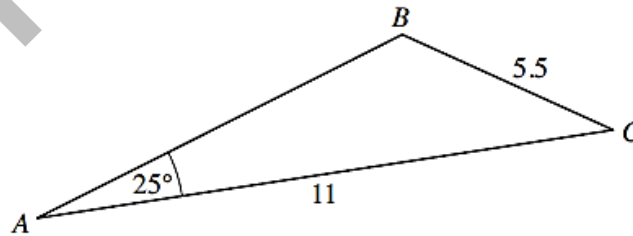
In the diagram, $AC = 11\,\text{cm}$, $BC = 5.5\,\text{cm}$ and $B\hat{A}C = 25°$.
It is given that $A\hat{B}C$ is an **obtuse** angle.

Calculate $A\hat{B}C$.   Answer .... [4]
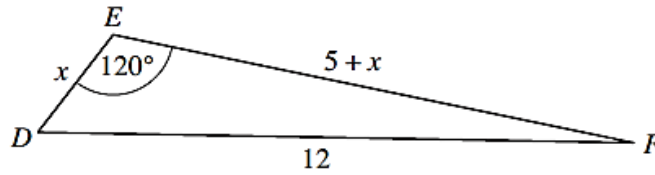
In the diagram, $DF = 12$ cm, $DE = x$ centimetres and $EF = (5 + x)$ centimetres.

(i) Form an equation in x and show that it reduces to $3x^2 + 15x - 119 = 0$. [3]

(ii) Solve the equation $3x^2 + 15x - 119 = 0$, giving each answer correct to 3 decimal places.

Answer $x = $ .... or .... [4]

(iii) Find the length of EF in millimetres, correct to the nearest millimetre.

Answer .... mm [1]

Answers: (a) 122° (b)(ii) 4.276 and -9.276 (iii) 93

50
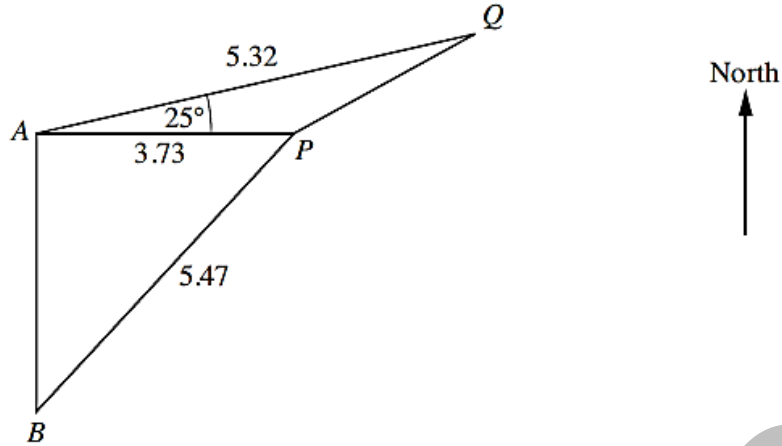
The diagram shows four points, A, B, P and Q, at sea.
B is due South of A and P is due East of A.
$AP = 3.73$ km, $BP = 5.47$ km, $AQ = 5.32$ km and $P\hat{A}Q = 25°$.

(a) Calculate $A\hat{B}P$.

Answer ....[2]

(b) Calculate PQ.

Answer ....km [4]

(c) A boat sailed in a straight line from Q to A.

(i) Find the bearing of A from Q.

Answer .... [1]

(ii) A lighthouse is situated at A.
The top of the lighthouse is 30 m above sea level.
Calculate the angle of depression of the boat from the top of the lighthouse when the boat is 100 m from A.

Answer .... [2]

Answers: **(a)** 43.0 **(b)** 2.50 **(c)(i)** 245° **(ii)** 16.7°

N11/22/Q2

51

(a) In triangle PQR, $R\hat{P}Q = 18°$, $P\hat{Q}R = 90°$ and $PR = 4.5\,\text{m}$.

Find PQ.

Answer .... m [2]
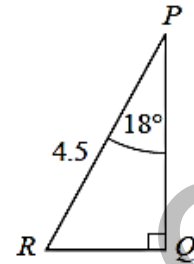
(b) In triangle ABC, $AB = 10.2\,\text{m}$, $AC = 6\,\text{m}$ and $A\hat{C}B = 90°$.

(i) Find $A\hat{B}C$.
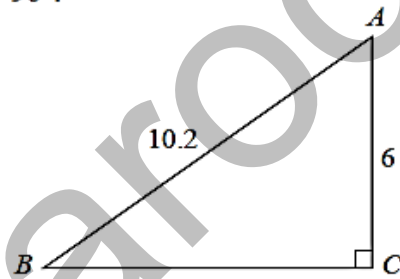
Answer .... [:

(ii)
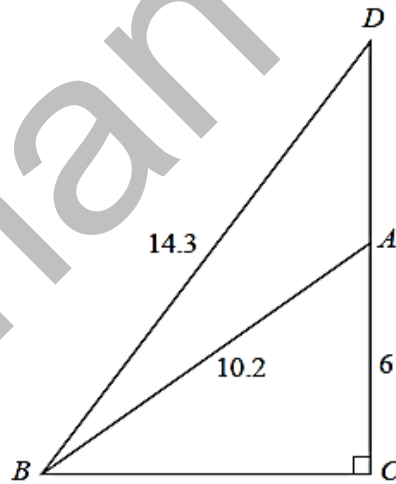
D is the point on CA produced such that $BD = 14.3\,\text{m}$.

Find AD.   *Answer* .... m [4]

Answer: (a) 4.28 (b)(i) 36.0° (b)(ii) 5.68 or 5.69   **N12/21/Q1**

324
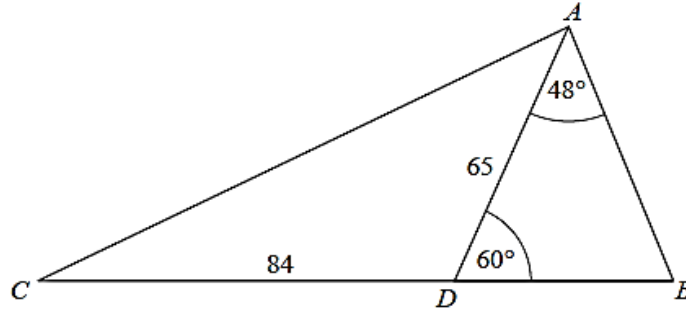
A, B, C and *D* are four points on horizontal ground.
CDB is a straight line.
AD = 65 m and *CD* = 84 m.
DÂB = 48° and *AD̂B* = 60°.

(a) Calculate *AB*.

Answer ....m [3]

(b) Calculate the area of triangle *ACD*.

Answer .... m² [2]

(c) Calculate *AC*.

Answer ....m [4]

(d) A vertical tree of height 35 m stands at *A*.

P is the point on the line *BC* such that the angle of elevation from the line *BC* to the top of
the tree is greatest.

Calculate this angle of elevation.   Answer .... [3]

Answer. (a) 59.2 (b) 2360 (c) 129 (d) 31.9°   **N12/21/Q9**

53   **(a)**
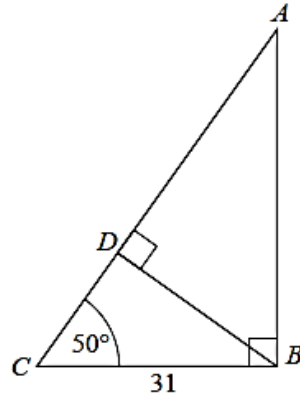
In the triangle ABC, $A\hat{B}C = 90°$, $A\hat{C}B = 50°$ and $BC = 31$ m.
D is the point on AC such that $B\hat{D}A = 90°$.

(i)   Show that $CD = 19.93$ m, correct to 2 decimal places.   [2]

(ii) Calculate AD.

Answer ....m [3]

(b)
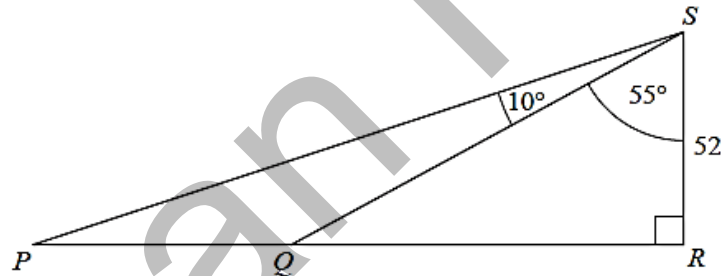
Two boats are at the points P and Q.
RS is a vertical cliff of height 52 m.
$P\hat{S}Q = 10°$ and $Q\hat{S}R = 55°$.

(i)   State the angle of depression of P from S.

Answer ....[1]

(ii) Calculate the distance, PQ, between the boats.

Answer ....m [3]

Answer: (a)(i) 19.93 from correct rounding (ii) 28.3 (b)(i) 25° (ii) 37.2 or 37.3   **N13/21/Q6**

326

54 **(a)**
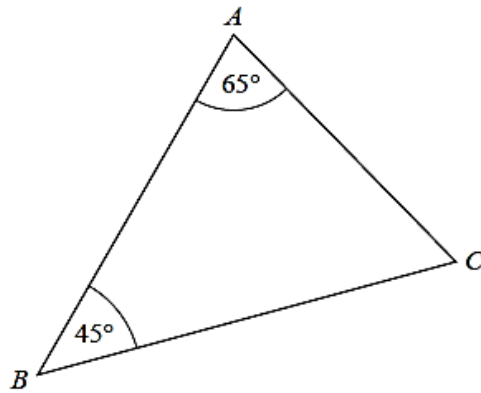
In triangle ABC, $A\hat{B}C = 45°$ and $B\hat{A}C = 65°$.
AC is 5 cm shorter than BC.

(i) Show that $BC = \dfrac{5\sin 65}{\sin 65 - \sin 45}$. [3]

(ii) Find the length of BC.

Answer .... cm [1]

(b)
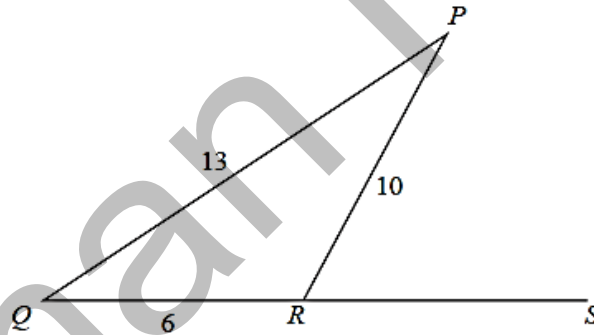
In triangle PQR, $PQ = 13$ cm, $QR = 6$ cm and $RP = 10$ cm.
QR is produced to S.

(i) Find the value of $\cos P\hat{R}Q$, giving your answer as a fraction in its lowest terms.

Answer .... [3]

(ii) Hence write down the value of $\cos P\hat{R}S$. Answer .... [1]

(c)
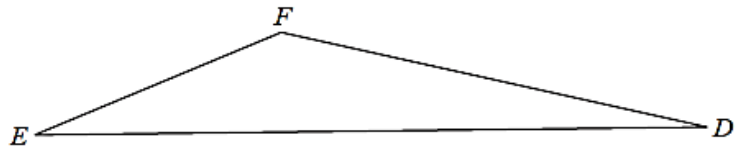
Triangle *DEG* has the same area as triangle *DEF*, but is not congruent to triangle *DEF*.
The point *G* is lower than *DE* and *GE* = *EF*.

Draw the triangle *DEG* in the diagram above.   [1]

(d) In triangle *LMN*, $L\hat{M}N = 30°$ and *ML* = 2*MN*.

When the area of triangle *LMN* is 18 cm^2, calculate *MN*.

Answer .... cm [3]

N13/21/Q12

55
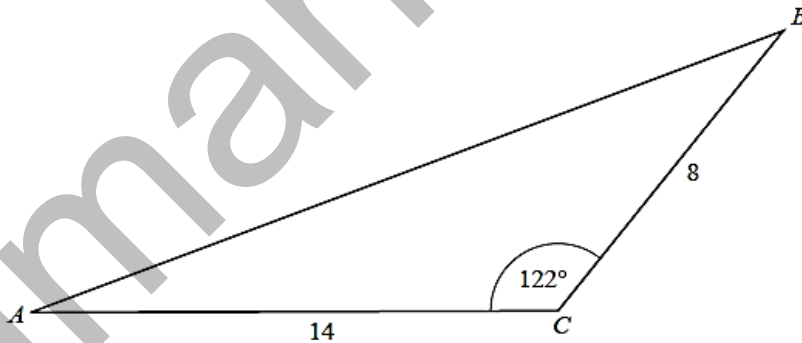
In triangle *ABC*, *AC* = 14 cm, *BC* = 8 cm and $A\hat{C}B = 122°$.

(a) Show that *AB* = 19.5 cm, correct to 3 significant figures.   [4]

(b) Calculate $A\hat{B}C$.   *Answer* .... [3]

11

(c) A rhombus, *BDEC,* of area 52 cm^2 and sides 8 cm is placed next to triangle *ABC* as shown in the diagram.

328
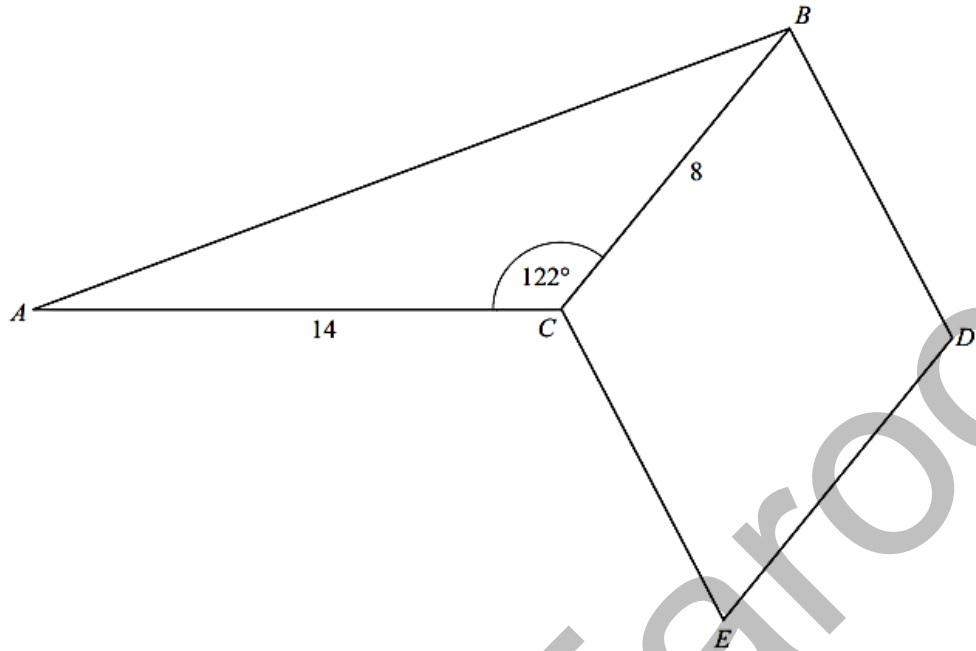
Given that $B\hat{C}E$ is an obtuse angle, calculate the **reflex angle** $A\hat{C}E$.

Answer .... [4]

56 **(b)**
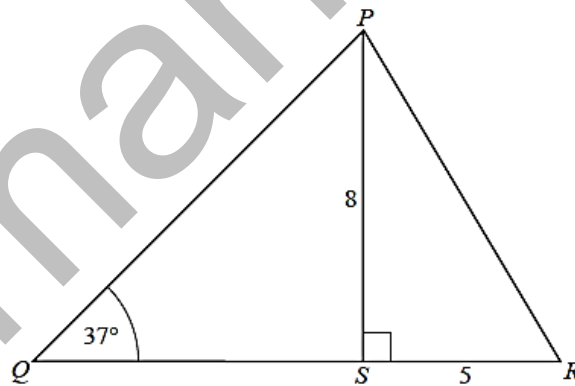
The diagram shows a triangle PQR with $P\hat{Q}R = 37°$.

S is the point on QR such that $P\hat{S}R = 90°$, $PS = 8$ cm and $SR = 5$ cm.

Calculate

(i) PR,
Answer .... cm [2]

(ii) the shortest distance from S to PQ.
Answer .... cm [3]

57

(a) *ABCDE* is a pentagon with one line of symmetry.
$BC = DE = 10$ cm, $DC = 30$ cm and $B\hat{C}D = C\hat{D}E = 90°$.
The shortest distance between *A* and *DC* is 22 cm.
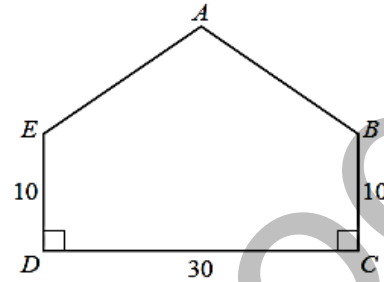
(i) Calculate *AB*.

Answer .... cm [2]

(ii) Calculate $A\hat{B}C$.

Answer .... [3]

(b) In triangle *PQR*, $PQ = 7$ cm, $PR = 9$ cm and $P\hat{Q}R = 65°$.

Calculate $P\hat{R}Q$.

Answer .... [3]

58
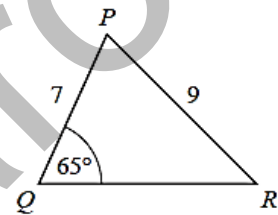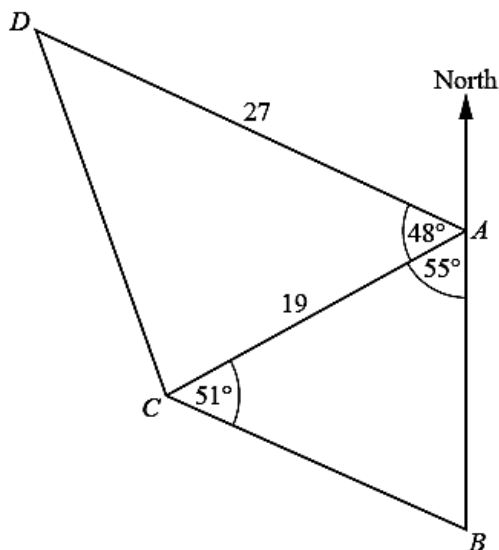
The diagram shows the positions of four islands at A, B, C and D.
A is due north of B.
$D\hat{A}C = 48°$, $C\hat{A}B = 55°$ and $B\hat{C}A = 51°$.
$AC = 19$ km and $AD = 27$ km.

(a) Calculate the bearing of D from A.

Answer .... [1]

(b) Calculate the bearing of A from C.

Answer .... [1]

(c) Calculate the distance between A and B.

Answer .... km [3]

(d) Calculate the distance between D and C.

Answer .... km [3]

(e) A boat leaves D and sails, at a constant speed, in a straight line to A.
It takes 3 hours and 36 minutes to sail from D to A.
X is the point on DA that is closest to C.

Calculate the time, correct to the nearest minute, the boat takes to travel from D to X.

Answer .... [4]

Answers: (a) 283 (b) 055 (c) 15.4 (d) 20.1 (e) 114 minutes   **N16/21/Q7**

331

59

North

B

25

A 220°

38

C
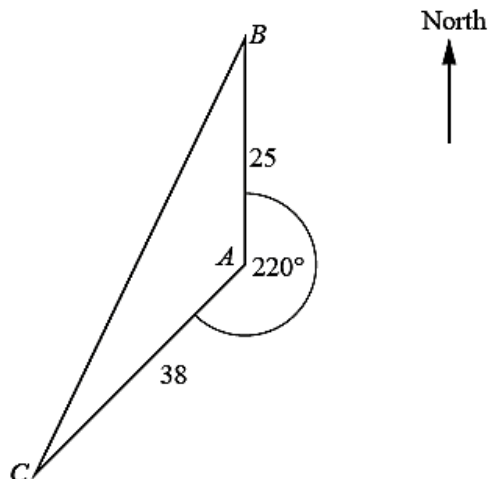
The diagram shows the positions of three towns, *A*, *B* and *C*.
B is due north of *A* and the bearing of *C* from *A* is 220°.
AB = 25 km and *AC* = 38 km.

(a) Find the bearing of *A* from *C*.

Answer .... [1]

(b) Show that *BC* = 59.4 km correct to 3 significant figures.

[3]

(c) Calculate the bearing of *C* from *B*.

Answer .... [4]

Answers: **(a)** 040 **(b)** 59.4 correctly derived **(c)** 204.3

N17/21/Q3

60

(b)

A 9 B

55°

5 35°

R

D P Q C
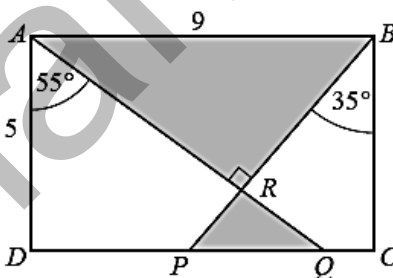
In rectangle *ABCD*, *AB* = 9 cm and *AD* = 5 cm.
$D\hat{A}Q = 55°$, $C\hat{B}P = 35°$ and *AQ* is perpendicular to *BP*.

(i) Calculate *AQ*.

Answer .... cm [2]

(ii) Calculate *AR*.

Answer .... cm [2]

(iii) Calculate the area of triangle *ARB*.

Answer .... cm² [2]

(iv) Calculate the total area shaded in the rectangle.

Answer .... cm² [3]

N17/21/Q11

332

61 **(a)**
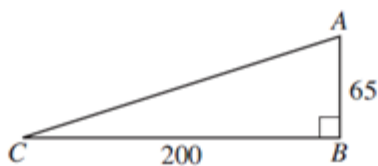
From the top of a vertical tower, AB, an observer sees a car at C.
$AB = 65\,\text{m}$ and $CB = 200\,\text{m}$.

Calculate $C\hat{A}B$. [2]

(b)
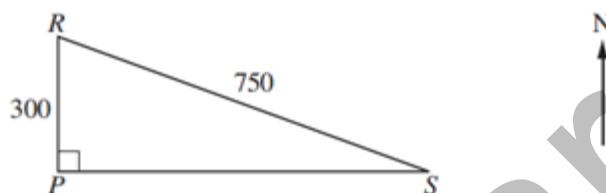
The diagram shows three positions at sea, R, P and S.
R is due north of P and S is due east of P.
$RP = 300\,\text{m}$ and $RS = 750\,\text{m}$.

(i) A boat sailed at a constant speed of $5\,\text{km/h}$ from R to S.
It was at R at 2256.

Find the time it reached S. [3]

(ii) Calculate the bearing of S from R. [3]

Answers: **(a)** 72.0°,   **(b)(i)** 23 05,   **(ii)** 114°.   **N07/2/Q1**

333

Bearings Paper 1

1

The diagram shows a lighthouse, *L*, and two ports *P* and *Q*.
Q is due east of *L* and $P\hat{L}Q = 80°$.
P and *Q* are each 10 km from *L*.
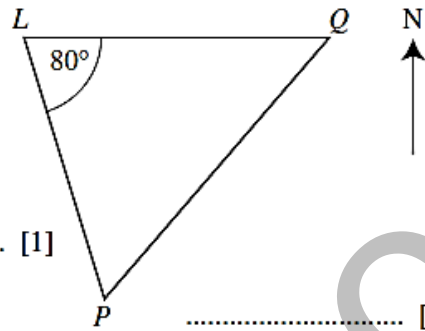
Find

(a) $L\hat{Q}P$, *Answer* (a) .... [1]

(b) the bearing of *Q* from *P*, ... [1]

(c) the bearing of *L* from *P*.   (b) .... [1]

2
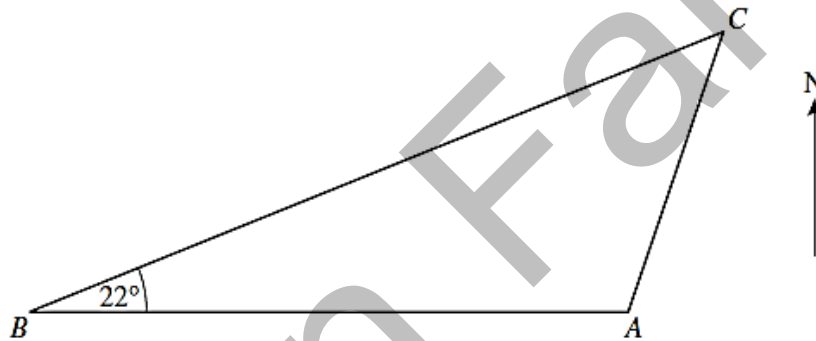
A, *B* and *C* are three ships.
B is due West of *A*.

(a) Given that $A\hat{B}C = 22°$, write down the bearing of *C* from *B*.

(b) By using your protractor, find the bearing of *A* from *C*.

Answer (a) .... [1]

(b) .... [2]

Answers: **(a)** 068°; **(b)** 199° to 200°.

3

A is due North of *O*.

(a) A ship sailed from *O* to *B*, where $A\hat{O}B = 12°$.
Write down the bearing of *B* from *O*.

(b) At *B*, the ship turned and sailed to *C*, where $O\hat{B}C = 50°$.
Calculate the bearing of *C* from *B*.

Answer (a) ....[1]

(b) .... [1]
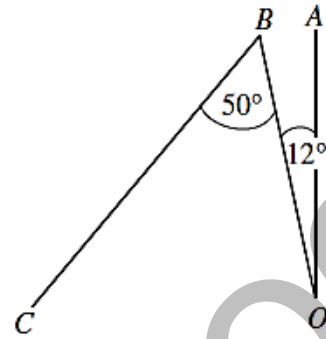
4
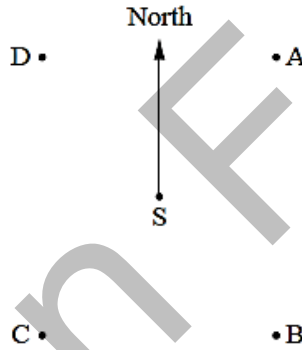
The bearing of a lighthouse from a ship, S, is 220°.
The position of S is marked on the diagram.

(a) Which of the four points A, B, C or D is a possible position of the lighthouse?

Answer (a) .... [1]

(b) Write down the bearing of S from the lighthouse. *Answer (b)* .... [1]

5

The scale drawing shows three towns, A, B and C.
The scale of the drawing is 1 cm to 25 km.
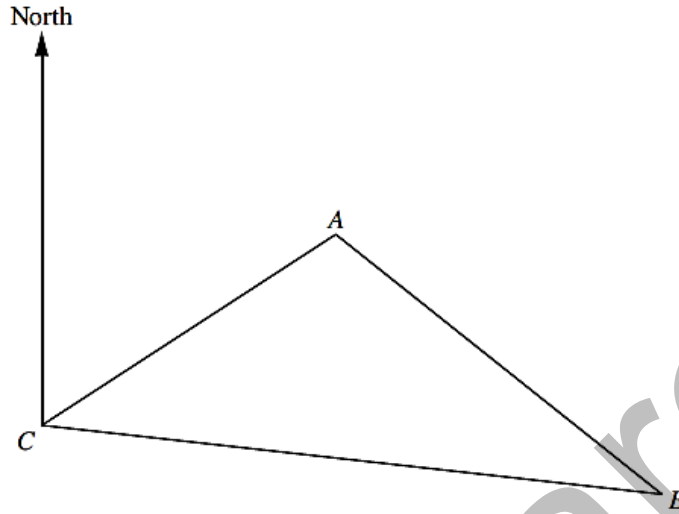
(a) Measure the bearing of A from C.   *Answer* .... [1]

(b) Find the bearing of C from A.   *Answer* .... [1]

(c) Find the actual distance, in kilometres, from B to C.   *Answer* .... km [1]

Answer. **(a)** 057° **(b)** 237° **(c)** 237.5   **J11/12/Q16**

6
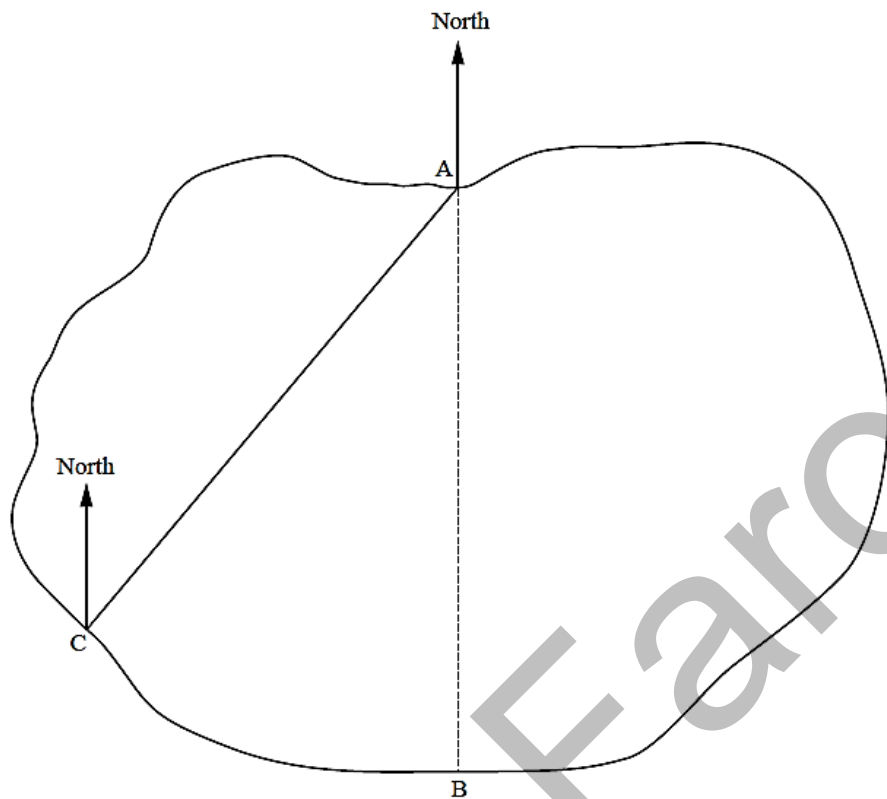
The diagram shows a map of a lake.
Three points A, B and C are on the edge of the lake.

(a) A ship sails due south from A to B.

Write down the bearing of B from A.   *Answer* .... [1]

(b) A yacht sails from A to C.

Measure and write down the bearing of C from A.   *Answer* .... [1]

(c) A cruiser sails from C to D on a bearing of 105°.

Work out the bearing of C from D.   *Answer* .... [1]

Answers: **(a)** 180° **(b)** 220° **(c)** 285°   **J14/11/Q11**

7
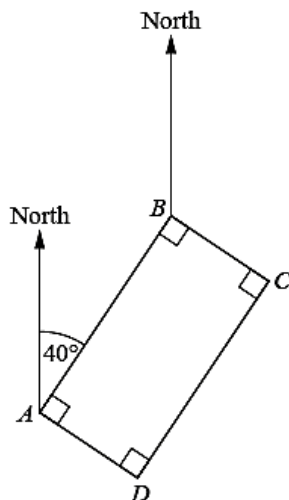
Four oil-rigs are positioned at the vertices of a rectangle *ABCD*.
The bearing of *B* from *A* is 040°.

(a) Find the bearing of

    (i) *A* from *B*,   *Answer* .... [1]

    (ii) *C* from *B*,   *Answer* .... [1]

    (iii) *C* from *D*.   *Answer* .... [1]

(b) A supply helicopter is due to arrive at *D* at 8.15 a.m.
It leaves its base at 7.33 a.m. and takes 49 minutes to fly to *D*.

How many minutes late does it arrive at *D*?

                                                      Answer .... [1]

Answers: **(a)(i)** 220° **(ii)** 130° **(iii)** 040° **(b)** 7 mins   **J15/11/Q20**

8

A, B and *C* are three towns.
C is equidistant from *A* and *B*.
The bearing of *C* from *A* is 132° and *BÂC* = 75°.

Find

(a) (i)   the acute angle *ACB*,

    (ii)   the reflex angle *ACB*,

(b)   the bearing of *A* from *C*,

(c)   the bearing of *A* from *B*.
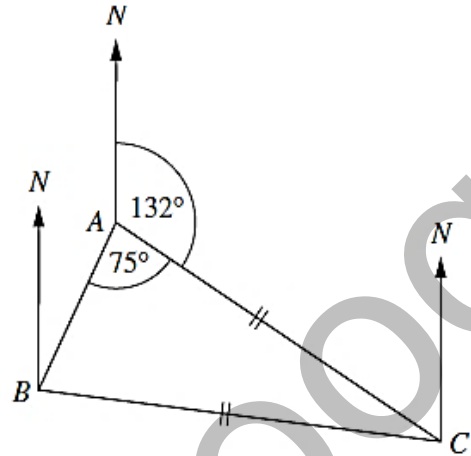
Answer (a) (i) .... [1]

       (ii) .... [1]

    (b) .... [1]

    (c) .... [1]

Answers: **(a)(i) 30°, (ii) 330°; (b) 312°; (c) 027°.**

9

The diagram shows the positions of *A* and *B*.

Find the bearing of

(a)   *A* from *B*,   *Answer (a)* .... [1]

(b)   *B* from *A*.   *(b)* .... [1]

Answers: **(a) 300°; (b) 120°**

10
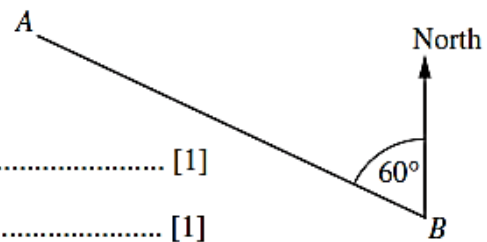
The diagram shows the positions of a harbour, H, and a lighthouse, L.
A boat is anchored at B where $L\hat{H}B = 108°$.

North

H | 125°

108°

L B
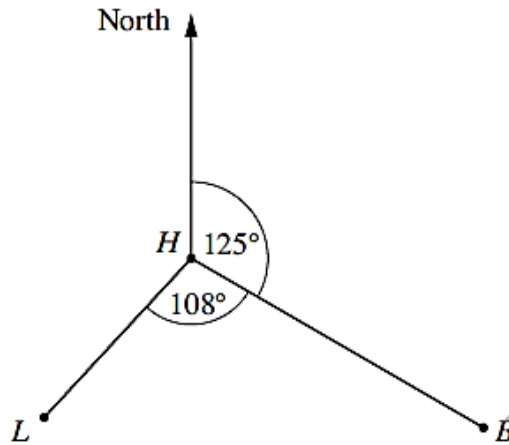
(a) Given that the bearing of B from H is 125°, find the bearing of

   (i) L from H,   *Answer* *(a)*(i) ....[1]

   (ii) H from B.   (ii) ....[1]

(b) At 7 30 a.m. the boat set sail in a straight line from B to H at an average speed of 25 km/h. Given that $BH = 70$ km, find the time at which the boat reaches the harbour.

                                                    (b) ....[2]

Answers: (a)(i) 233°,   (ii) 305°,   (b) 10 18 a.m.   **N08/1/Q18**

11

In the diagram, the triangle ABC is equilateral.
The bearing of B from A is 075°.

North

75°

A

B

C
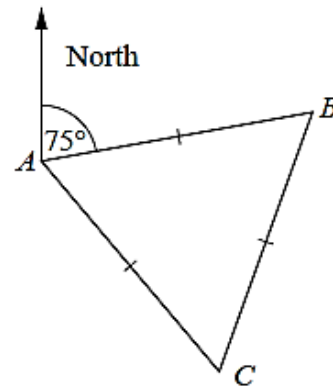
(a) Find the bearing of C from A.

   Answer ....[1]

(b) Find the bearing of C from B.

   Answer ....[1]

Answers: (a) 135 (b) 195   **N12/11/Q10**

12 In the diagram, the triangle *ABC* is equilateral.

North

A

B C
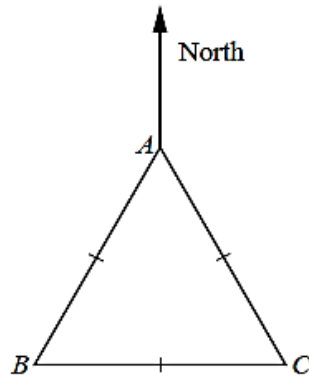
C is due East of *B*.

(a) Find the bearing of *B* from *A*.

Answer ....[1]

(b) Find the bearing of *A* from *C*.

Answer ....[1]

(c) A boat sails around a course represented by triangle *ABC*.
It started at 13 38 and finished at 14 21.

How many minutes did it take?   Answer ....[1]

Answers: (a) 210° (b) 330° (c) 43

N13/11/Q10

341

Mensuration Paper 1

1 **(a)**
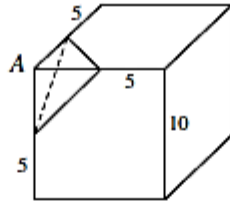
The diagram shows a 10 cm cube.

(i) A triangular pyramid is cut from the corner of the cube at A.
The cut is made halfway along each of the edges meeting at A as shown.
Calculate the volume, in cubic centimetres, of the pyramid.

[The volume of a pyramid $= \frac{1}{3} \times$ area of base $\times$ height]

(ii) From another 10 cm cube, shown in the answer space, a second similar pyramid is cut from the corner at P.
The volume is 8 times the volume of the first pyramid.
On the diagram in the answer space, draw the lines where the cut is made.

Answer *(a)* (i) .... cm³ [2]

(ii)
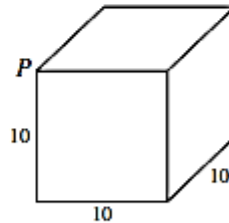
[1]

(b) Another 10 cm cube is cut as shown.

A prism containing the corners B and C is removed.
Calculate the volume which remains.
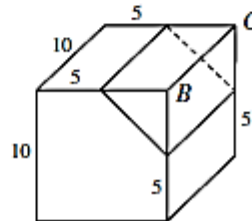
Answer *(b)* .... cm³ [2]

2

The vertices of the square *ABCD* lie on a circle of radius *r* cm.

(a) Show that the length, *l* cm, of a side of the square is $r\sqrt{2}$ cm. [1]

(b) By comparing the perimeter of the square and the circumference of the circle, or otherwise, show that $\sqrt{2} < \dfrac{\pi}{2}$. [2]

(c) What special kind of numbers are $\sqrt{2}$ and π? (c) .... [1]
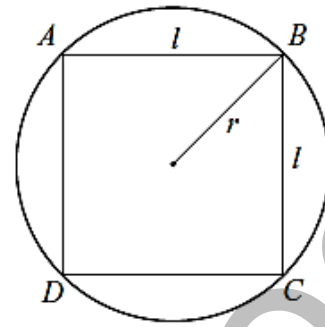
Answer. (c) Irrational.

3

A pendulum of length 105 cm is suspended from *O*.

Its end swings 3° on either side of the vertical from *A* to *B*.

Taking $\pi = \dfrac{22}{7}$, calculate the length of the arc *AB*.

Answer .... cm [2]
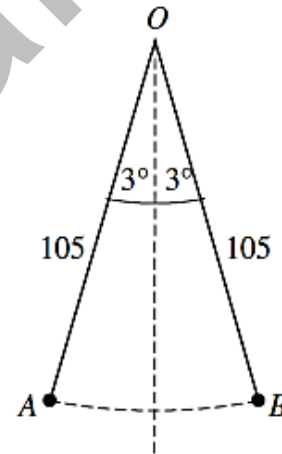
Answer. 11 cm.

4

The base of a pyramid is a square with diagonals of length 6 cm.
The sloping faces are isosceles triangles with equal sides of length 7 cm.
The height of the pyramid is $\sqrt{l}$ cm.

Calculate *l*.
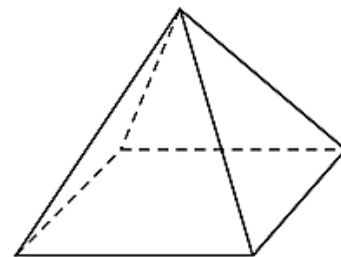
Answer l = ... [3]

Answer. 40.

5

OAB is a sector of a circle with centre O and radius $8\,\text{cm}$.
$A\hat{O}B = x°$.

(a) Write down an expression, in terms of x and π, for the area of the sector OAB.
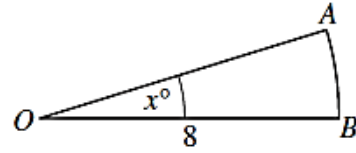
(b) PQR is a semicircle of radius $4\,\text{cm}$.
The area of the sector OAB is $\frac{1}{3}$ of the area of this semicircle.
Calculate the value of x.
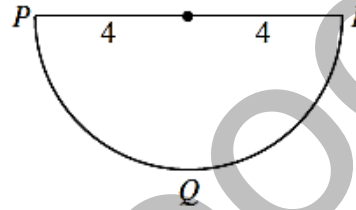
Answer (a) ....cm^2 [1]

(b) x = ... [2]

Answers: (a) $\dfrac{8\pi x}{45}$ cm^2; (b) $15°$.

6

$ABCDE$ is a pyramid.
The base $BCDE$ is a square of side $10\,\text{cm}$.
The sloping faces are isosceles triangles.
$AB = AC = AD = AE = 13\,\text{cm}$.

(a) Calculate the area of the sloping face ABC.

(b) The pyramid $ABCDE$ is joined to an identical pyramid $BCDEF$ to form the solid $ABCDEF$.

   (i) Calculate the surface area of the solid $ABCDEF$.

   (ii) Describe fully the locus of all points which are equidistant from A and F.

Answer (a) ....cm^2 [2]

(b)(i) ....cm^2 [1]

344

ABCDE is a pyramid.
The base *BCDE* is a square of side 10 cm.
The sloping faces are isosceles triangles.
$AB = AC = AD = AE = 13$ cm.

(a) Calculate the area of the sloping face *ABC*.

(b) The pyramid *ABCDE* is joined to an identical pyramid *BCDEF* to form the solid *ABCDEF*.

   (i) Calculate the surface area of the solid *ABCDEF*.

   (ii) Describe fully the locus of all points which are equidistant from *A* and *F*.

Answer (a) ....cm² [2]

(b)(i) ....cm² [1]

(ii) ....

.... [1]

Answers: **(a)** 60 cm²; **(b)(i)** 480 cm², **(ii)** the plane *BCDE*.

---

7 In the diagram, *ABCD* is a diameter of the circle centre *P*.
$AB = BC = CD = 2x$ centimetres.

(a) Find an expression, in terms of *x* and π, for the circumference of this circle.

(b) The perimeter of the shaded region consists of two semicircles whose diameters are *AB* and *CD*, and two semicircles whose diameters are *AC* and *BD*.
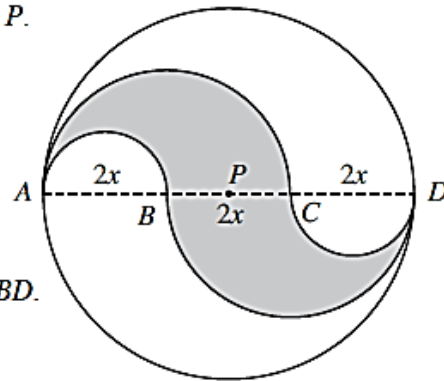
Find an expression, in terms of *x* and π, for the area of the shaded region.

Answer (a) ....cm [1]

(b) .... cm² [2]

Answer: **(a)** 6πx cm   **(b)** 3πx² cm²

8

The diagram shows a solid cuboid with base 10 cm by 6 cm.
The height of the cuboid is x centimetres.
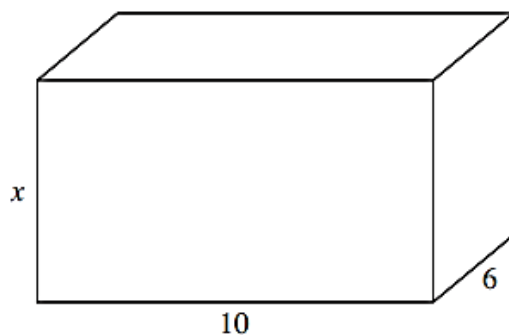
(a) Find an expression, in terms of x, for the total surface area of the cuboid.

(b) The total surface area of the cuboid is $376\,cm^2$.

Form an equation in x and solve it to find the height of the cuboid.

Answer (a) .... cm^2 [1]

(b) .... cm [2]

Answers: **(a)** 32x + 120 cm² **(b)** 8 cm   **J08/1/Q7**

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9
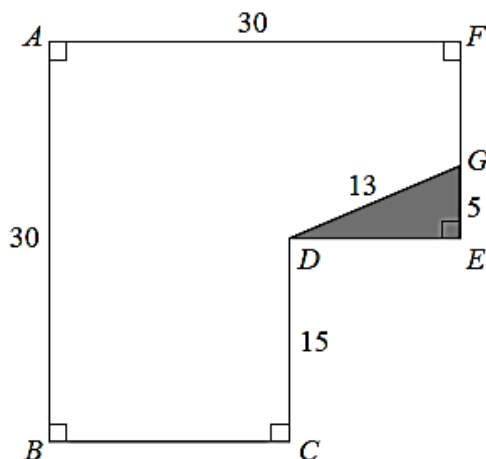
ABCDEF represents an L-shaped piece of glass with *AB = AF = 30* cm and *CD = 15* cm. The glass is cut to fit the window in a door and the shaded triangle *DEG* is removed. *DG = 13* cm and *EG = 5* cm.

(a) Show that DE = 12 cm. ....[1]

(b) For the remaining piece of glass *ABCDGF*, find

    (i) its perimeter,   *Answer* *(b)*(i) .... cm **[2]**

    (ii) its area.   *Answer* *(b)*(ii) .... cm² **[2]**

  (c) State the value of cos $D\hat{G}F$.   *Answer* *(c)* .... [1]

Answer. **(a)** $\sqrt{13^2 - 5^2}$ evaluated or $13^2 = 12^2 + 5^2$ confirmed   **(b)(i)** 116   **(ii)** 690   **(c)** $-\dfrac{5}{13}$ **J09/1/Q22**

10

OAB is the sector of a circle of radius *r* cm.
$A\hat{O}B = 60°$.

Find, in its simplest form, an expression in terms of *r* and π for

(a) the area of the sector,
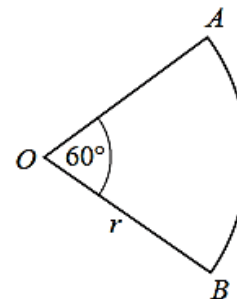
                                         Answer *(a)* .... cm² [1]

(b) the perimeter of the sector.   *Answer* *(b)* .... cm [2]

                                                             J10/12/Q18

Answer. **(a)** $\dfrac{\pi r^2}{6}$ **(b)** $2r + \dfrac{\pi r}{3}$

347

11 The diagram shows a window made up of a large semicircle and a rectangle.
The large semicircle has 4 identical sections, A, B, C, D, and a small semicircle, E.
The rectangle has three identical square sections, F, G and H.
The side of each square is 20 cm.
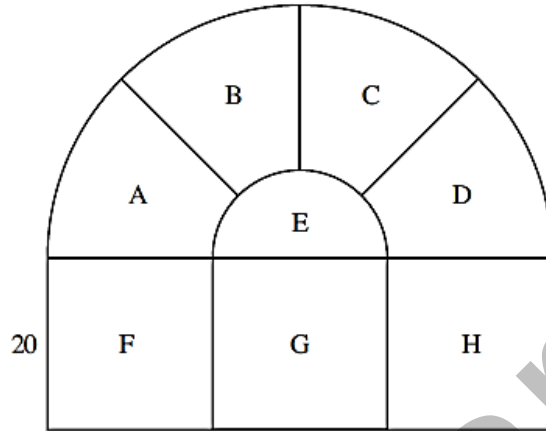
Find an expression, in the form $a + b\pi$, for

(a) the area of the whole window,

Answer ....cm^2 [2]

(b) the perimeter of section B.

Answer ....cm [3]

Answers: (a) $1200 + 450\pi$ (b) $40 + 10\pi$ **J11/11/Q22**

12
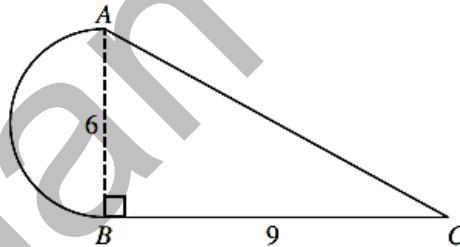
ABC is a right-angled triangle with $AB = 6$ cm and $BC = 9$ cm.
A semicircle of diameter 6 cm is joined to the triangle along AB.

Find an expression, in the form $a + b\pi$, for the **total** area of the shape.

Answer ....cm^2 [2]

Answer: $\dfrac{9\pi}{2} + 27$ **J11/12/Q6**

13 In this shape all the lengths are in centimetres.
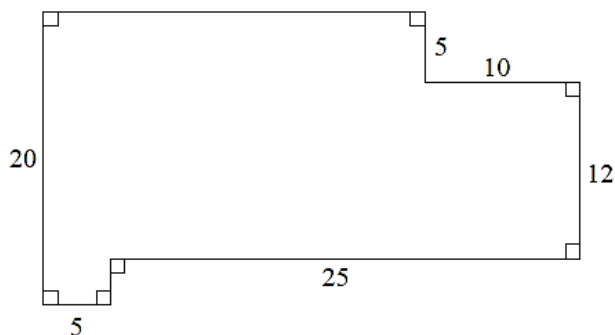
Work out

(a) the perimeter,   *Answer* .... cm [1]

(b) the area.   *Answer* .... cm^2 [1]

Answer: (a) 100 (b) 475   **J13/11/Q1**

14 Choose a quadrilateral from the list to complete each statement.

Kite	Parallelogram	Rectangle	Rhombus	Square	Trapezium

(a) A .... has four equal sides and four angles of 90°.   [1]

(b) A .... has just one pair of parallel sides.   [1]

(c) A .... has just one pair of opposite angles equal and its diagonals bisect at 90°.   [1]

Answer: (a) square (b) trapezium (c) kite   **J13/11/Q11**

15 The diagram shows the metal cover for a circular drain.
Water drains out through the shaded sections.
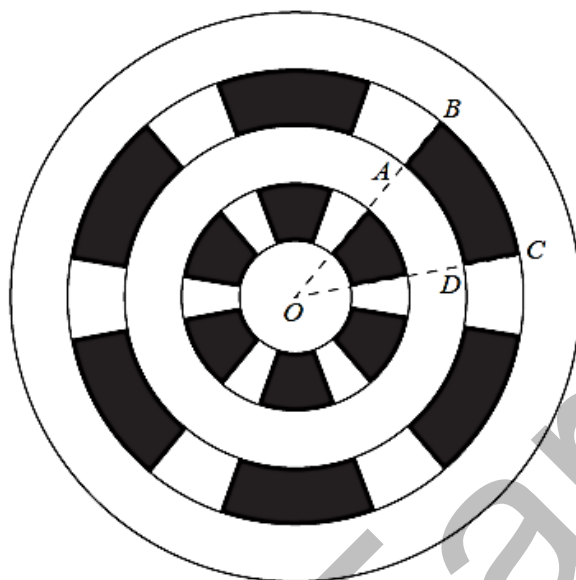
O is the centre of circles with radii 1 cm, 2 cm, 3 cm, 4 cm and 5 cm.
The cover has rotational symmetry of order 6 and $B\hat{O}C = 40°$.

(a) Calculate the area of the shaded section $ABCD$, giving your answer in terms of π.

Answer ....cm² [2]

(b) The total area of the metal (unshaded) sections of the cover is $\dfrac{55}{3}\pi$ cm².

   (i) Calculate the total area of the shaded sections, giving your answer in terms of π.

Answer ....cm² [1]

   (ii) Calculate the fraction of the total area of the cover that is metal (unshaded).
Give your answer in its simplest form.

Answer ....[1]

J13/11/Q19

16
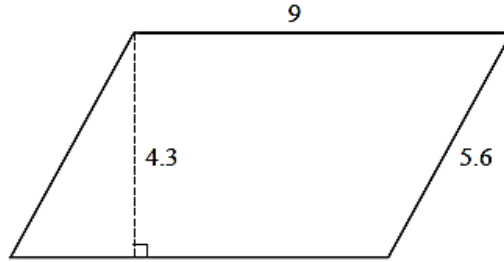
The diagram shows a parallelogram with lengths as marked.
All the lengths are in centimetres.

(a) Calculate the perimeter of the parallelogram.

Answer ....cm [1]

(b) Calculate the area of the parallelogram. Answer ....cm² [1]

Answers: (a) 29.2 (b) 38.7

---

17 A thin piece of wire is shaped into a figure five as shown.
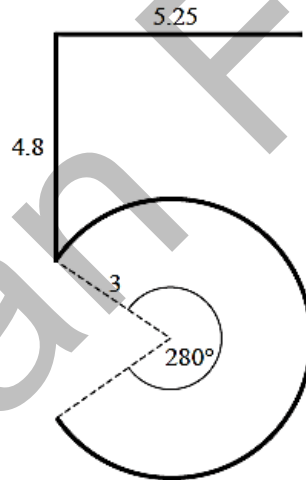
The shape has two straight sections of length 5.25 cm and 4.8 cm.
The curved part is the arc of the major sector of a circle, radius 3 cm.
The angle of the major sector is 280°.

The total length of wire needed to make the figure is $(a + b\pi)$ cm.

Find the values of a and b.

Answer a = ...

b = ... [2]

Answers: a = 10.05 b = 14 / 3

18   [The volume of a sphere is $\frac{4}{3}\pi r^3$] [The volume of a cone is $\frac{1}{3}\pi r^2 h$]

A cone is removed from a solid wooden hemisphere of radius 3 cm.

The cone has radius 3 cm and height 2 cm.

The volume of wood remaining is $k\pi$ cm^3.

Find k.
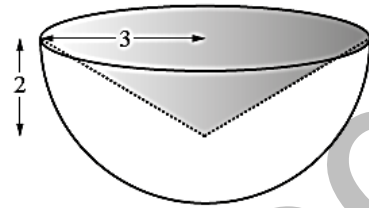
Answer $k =$ .... [3]

Answer: 12

19   **(a)** Find the perimeter of the shape below.
All the angles are right angles.
All the lengths are in centimetres.
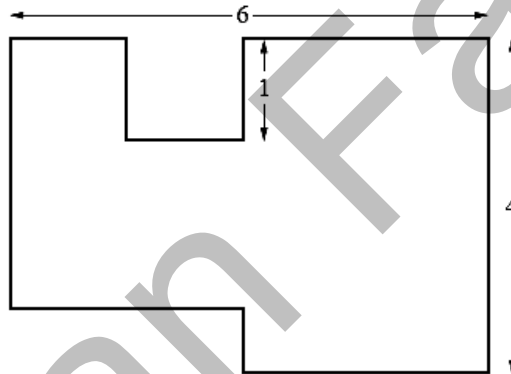
Answer .... cm [1]

(b) On the grid below draw a trapezium with height 4 cm and area 18 cm^2.
One side of the trapezium has been drawn for you.
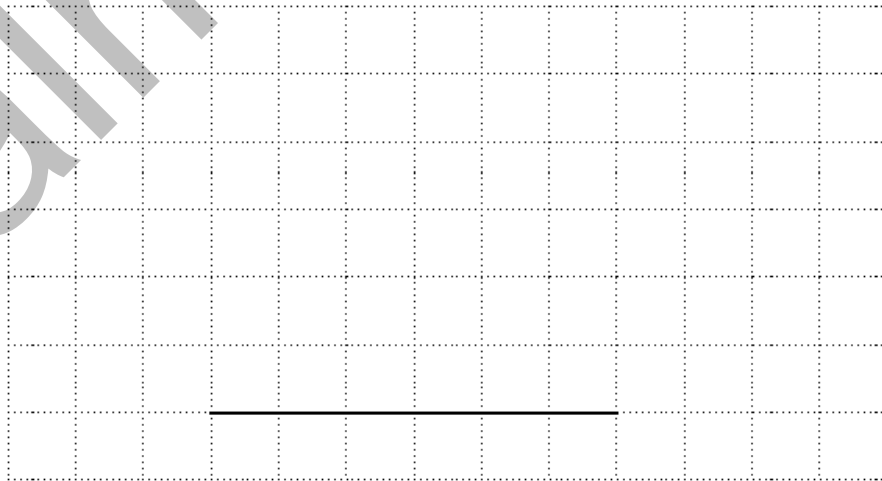
[1]

20 [Volume of a pyramid = $\frac{1}{3}$ × base area × perpendicular height]
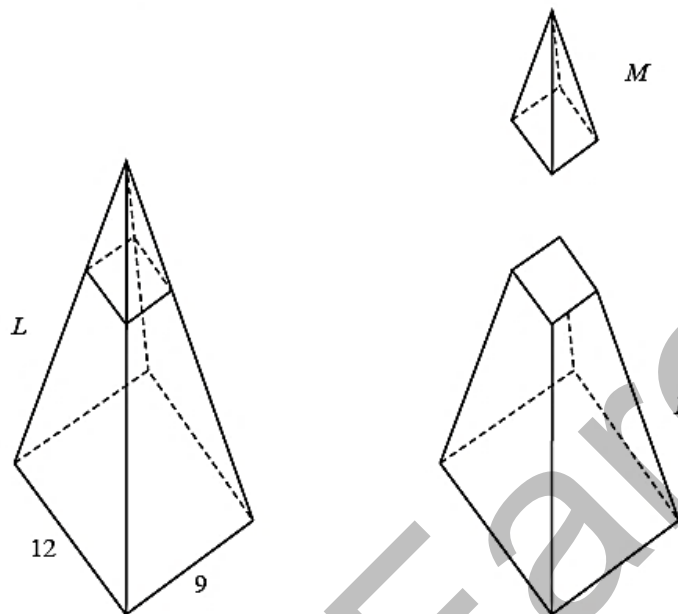
The diagrams show a solid pyramid L cut into two parts, M and N, by a plane parallel to its base.
The base of pyramid L is a rectangle 9 cm by 12 cm.
The perpendicular height of pyramid L is 30 cm.

(a) Work out the volume of pyramid L. 　　　　Answer ....cm³ [1]

(b) The perpendicular height of pyramid M is $\frac{1}{3}$ of the perpendicular height of pyramid L.

　　(i) Express the volume of M as a fraction of the volume of L.

　　　　　　　　　　　　　　　Answer ....[1]

　　(ii) Calculate the volume of the solid N. 　　Answer ....cm³ [2]

Answers: **(a)** 1080 **(b) (i)** $\frac{1}{27}$ **(ii)** 1040

---

21 A block of wood is a cuboid, 10 cm by 6 cm by 2 cm.

Find

(a) its volume,

(b) its surface area.
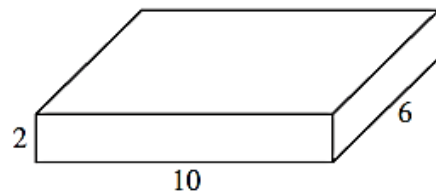
Answer (a) .... [1]

(b) .... [1]

Answers: **(a)** 120 cm³; **(b)** 184 cm².　　　　　**N04/Q7**

353

22 A cuboid is shown in the diagram.
The volume of the cuboid is 90 000 cm³.
Find the height of the cuboid.

wer ....cm [2]

50 cm

60 cm

Answer: 30.

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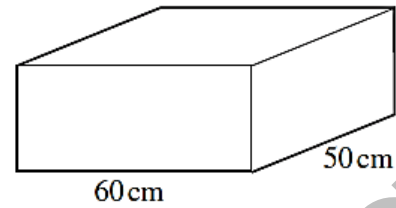
23 The diagram shows a container consisting of
two cylinders fastened together.
The lower cylinder has radius r centimetres
and height $2h$ centimetres.
The upper cylinder has radius $2r$ centimetres
and height h centimetres.
Water was poured into the container at a
constant rate.
The container was filled in 12 minutes.
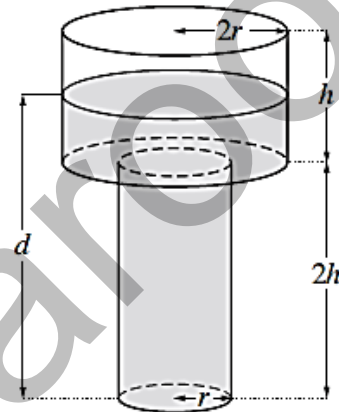
(a) Calculate the time taken to fill the
lower cylinder.

Answer (a) .... minutes [1]

(b) On the axes in the answer space, draw the graph showing how the depth,
d centimetres, of water, changes during the 12 minutes.

Answer (b)

[2]

Depth
(d cm)

$3h$

$2h$

h

0   2   4   6   8   10   12

Time (minutes)
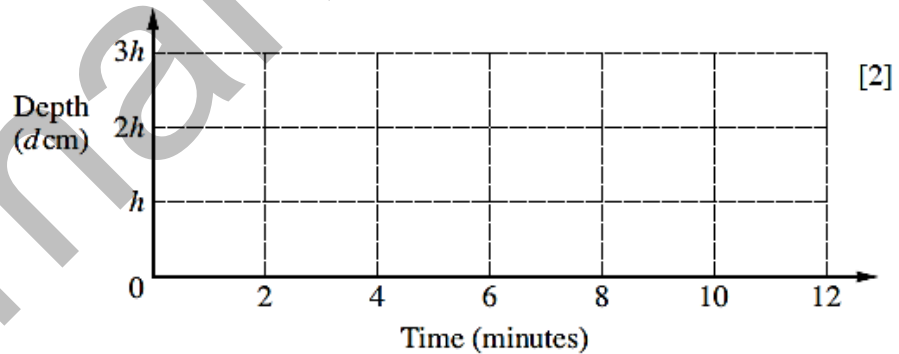
Answer: **(a)** 4 minutes   **(b)** straight line from (0,0) to (4, 2h); straight line from (4, 2h) to (12, 3h)

24
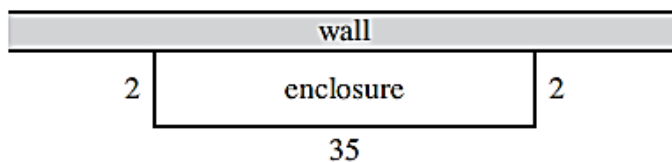
A farmer wishes to build a rectangular enclosure against a straight wall.
He has 39 identical fence panels, each 1 metre long.
One possible arrangement, which encloses an area of $70\,\text{m}^2$, is shown in the diagram and recorded in the table below.

Find the length of the enclosure which would contain the largest area.
Write down this length and the largest area.
Record all your trials in the table.
Marks will be awarded for clear, appropriate working.

Width (m)	2				
Length (m)	35				
Area (m²)	70				

Answer Length = ...m

Area = ...m² [3]

Answer: Length = 19 m; Area = 190 m²

25

The diagram shows a sector of a circle, centre O.
The radius of the circle is 9 cm and the sector angle is $100°$.
Taking the value of π to be 3.14, calculate
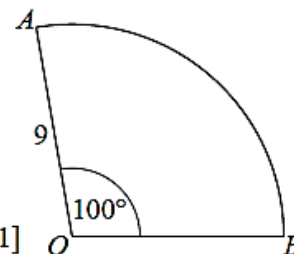
(a) the length of the arc AB, *Answer (a)* .... cm [2]

(b) the perimeter of the sector. *Answer (b)* .... cm [1]

Answers: (a) 15.7 (b) 33.7

26 The diagram shows a shape made from thin wire.
The shape is formed from 3 identical sectors of a circle, each with an angle of 40°.
The angle between each pair of sectors is $x°$.
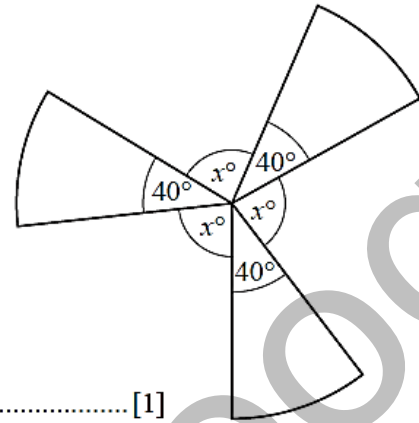
(a) State the order of rotational symmetry of the shape.

Answer (a) ....[1]

(b) Calculate the value of x.   Answer (b) x = ...[1]

(c) **In this part take the value of π to be 3.**
Given that the **total** length of the wire is 60 cm, calculate the radius of one of the sectors.   Answer (c) .... cm [3]

Answer: (a) 3 (b) 80 (c) $7\frac{1}{2}$

---

27 The diagram shows a solid prism of length 20 cm.
The cross-section, $ABCD$, is a trapezium.

$AB = 2$ cm, $BC = 5$ cm, $CD = 6$ cm, $DA = 3$ cm and angle $ADC = 90°$.
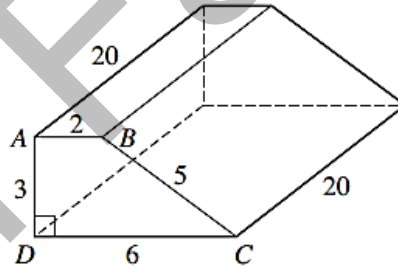
(a) Calculate the area of trapezium $ABCD$.   Answer .... cm² [1]

(b) Calculate the **total** surface area of the prism.   Answer .... cm² [2]

Answers: (a) 12 (b) 344

356

28 The diagram shows the plan of a rectangular garden, measuring 11 m by 9 m.
All the angles are right angles.
The pond is a square of side 4 m.
The paths are 1 m wide.
The remainder of the garden is a lawn.
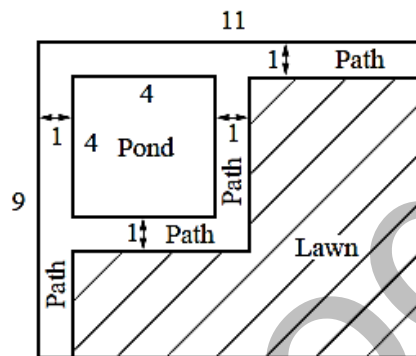
(a) Find the perimeter of the lawn.

Answer .... m [1]

(b) Find the total area of the paths.

Answer .... m² [1]

(c) The paths are paved with square tiles of side 50 cm.

How many tiles are used?

Answer ....[1]

Answers: (a) 36 (b) 28 (c) 112

---

29 [Volume of a cone $= \frac{1}{3}\pi r^2 h$]

Cone 1 has radius $2x$ cm and height $7x$ cm.

Cone 2 has radius x cm and height $4x$ cm.   *Answer* ....cm³ [3]

Find an expression, in terms of π and x, for the **difference** in the volume of the two cones.
Give your answer in its simplest form.

Answer. $8\pi x^3$

30
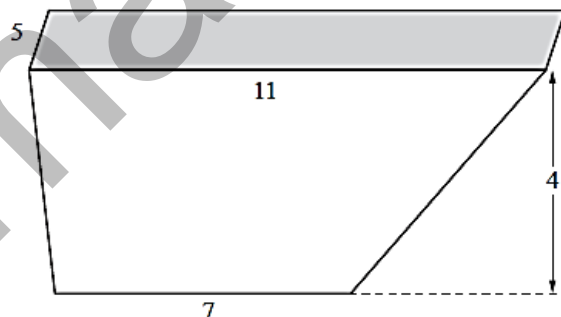
The diagram shows a scoop used for measuring washing powder.
The scoop is a prism. Its cross-section is a trapezium.
The trapezium has height 4 cm and parallel sides of length 7 cm and 11 cm.
The width of the scoop is 5 cm.

(a) Show that the volume of the scoop is 180 cm³.   [1]

(b) A scoop used in industry is geometrically similar to the scoop above.
It has a volume of 22.5 litres.

357

Calculate the height of the industrial scoop.   *Answer* ....cm [3]

31
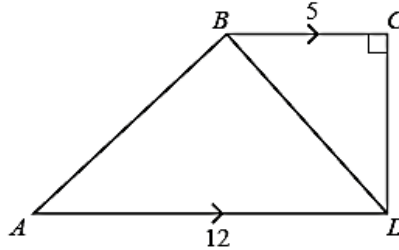
ABCD is a quadrilateral with *BC* parallel to *AD*.
CD is perpendicular to *BC*.
BC = 5 cm and *AD* = 12 cm.
The area of triangle *BCD* is 20 cm².

(a) Find *CD*.   *Answer* .... cm [1]

(b) Find the area of triangle *ABD* .   *Answer* ....cm² [1]

32
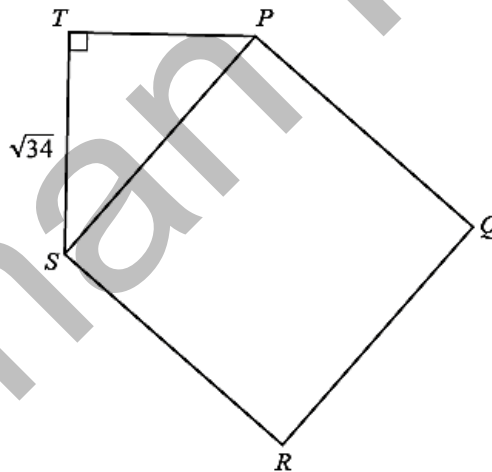
The diagram shows a square *PQRS* and a right-angled triangle *PST*.
The area of the square is 50 cm².
ST = √34 cm.

Calculate *PT*.   *Answer* .... cm [2]

Mensuration Paper 2

1   [The value of π is 3.142, correct to three decimal places.]

[The volume of a sphere is $\frac{4}{3}\pi r^3$.]
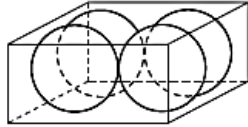
Diagram I   Diagram II

The diagrams show two ways of packaging 4 identical balls.
The radius of each ball is 3 cm.

Diagram I shows a closed rectangular box with a square base.
Each ball touches the top, the bottom and two sides of the box.
Each ball also touches two other balls.

Diagram II shows a closed cylinder.
The balls touch the ends and the side of the cylinder.

(a) (i)   Write down the dimensions of the rectangular box.   [1]

    (ii)   Calculate the total surface area of the outside of this box.   [2]

(b) Calculate the total surface area of the outside of the cylinder.   [2]

(c) Calculate the total volume of the 4 balls.   [2]

(d) Calculate, correct to three decimal places, the value of

$$\frac{\text{volume of the cylinder}}{\text{volume of the box}} .$$ [2]

(e) Hence state which of the two containers has more space **not** occupied by the balls.   [1]

J02/2/Q5

2 [The value of π is 3.142, correct to three decimal places.]

Diagram I shows a sector, *ADB*, of a circle.
The centre of the circle is *C* and its radius is 6 cm.
$A\hat{C}B = 120°$.

(a) Calculate the length of the major arc *ADB*.

Diagram I

[2]
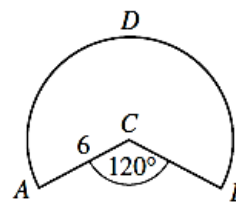
(b) Two tangents are drawn to touch the circle at *A* and *B*.
The tangents meet at *T*, to form the shape shown in
Diagram II.

   (i) Explain why $A\hat{T}B = 60°$. [1]

  (ii) Calculate the length of *AT*. [2]

 (iii) Calculate the perimeter of the shape *ADBT*. [2]
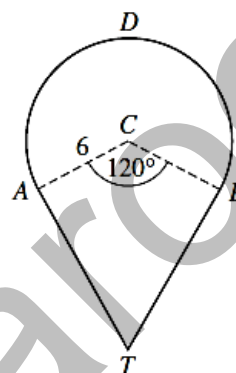
Diagram II

(c) Four of the shapes shown in Diagram II
are arranged to form the figure shown
in Diagram III.

This figure has rotational symmetry
of order 4.
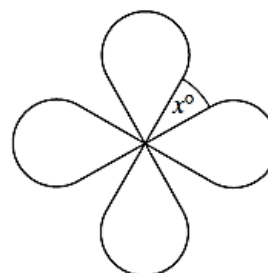
Diagram III

   (i) Write down the number of lines of symmetry in this figure. [1]

  (ii) Calculate the angle (marked as *x*° in Diagram III) between each shape. [1]

 (iii) Every second, the figure turns through 40° about its centre.
Calculate the time it takes to make 108 revolutions.
Give your answer in minutes and seconds. [3]

3

[The value of π is 3.142, correct to three decimal places.]
[The surface area of a sphere is $4\pi r^2$.]
[The volume of a sphere is $\frac{4}{3}\pi r^3$.]

A closed container is made by joining together
a cylinder of radius 9 cm and a hemisphere
of radius 9 cm as shown in Diagram I.
The length of the cylinder is 18 cm.
The container rests on a horizontal
surface and is exactly half full of water.
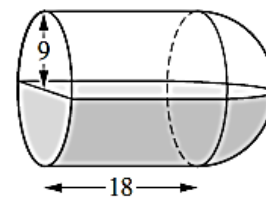
Diagram I

(a) Calculate the surface area of the inside of the container that is in contact with the water.
Give your answer correct to the nearest square centimetre. [4]

(b) Show that the volume of the water is 972π cm^3. [2]

(c) The container is held with
its axis vertical, the hemisphere
being at the bottom, as shown
in Diagram II.

Calculate the depth of the water.
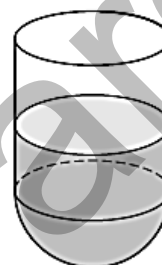
Diagram II [4]

(d) The container is now placed with its
circular end on a horizontal
surface as shown in Diagram III.

Find the depth of the water.
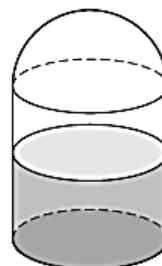
Diagram III [2]

Answers: (a) 891cm^2; (b) $\frac{1}{2}.\pi.9^2.18 + \frac{1}{2}.\frac{2}{3}.\pi.9^3 = 972\pi$; (c) 15cm; (d) 12cm.   J03/2/Q7

4

[The surface area of a sphere is $4\pi r^2$.]

[The volume of a cone is $\frac{1}{3} \times$ base area $\times$ height.]

[The area of the curved surface of a cone of radius r and slant height l is πrl.]
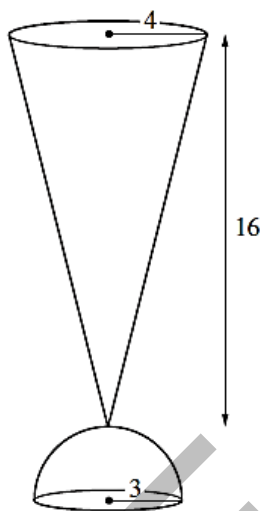
A drinking glass consists of a hollow cone attached to a solid hemispherical base as shown in the diagram.
The hemisphere has a radius of 3 cm.
The radius of the top of the cone is 4 cm and the height of the cone is 16 cm.

(a) Calculate the total surface area of the solid hemispherical base. [3]

(b) Calculate the curved surface area of the outside of the cone. [3]

(c) (i) The cone contains liquid to a depth of d centimetres.

      Giving your reasons, show that the radius of the surface of the liquid is $\frac{1}{4}d$ centimetres. [1]

  (ii) The cone is completely filled with liquid.

      Calculate the volume of the liquid. [2]

 (iii) Half of the volume of the liquid from the full cone is now poured out.

      Using the answers to parts (i) and (ii), find the depth of the liquid that remains in the cone. [3]

Answers: **(a)** 84.8 cm²; **(b)** 207 cm²; **(c)(ii)** 268 cm³, **(iii)** 12.7 cm.

J04/2/Q7

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5   [The volume of a pyramid is $\frac{1}{3} \times$ base area $\times$ height.]

[The volume of a sphere is $\frac{4}{3}\pi r^3$.]

Morph made several different objects from modelling clay.
He used $500\,cm^3$ of clay for each object.

(a) He made a square-based cuboid of height 2 cm.

Calculate the length of a side of the square.   [2]

(b) He made a pyramid with a base area of $150\,cm^2$.

Calculate the height of the pyramid.   [2]

(c) He made a sphere.

Calculate the radius of the sphere.   [2]

(d) He wrapped the clay around the curved surface
of a hollow cylinder of height 6 cm.

The thickness of the clay was 1.5 cm.

Calculate the radius of the hollow cylinder.
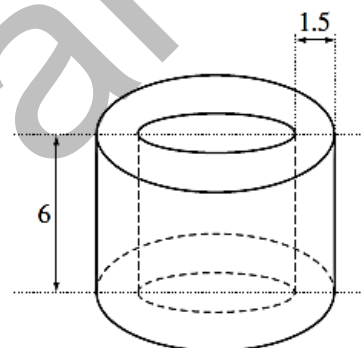
[4]

(e) He made a cone.

Then he cut through the cone, parallel to its
base, to obtain a small cone and a frustum.

The height of the small cone was two-fifths
of the height of the full cone.

Use a property of the volumes of similar
objects to calculate the volume of clay in
the small cone.   [2]
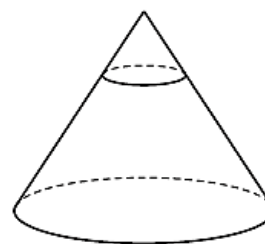
Answers: **(a)** 15.8 cm; **(b)** 10 cm; **(c)** 4.92 cm; **(d)** 8.09 cm; **(e)** 32 cm³.   **J05/2/Q7**

6 (a) A solid cuboid measures 7 cm by 5 cm by 3 cm.
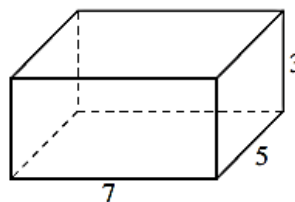
    (i) Calculate the total surface area of the cuboid. [2]

    (ii) A cube has the same volume as the cuboid.
        Calculate the length of an edge of this cube. [2]

  (b) [The volume of a cone is $\frac{1}{3} \times$ base area $\times$ height.]

     [The area of the curved surface of a cone of radius r and slant height l is $\pi r l$.]

    A solid cone has a base radius of 8 cm and a height of 15 cm.

    Calculate
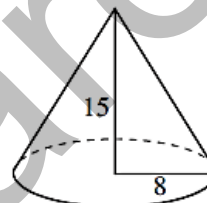
    (i) its volume, [2]

    (ii) its slant height, [1]

    (iii) its curved surface area, [2]

    (iv) its **total** surface area. [1]

Answers: (a)(i) 142 cm², (ii) 4.72 cm; (b)(i) 1010 cm³, (ii) 17 cm (iii) 427 cm² (iv) 628 cm².　**J06/2/Q2**

7 (b) The diagram shows a circle, centre O, with the sector POQ shaded.
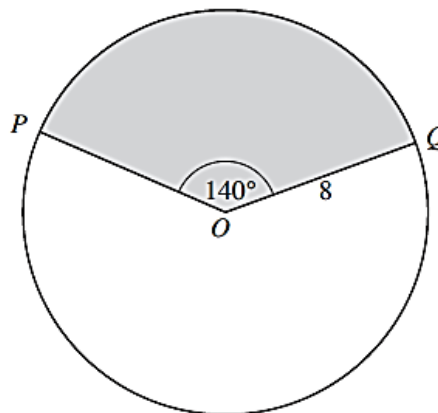
    Given that $P\hat{O}Q = 140°$ and the radius of the circle is 8 cm, calculate

    (i) the area of the shaded region, [2]

    (ii) the total perimeter of the **unshaded** region. [3]

(b)(i) 78.2 cm², (ii) 46.7 cm.　**J06/2/Q3b**

8   **(a)** [The volume of a sphere is $\frac{4}{3}\pi r^3$.]
             [The surface area of a sphere is $4\pi r^2$.]
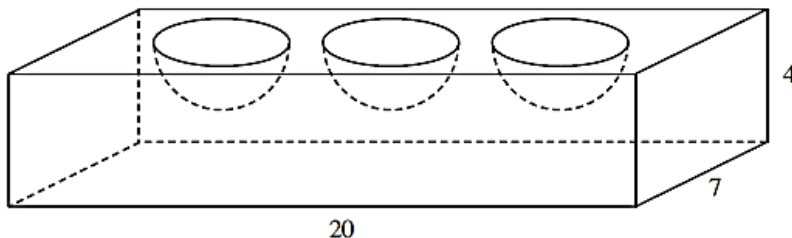
A wooden cuboid has length 20 cm, width 7 cm and height 4 cm.
Three hemispheres, each of radius 2.5 cm, are hollowed out of the top of the cuboid, to leave the block as shown in the diagram.

    (i)   Calculate the volume of wood in the block.   [3]

    (ii) The four vertical sides are painted blue.
           Calculate the total area that is painted blue.   [1]

    (iii) The inside of each hemispherical hollow is painted white.
           The flat part of the top of the block is painted red.
           Calculate the total area that is painted

           (a)   white,   [2]

           (b)   red.   [2]

    (b) The volume of water in a container is directly proportional to the cube of its depth.
         When the depth is 12 cm, the volume is 576 cm^3.
         Calculate

      (i)   the volume when the depth is 6 cm,   [2]

      (ii) the depth when the volume is 1300 cm^3.   [2]

Answer: **(a)(i)** 462 cm^3, **(ii)** 216 cm^2, **(iii)(a)** 118 cm^2, **(b)** 81.1 cm^2; **(b)(i)** 72 cm^3, **(ii)** 15.7 cm.   **J07/2/Q7**

---

9   A, B, C, D and E are five different shaped blocks of ice stored in a refrigerated room.

    (a) At 11 p.m. on Monday the cooling system failed, and the blocks started to melt.
         At the end of each 24 hour period, the volume of each block was 12% less than its volume at the start of that period.

      (i)   Block A had a volume of 7500 cm^3 at 11 p.m. on Monday.

          Calculate its volume at 11 p.m. on Wednesday.   [2]

      (ii) Block B had a volume of 6490 cm^3 at 11 p.m. on Tuesday.

          Calculate its volume at 11 p.m. on the previous day.   [2]

      (iii) **Showing your working clearly**, find on which day the volume of Block C was half its volume at 11 p.m. on Monday.   [2]

    (b) [The volume of a sphere is $\frac{4}{3}\pi r^3$.]
         [The surface area of a sphere is $4\pi r^2$.]

      At 11 p.m. on Monday Block D was a **hemisphere** with radius 18 cm.

Calculate

   (i)   its volume,   [2]

   (ii)   its **total** surface area.   [2]

(c) As Block E melted, its shape was always **geometrically similar** to its original shape.
It had a volume of $5000 \, cm^3$ when its height was 12 cm.

Calculate its height when its volume was $1080 \, cm^3$.   [2]

Answers: **(a) (i)** 5810,   **(ii)** 7375,   **(iii)** Sunday,   **(b) (i)** 12200,   **(ii)** 3050,   **(iii)** 7.2.   **J08/2/Q7**

10   (b) [The volume of a sphere is $\frac{4}{3}\pi r^3$.]

The ball is a sphere of volume 96 cm^3.

Calculate its radius.   [2]

(b) 2.84 cm.   **J09/2/Q3b**

11   (a) When a solid rectangular wooden block of oak floats, 60% of its height is under water.

    (i)   What fraction of its height is **above** water?   [1]

    (ii)   A block of oak has length 60 cm,
breadth 50 cm and height
h centimetres.

           It floats with 15 cm of its height
under water.
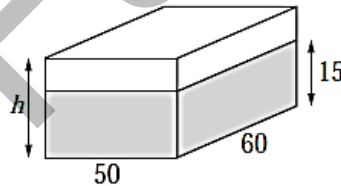
           (a) Find the value of h.   [1]

           (b) In the diagram, the shaded region represents part of the surface area of the block that is in
contact with the water.

                Calculate the **total** surface area of the block that is in contact with the water.   [2]

(b)
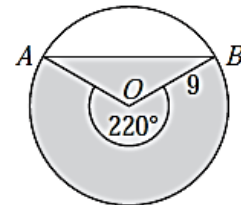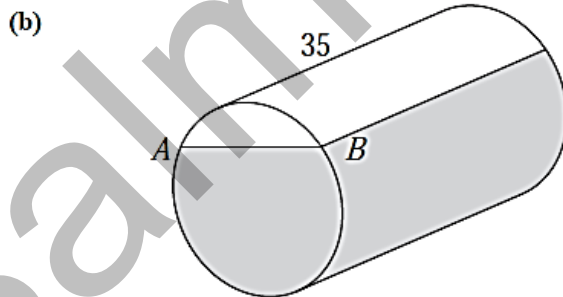
A solid cylinder, made from a different type of wood, floats in water.
The shaded region represents part of the surface of the cylinder that is in contact with the water.
The right hand diagram shows the circular cross-section of one end.
The centre of the circle is O and the water level reaches the points A and B on the circumference.
Reflex angle $AOB = 220°$.
The cylinder has radius 9 cm and length 35 cm.

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Calculate

(i) the area of the **curved surface** of the cylinder that is in contact with the water, [2]

(ii) the surface area of **one end** of the cylinder that is in contact with the water, [4]

(iii) the distance between the water level *AB* and the top of the cylinder. [2]

Answers: (a)(i) ⅖,   (ii)(a) 25 cm,   (ii)(b) 6 300 cm²;   (b)(i) 1 210 cm², (ii) 182 cm², **J09/2/Q7**
(iii) 5.92 cm.

12
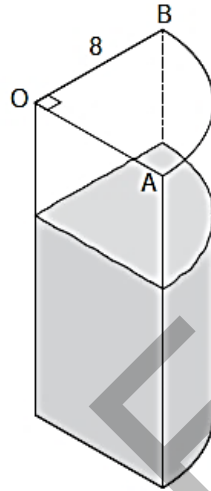
Diagram I

Diagram I shows a vessel in the shape of a prism.
The cross-section *OAB* is a sector of a circle of radius 8 cm and $A\hat{O}B = 90°$.

(a) **(i)** Calculate the perimeter of the sector *OAB*. [3]

(ii) The vessel, which stands on a horizontal table, contains 800cm³ of water, shown shaded in the diagram.

Calculate the depth of the water in the vessel. [3]

(b)
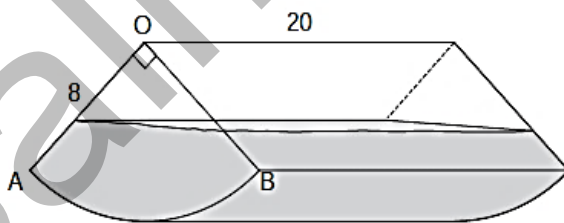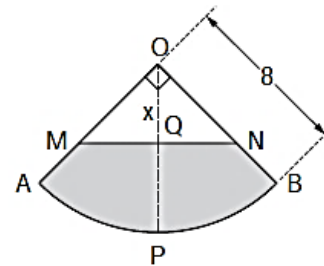
Diagram II   **Diagram III**

The vessel is now placed so that its curved surface is in contact with the horizontal table as shown in Diagram II.

Diagram III shows the cross-section of the vessel with the shaded section representing the water.
P is the midpoint of the arc *AB* and is in contact with the table.
Q is the point on the water surface, *MN*, which is vertically above *P*.

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It is given that $OQ = x$ centimetres.

(i) (a) Write down an expression, in terms of x, for MN.   [1]

    (b) Show that the shaded area in Diagram III is $(16\pi - x^2)$ square centimetres.   [2]

(ii) The vessel still contains $800\,cm^3$ of water.

    Given that the length of the vessel is $20\,cm$, find the value of x.   [3]

Answers: (a)(i) 28.6 cm   (ii) 15.9 cm   (b)(i)(a) 2x   (ii) 3.20 cm   **J10/21/Q9**

13
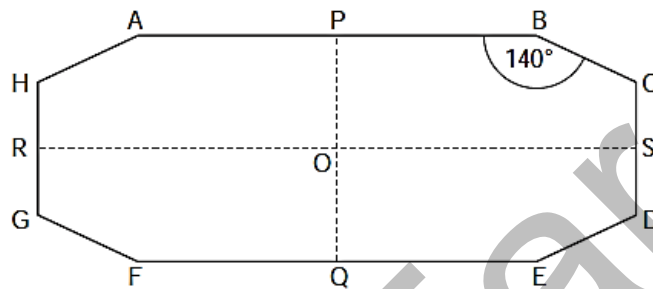
$ABCDEFGH$ is an octagon with exactly two lines of symmetry.
These are shown by the dashed lines PQ and RS which intersect at O.
$A\hat{B}C = 140°$.

(a) Find

    (i) $E\hat{F}G$,   [1]

    (ii) $F\hat{G}H$.   [2]

(b) The diagram shows part of the octagon.

    T is the point such that PT and TS are perpendicular.
    $PB = 50\,cm$, $BT = 23\,cm$ and $CS = 20\,cm$.
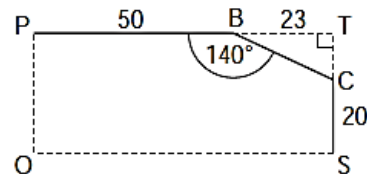
    (i) Calculate CT.   [2]

    (ii) Calculate the area of the pentagon $PBCSO$.   [3]

    (iii) Hence find the area of the octagon $ABCDEFGH$.   [1]

    (iv)
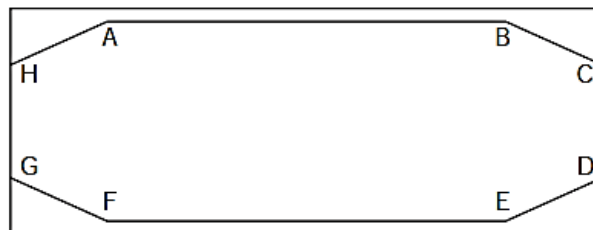
The octagonal shape *ABCDEFGH* is to be cut from a rectangular piece of card where the length and breadth, measured in centimetres, are integers.
The remaining card is wasted.
The card that is wasted must be kept to a minimum.

Find

(a) the length and breadth of the rectangular piece of card,   [2]

(b) the area of the card that is wasted.   [1]

14   (b) In triangle *WXY*, *WX* = 24 cm, *WY* = 17 cm
and $X\hat{W}Y = 55°$.
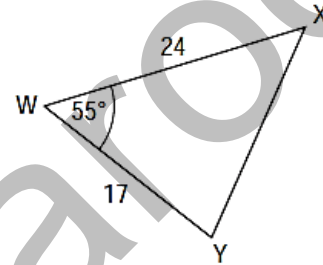
Diagram I

(i) Calculate

(a) the area of triangle *WXY*,   [2]

(b) *XY*.   [4]

(ii) [Volume of a pyramid = $\frac{1}{3}$ × base area × height]

The triangle *WXY* shown in Diagram I forms the
horizontal base of the triangular pyramid *VWXY*,
shown in Diagram II.

The vertex *V* is vertically above *Z*, a point on *WX*.
WV = 15 cm and $WZ = \frac{1}{4}$ *WX*.
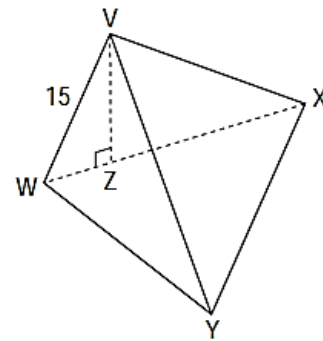
Diagram II

(a) Calculate *VZ*.   [2]

(b) Hence find the volume of the pyramid.   [1]

15 [Volume of a cone = $\frac{1}{3}\pi r^2 h$]
[Curved surface area of a cone = $\pi r l$]

Diagram I shows a solid cone with C
as the centre of its base.
B is the vertex of the cone and A is a point
on the circumference of its base.
$AC = 9$ cm and $BC = 12$ cm.
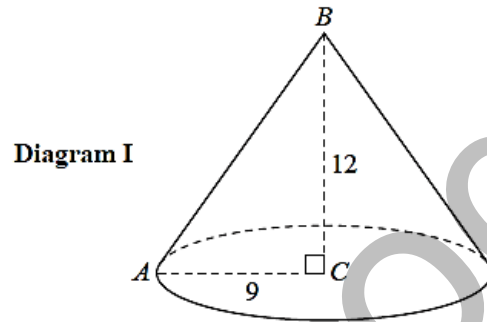
Diagram I

(a) Calculate

    (i) AB,   [2]

    (ii) the **total** surface area of the cone,   [2]

    (iii) the volume of the cone.   [2]

(b) The cone in Diagram I is cut, parallel
to the base, to obtain a small cone shown
in Diagram II and a frustum shown in
Diagram III.
Y is the centre of the base of the small cone.
X is the point on the circumference of this
base and on the line AB such that
$XY = 3$ cm.
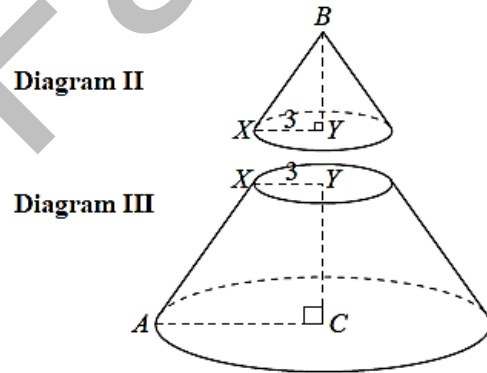
Diagram II

Diagram III

Calculate

    (i) BY,   [1]

    (ii) AX,   [1]

    (iii) the circumference of the base of the small cone,   [2]

    (iv) the volume of the frustum.   [2]

Answers: **(a)(i)** 15 cm,   **(ii)** 679 cm²,   **(iii)** 1020 cm³; **(b)(i)** 4 cm,   **(ii)** 10 cm,   **J10/22/Q12**
        (iii) 18.8 cm,   **(iv)** 980 cm³.

16 **(a)**
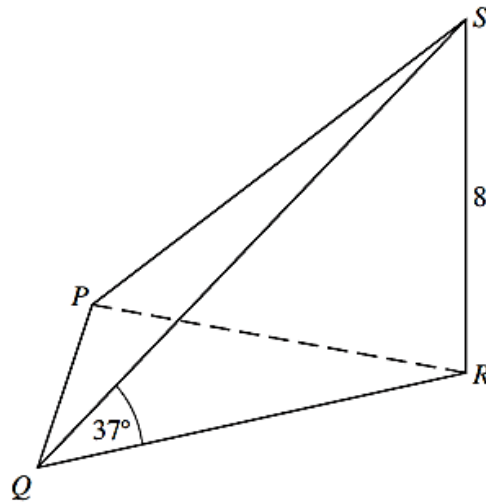
PQRS is a triangular-based pyramid.
RS is perpendicular to the base *PQR*.
RS = 8 cm and $R\hat{Q}S = 37°$.

(i)   Find *QR*.

Answer .... cm [2]

(ii)
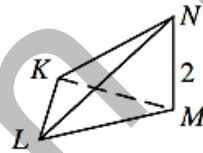
Pyramid *KLMN* is similar to pyramid *PQRS*.
MN = 2 cm and the volume of *KLMN* is 3 cm³.

Find the volume of *PQRS*.

Answer .... cm³ [2]

Answers: (a)(i) 10.6   (ii) 192   **J11/21/Q3**

17

A cylindrical candle has a height of 5 cm.
A is the centre of the top of the candle and *B* is the
centre of the base of the candle.
The wick runs from *B* through *A* and extends
12 mm above *A*.
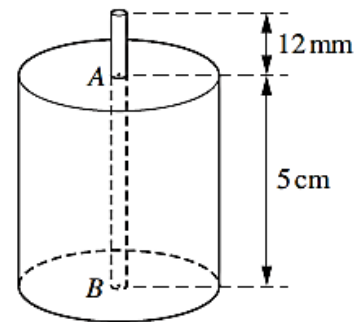
(a)   How many of these candles can be made using a 2 m length of wick?

Answer .... [2]

(b) The wick is in the form of a solid cylinder.
The volume of the wick inside the candle from A to B is $0.2\,\text{cm}^3$.

(i) Calculate the radius of the wick.   *Answer* .... mm [3]
Give your answer in millimetres.

(ii) One candle was made by pouring candle wax into a cylindrical mould so that it surrounded the wick.
This mould has an internal radius of $1.9\,\text{cm}$.

(a) Calculate the volume of candle wax required to make this candle.

Answer .... cm^3 [3]

(b) How many of these candles can be made using 3 litres of candle wax?

Answer .... [2]

(c)
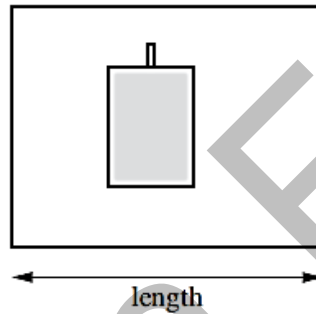
length

One of these candles is placed on a rectangular piece of wrapping paper.
The paper is wrapped around the candle so that it covers the outside and there is an extra 1 cm for an overlap.

What is the length, in centimetres, of paper required to wrap one candle?

Answer .... cm [2]

Answers: (a) 32 (b)(i) 1.13 (ii)(a) 56.5 (b) 53 (c) 12.9   **J11/21/Q10**

18   **(a)** The formula for the area of a trapezium is $A = \frac{1}{2}h(c+d)$.

(i) Find an expression for c in terms of A, h and d.   *Answer* ....[2]

(ii)

372
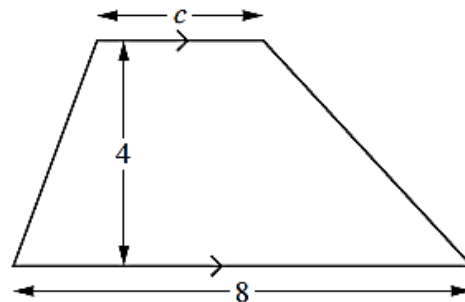
The diagram shows a trapezium with dimensions given in centimetres.
The perpendicular distance between the parallel lines is 4 cm.
The area of the trapezium is 22 cm^2.

Find c.   Answer ....[1]

19 [Volume of a cone = $\frac{1}{3}\pi r^2 h$]
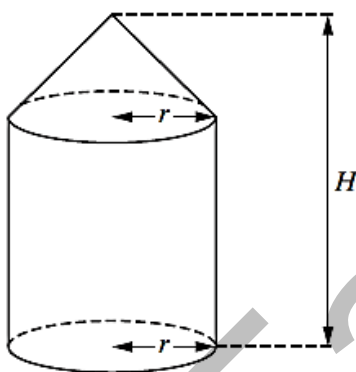
The solid above consists of a cone with base radius r centimetres on top of a cylinder of radius r centimetres.
The height of the cylinder is twice the height of the cone.
The total height of the solid is H centimetres.

(a) Find an expression, in terms of π, r and H, for the volume of the solid.
Give your answer in its simplest form.

Answer .... [3]

(b) It is given that $r = 10$ and the height of the cone is 15 cm.

(i) Show that the slant height of the cone is 18.0 cm, correct to one decimal place. [2]

(ii) Find the circumference of the base of the cone. Answer ....cm [2]

(iii) The curved surface area of the cone can be made into the shape of a sector of a circle with angle $\theta°$.

Show that θ is 200, correct to the nearest integer.
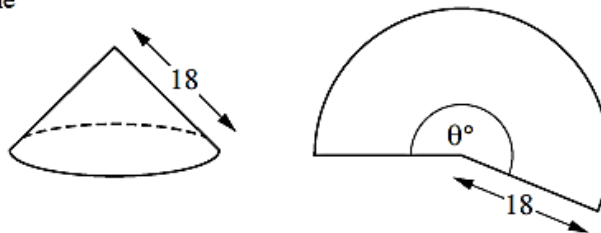
[2]

(iv) Hence, or otherwise, find the **total** surface area of the solid.

Answer ....cm^2 [3]

373

20   4   The entrance to a zoo has this sign above it.
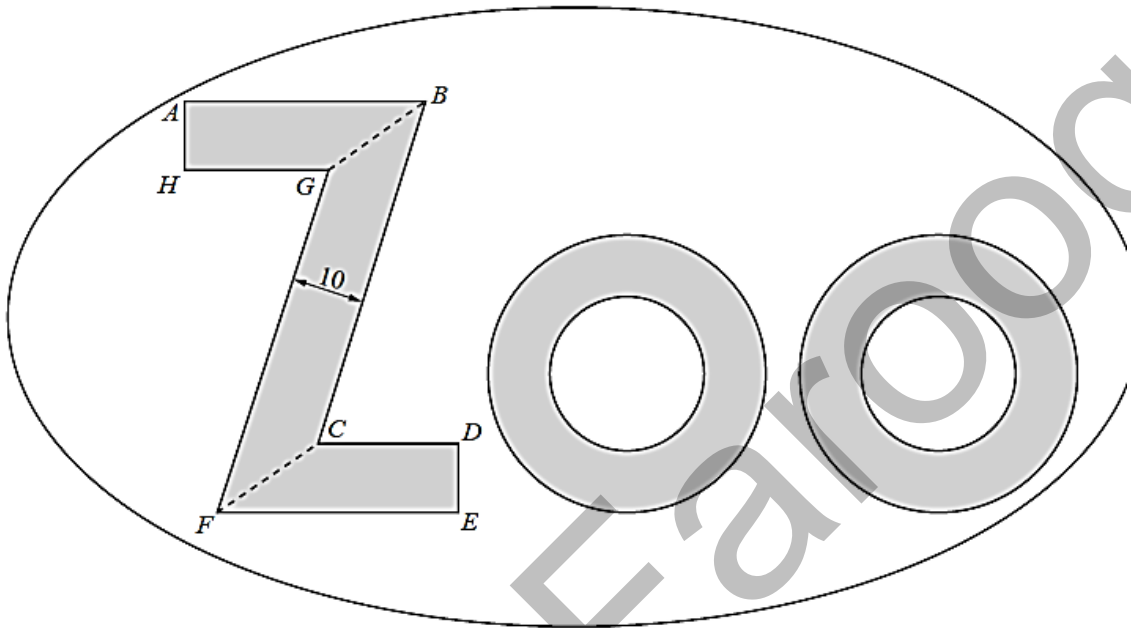
(a) The letter Z has rotational symmetry order 2 and *DE* is perpendicular to *FE* and *CD*.
$CD = 35$ cm, $FE = 50$ cm, $DE = 10$ cm and $BC = 81$ cm.
The perpendicular distance between *BC* and *GF* is 10 cm.

Calculate the area of the letter Z.   *Answer* .... cm^2 [3]

(b) The shaded area of one letter O is 1206 cm^2.
The radius of the unshaded inner circle is 15 cm.

Calculate the radius of the outer circle.   *Answer* .... cm [3]

(c) The sign above the exit of the zoo is geometrically similar to the one above the entrance.
The radius of the inner circle of the letter O on the sign above the exit is 10 cm.

(i) The length of the base of the letter Z on the sign above the entrance is 50 cm.

Calculate the length of the base of the letter Z on the sign above the exit.

Answer .... cm [1]

(ii) The area of the sign above the entrance is A cm^2.
The area of the sign above the exit is kA cm^2.

Write down the value of k as a fraction in its simplest form.   *Answer* ....[2]

374

21 **(a)** Shape I is a cylinder with radius 4 cm and height h cm.
The volume of Shape I is 1500 cm³.

    (i) Find h.
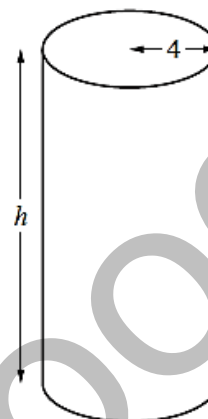
Shape I

Answer .... [2]

    (ii) Shape I is made by pouring liquid into a mould at a rate of 0.9 litres per minute.
Find the number of **seconds** it takes to pour this liquid into the mould.

Answer .... seconds [1]

(b) Shape II is a prism of length 8 cm with a triangular cross-section,
shown shaded.
Two sides of the shaded triangle are at right angles to each other and
have lengths $5x$ cm and $12x$ cm.

Given that Shape II also has a volume of 1500 cm³, find x.
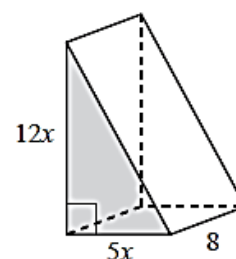
Shape II

Answer .... [2]

(c) Shape III is also a prism of length 8 cm with a triangular cross-section,
shown shaded.
Two sides of the shaded triangle are at right angles to each other and
have lengths $5y$ cm and $12y$ cm. The third side is of length $13y$ cm.
y satisfies the equation $4y^2 + 16y - 33 = 0$.

    (i) Factorise $4y^2 + 16y - 33$.

Shape III

375
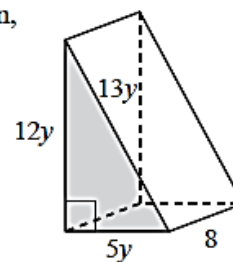
(ii) Hence solve the equation $4y^2 + 16y - 33 = 0$.

Answer $y = $ .... or .... [1]

(iii) Find the area of the shaded triangle. *Answer* .... cm^2 [1]

(iv) Find the total surface area of Shape III. *Answer* .... cm^2 [3]

(d) Find $\dfrac{\text{Volume of \textbf{Shape III}}}{\text{Volume of \textbf{Shape II}}}$ as a fraction in its simplest form.

Answer .... [1]

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22 **(a)** A candle is in the shape of a cylinder of radius 1.6 cm and height 7.5 cm.

(i) Calculate the volume of the candle. *Answer* .... cm^3 [2]

(ii) Six of these candles are packed into a box of height 7.5 cm as shown.

7.5
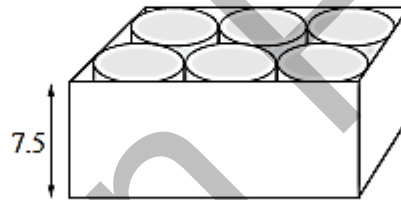
(a) Find the length and width of the box.

Answer length = ... cm

width = ... cm [1]

(b) Calculate the volume of empty space in the box.

Answer .... cm^3 [2]

---

23   [The volume of a cone $= \frac{1}{3}\pi r^2 h$] [The volume of a sphere $= \frac{4}{3}\pi r^3$]

7.6

4.5
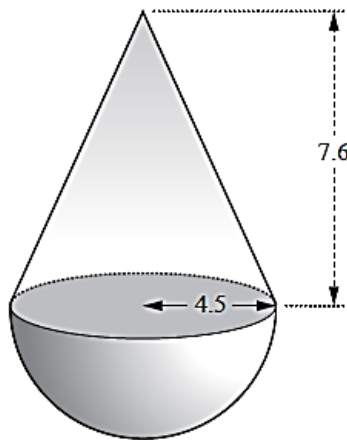
A solid is formed by joining a cone of radius 4.5 cm and height 7.6 cm to a hemisphere of radius 4.5 cm as shown.

(a)   Calculate the area of the circle where they are joined.

Answer ....cm² [2]

(b)   Calculate the total volume of the solid.   *Answer* ....cm³ [2]

(c)   Another solid of the same type is made by joining a cone of radius 5 cm and height *h* cm to a hemisphere of radius 5 cm.
The cone and hemisphere have equal volumes.

Calculate the height of the cone.   *Answer* ....cm [2]

Answers: (a) 63.6 to 63.62 (b) 352 to 353 (c) 10   **J15/21/Q4**

24   The diagram shows a sector *AOB* of a circle with centre *O* and radius 9.3 cm.
The angle of the sector is 260°.

(a) (i)   Calculate the length of the major arc *AB*.

A   B

9.3

O

260°
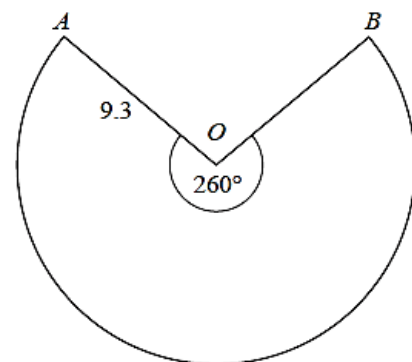
Answer .... cm [2]

(ii)   Calculate the area of the major sector *AOB*.   *Answer* ....cm² [2]

(b) A sector of radius 0.8 cm, centre O, is removed from the sector AOB as shown in Diagram I.
The shaded shape is used to make part of a conical funnel.
AD is joined to BC as shown in Diagram II.

Diagram I

Diagram II
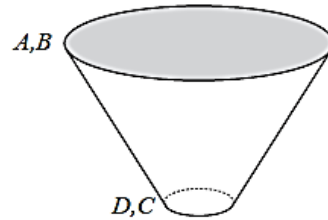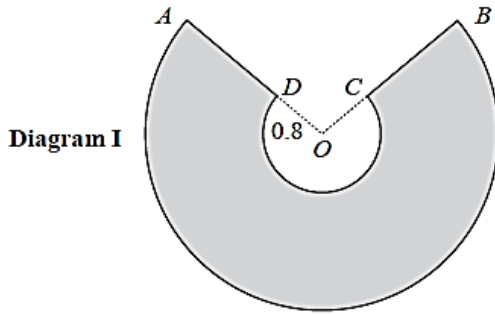
The circumference of the top of the conical funnel is the major arc AB, and the circumference of the bottom of the conical funnel is the major arc CD.

(i) Calculate the external surface area of this part of the funnel.

Answer .... cm² [2]

(ii) The funnel is completed by attaching an open cylinder of height 5 cm to the bottom of the conical part.

(a) Show that the radius of the cylinder is 0.578 cm, correct to 3 significant figures.
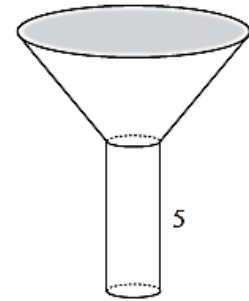
[2]

(b) Calculate the external curved surface area of this cylinder.

Answer .... cm² [2]

(c) Calculate the volume of this cylinder.   Answer .... cm³ [2]

J15/21/Q8

25

[The value of π is 3.142 correct to three decimal places.]
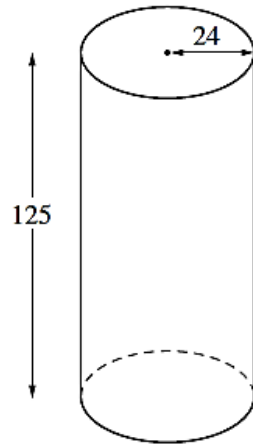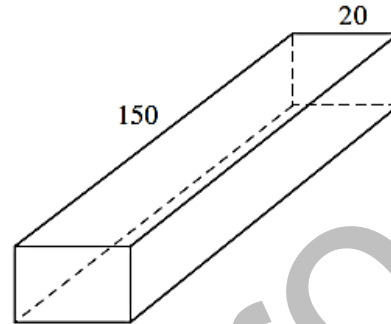
Diagram I   Diagram II

A water tank, shown in Diagram I, is a circular cylinder of radius 24 cm and height 125 cm.
It is open at one end and full of water.

(a) Calculate

    (i)   the volume, in litres, of water in the tank,   [3]

    (ii)   the **total** area, in square metres, of the outside of the open tank.   [3]

(b) Diagram II shows a rectangular trough of length 150 cm and width 20 cm.
The trough was completely filled with 48 000 cm^3 of water from the tank.

    Calculate the depth of the trough.   [2]

(c) After the trough had been filled, water started to leak from the tank.
In 2 hours 30 minutes it was found that 20 000 cm^3 ran out of the tank.

    Calculate the rate at which the level of water in the tank was falling.
Express your answer in centimetres per hour.   [3]

Answers: **(a)(i)** 226 litres, **(ii)** 2.07m^2; **(b)(i)** 16cm, **(ii)** 4.42cm/h.   **N01/2/Q5**

26 [The value of π is 3.142 correct to three decimal places.]
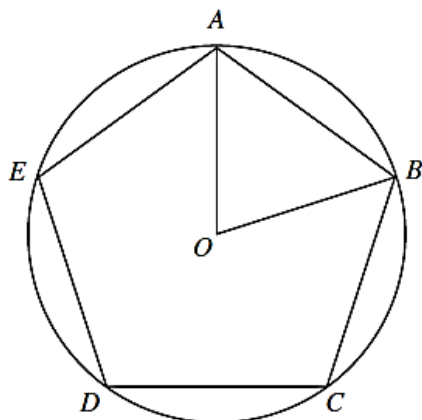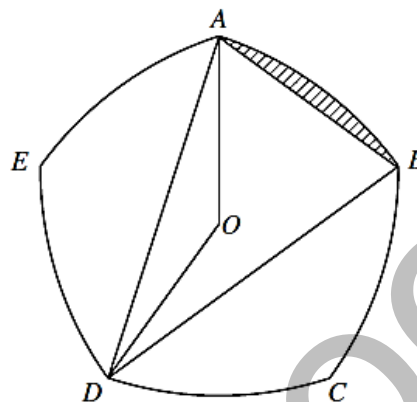
Diagram I

Diagram II

Diagram I shows a regular pentagon, *ABCDE*, inscribed in a circle of radius 1.5 cm and centre *O*.

(a) Explain why $A\hat{O}B = 72°$.   [1]

(b) Calculate the area of the pentagon *ABCDE*.   [3]

(c) Diagram II shows a design for a new coin.
    The vertices of the regular pentagon *ABCDE* are joined by circular arcs whose centres are the opposite vertices.
    For example, the arc *AB* has centre *D* and radius *DA*.

   (i) Explain why $A\hat{D}B = 36°$.   [1]

  (ii) Show that the length of *DA* is approximately 2.85 cm.   [2]

 (iii) Calculate the area of triangle *DAB*.   [2]

 (iv) Calculate the area of the segment shaded in Diagram II.   [2]

  (v) Calculate the area of the face *ABCDE* of the coin.   [1]

Answers: **(b)** 5.35cm²; **(c)(iii)** 2.39cm², **(iv)** 0.165cm², **(v)** 6.17cm².   **N01/2/Q10**

380

27

[The value of π is 3.142 correct to three decimal places.]
[The area of the curved surface of a cone of radius r and slant height l is $\pi r l$.
The volume of a cone is $\frac{1}{3} \times$ base area $\times$ height.]
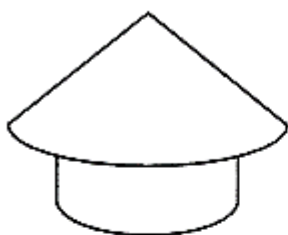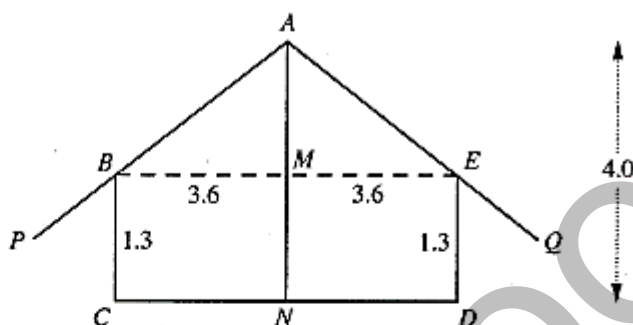
Diagram I

Diagram II

Diagram I shows a traditional hut which consists of a circular cylinder with an overhanging roof.
The roof is the curved surface of a cone and is supported by a central vertical pole.

Diagram II shows a vertical cross-section of the hut.
BE and CD are horizontal.

$AN = 4.0$ m, $BM = ME = 3.6$ m and $BC = DE = 1.3$ m.

(a) Show that $AB = 4.5$ m. [1]

(b) Calculate

    (i) the volume of the inside of the hut, [3]

    (ii) the **total** surface area of the inside of the hut (including the floor). [4]

(c) The sun is directly overhead.

    The shadow of the overhanging section of the roof on the ground is a circular ring around the hut.

    $AP = AQ = 5.5$ m.

    Calculate

    (i) PQ, [2]

    (ii) the area of the circular ring of shadow **outside** the hut. (Ignore the thickness of the walls.) [2]

Answers: **(b)(i)** 89.6 m³, **(ii)** 121 m²; **(c)(i)** 8.80 m, **(ii)** 20.1 m².

N02/2/Q9

381

28
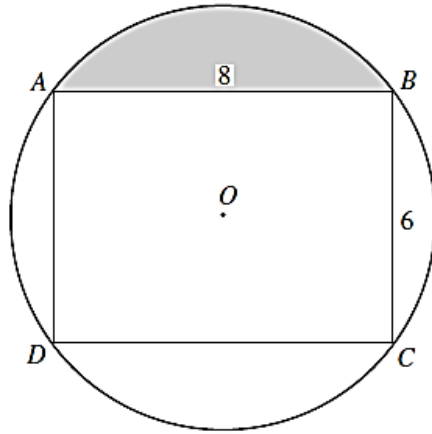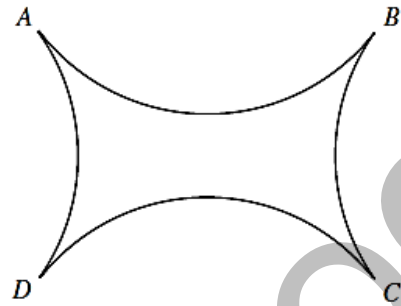
Diagram I   **Diagram II**

ABCD is a rectangle in which *AB* = 8 cm and *BC* = 6 cm.

A circular piece of wire, centre *O*, passes through the vertices of the rectangle as shown in Diagram I.

(a) Show that the radius of the circular wire is 5 cm.   [1]

(b) Show that angle *AOB* = 106.3°, correct to 1 decimal place.   [2]

(c) Calculate the area of the shaded segment.   [3]

(d) The circular wire is cut at *A*, *B*, *C* and *D*, and the four pieces joined to form the shape shown in Diagram II.

Calculate the area enclosed by the wires in Diagram II.   [3]

Answers: **(c)** 11.2 cm²; **(d)** 17.5 cm².

29   [The volume of a pyramid $= \frac{1}{3} \times$ base area × height.]
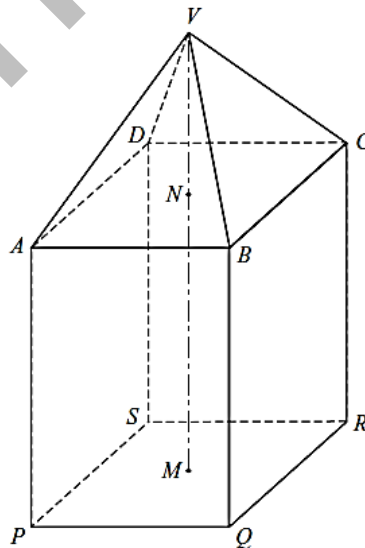
The diagram shows a solid traffic bollard.
It consists of a square-based pyramid, *VABCD*, attached to a cuboid, *ABCDPQRS*.
The vertical line, *VNM*, passes through the centres, *N* and *M*, of the horizontal squares *ABCD* and *PQRS*.

AB = *BC* = 60 cm and *VN* = 40 cm.

(a) Calculate

    (i)   *VA*,   [2]

    (ii) angle *VAN*,   [2]

    (iii) angle *VAP*.   [1]

(b) Given also that *AP* = *BQ* = *CR* = *DS* = 80 cm, calculate

    (i)   the volume of the bollard,   [2]

    (ii) the total surface area of the sides and top of the bollard.   [3]

(c) The highway authority needs to paint the sides and tops of 17 of these bollards.
    The paint is supplied in tins, each of which contains enough paint to cover 8 m^2.

    Find the number of tins of paint needed.   [2]

Answers: (a)(i) 58.3 cm, (ii) 43.3°, (iii) 133.3°; (b)(i) 336 000 cm^3, (ii) 25 200 cm^2; (c) 6.   **N03/2/Q7**

30
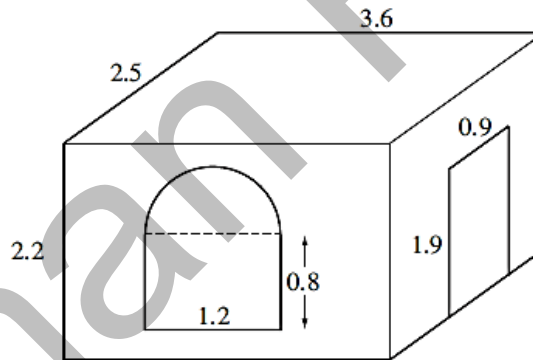
A room has length 3.6 m, width 2.5 m and height 2.2 m.
It has one door which is a rectangle of width 0.9 m and height 1.9 m.
It has one window which is a rectangle of width 1.2 m and height 0.8 m, with a semicircle on one of its longer sides.

(a) (i) Calculate the area of the window.   [2]

    (ii) Show that the area of the walls, correct to three significant figures, is 23.6 m^2.   [2]

(b) Tiles are to be fixed to the walls inside the room.
    Eileen estimated the number of tiles needed to cover the walls inside the room in the following way.
    She first increased the area, 23.6 m^2, by 12% and then calculated the number of tiles that she needed to cover this total area.
    Each tile is a square of side 25 cm.

    (i)   Find the number of tiles that she needed.   [3]

383

(ii) The tiles are sold in boxes, each containing 20 tiles.
Each box of tiles costs $15.

Calculate the cost of the boxes of tiles that she bought.   [2]

(iii) When the shopkeeper sold the tiles at $15 per box, he made a profit of 20%.

Calculate the profit that he made on each box.   [3]

Answers: **(a)(i)** 1.53 m²; **(b)(i)** 423, **(ii)** $330, **(iii)** $ 2.50.

---

31   [The surface area of a sphere = $4\pi r^2$.]

[The volume of a sphere = $\frac{4}{3}\pi r^3$.]

[The area of the curved surface of a cone of radius r and slant height l is $\pi r l$.]

[The volume of a cone = $\frac{1}{3} \times$ base area $\times$ height.]
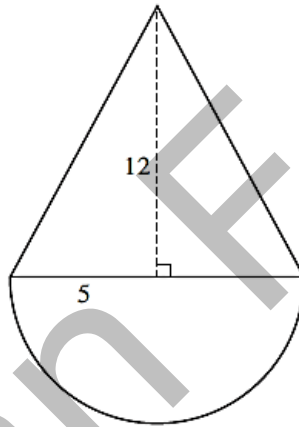
A solid cone has a base radius of 5 cm and height 12 cm.
A solid hemisphere has a radius of 5 cm.
A metal toy is formed by joining the plane faces of the cone and the hemisphere.

(a) Show that the length of the slant edge of the cone is 13 cm.   [1]

(b) Calculate

   (i)   the surface area of the toy,   [4]

   (ii) the volume of the toy.   [3]

(c) A solid metal cylinder has a radius of 1.5 m and height 2 m.
The cylinder was melted down and all of the metal was used to make a large number of these toys.

Calculate the number of toys that were made.   [4]

Answers: **(b)(i)** 361 cm², **(ii)** 576 cm³; **(c)** 24 500.

32

[Surface area of a sphere = $4\pi r^2$]

[Volume of a sphere = $\frac{4}{3}\pi r^3$]

A hot water tank is made by joining a hemisphere of radius 30 cm to an open cylinder of radius 30 cm and height 70 cm.
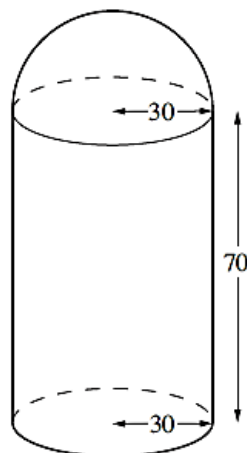
(a) Calculate the total surface area, including the base, of the outside of the tank. [4]

(b) The tank is full of water.

    (i) Calculate the number of litres of water in the tank. [3]

    (ii) The water drains from the tank at a rate of 3 litres per second.

        Calculate the time, in minutes and seconds, to empty the tank. [2]

    (iii)
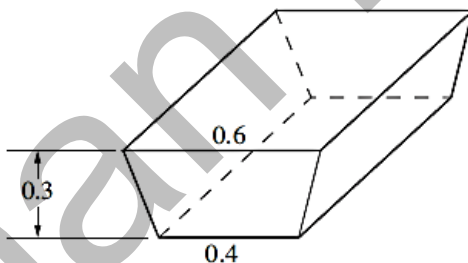
All of the water from the tank runs into a bath, which it just completely fills.

The bath is a prism whose cross-section is a trapezium.

The lengths of the parallel sides of the trapezium are 0.4 m and 0.6 m.

The depth of the bath is 0.3 m.

Calculate the length of the bath. [3]

Answer: (a) 21 700 cm² (b)(i) 254 litres (ii) 1 minute 25 s (iii) 1.70 m   **N06/2/Q7**

33

(a) **Diagram I** shows a design which consists of
7 congruent circles drawn inside a large circle.
The circles touch at all the points shown.
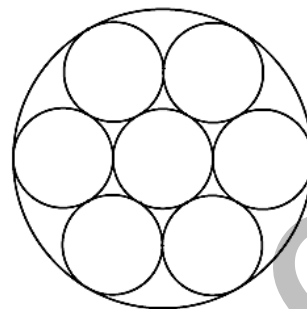
Diagram I

(i) State the order of rotational symmetry of this design. [1]

(ii) In **Diagram II**, two sections of the design have been shaded.
Each small circle has a radius of 5 cm.
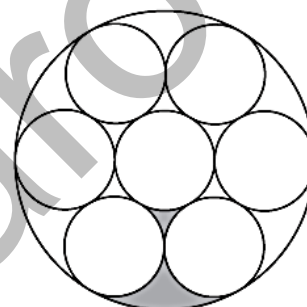
Diagram II

Calculate

  (a) the area of the large circle, [2]

  (b) the shaded area. [2]

(b) In **Diagram III**, circles, centres O, A and B,
are shown, together with the same shaded area.
O is also the centre of the large circle.
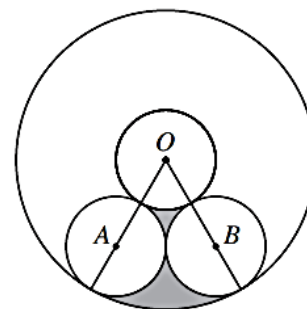
Diagram III

(i) Write down angle AOB. [1]

(ii) Calculate the **total** perimeter of the shaded area. [4]

Answers: (a)(i) 6, (ii)(a) 707 cm², (b) 26.2 cm², (b)(i) 60°, (ii) 54.2 cm.

34

(a) Compost for growing plants consists of 3 parts of soil to 2 parts of sand to 1 part of peat.

    (i) Calculate the number of litres of sand in a 75 litre bag of compost. [2]

    (ii) Compost is sold in 5 litre, 25 litre and 75 litre bags costing \$2, \$8.75 and \$27 respectively.

        Showing your working clearly, state which bag represents the best value for money. [2]

(b) [The volume of a cone $= \frac{1}{3} \times$ **base area $\times$ height.**]

The diagram shows a plant pot.
The open end of the plant pot is a circle of radius 10 cm.
The closed end is a circle of radius 5 cm.
The height of the plant pot is 12 cm.
The plant pot is part of a right circular cone of height 24 cm.
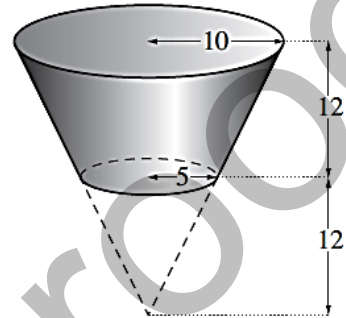
    (i) Calculate the volume of the plant pot.
       Give your answer in litres. [4]

    (ii) How many of these plant pots can be completely filled from a 75 litre bag of compost? [2]

   (iii) A smaller plant pot is geometrically similar to the original plant pot.
       The open end of this plant pot is a circle of radius 5 cm.
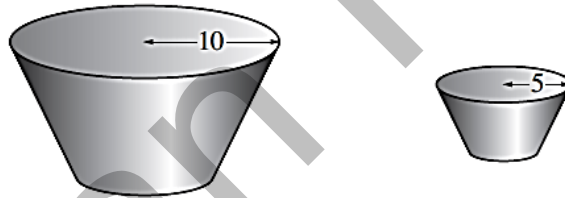
       How many of these plant pots can be completely filled from a 75 litre bag of compost? [2]

Answers: (a)(i) 25, (ii) 25 litre bag, (b)(i) 2.20, (ii) 34, (iii) 272.   **N07/2/Q7**

35

(a) The diagram shows a roll of material.
The material is wound onto a metal
cylinder whose cross-section is a circle
of radius 10 cm.
The shaded area shows the cross-section
of the material on the roll.
The outer layer of material forms the
curved surface of a cylinder of
radius 30 cm.
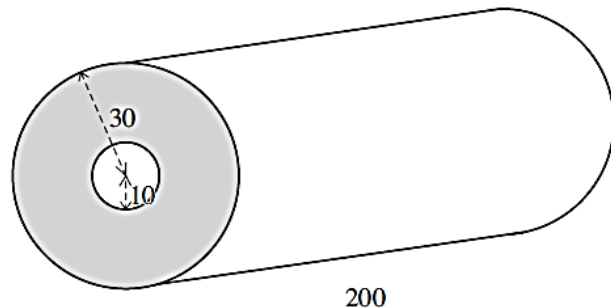
    (i) Calculate, in square centimetres, the area of the cross-section of the material on the roll (shaded on the diagram). [2]

    (ii) The material is 200 cm wide on the roll.

       Calculate, **in cubic metres**, the volume of the material. [2]

387

(iii) When unwound, the length of the material is 150 m.

Calculate the thickness of the material, giving your answer in millimetres. [2]

(b) The diagram shows a conical tent.
The diameter of the base is 3.5 m and the slant height is 3 m.

It is made from a flat piece of canvas that forms a sector of a circle of radius 3 m. The angle at the centre is $\theta°$.
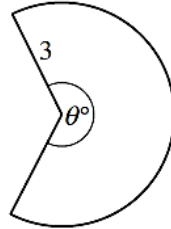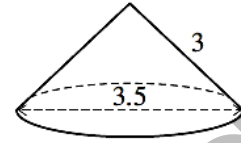
(i) Show that $\theta = 210$. [3]

(ii) As shown, the required shape is cut from a rectangular piece of canvas of width w metres.
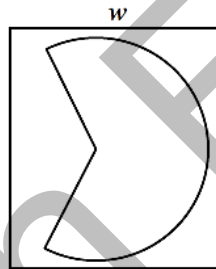
Given that w is a whole number, find its least possible value.
Show all your working. [3]

36 **(a)** A fuel tank is a cylinder of diameter 1.8 m.

   (i) The tank holds 25 000 litres when full.

   Given that $1m^3 = 1000$ litres, calculate the length of the cylinder.
   Give your answer in metres. [4]

   (ii)
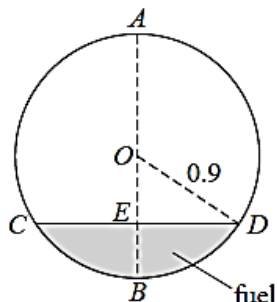
   The diagram shows the cross-section of the cylinder, centre O, containing some fuel.
   CD is horizontal and is the level of the fuel in the cylinder.
   AB is a vertical diameter and intersects CD at E.

   Given that E is the midpoint of OB,

   (a) show that $E\hat{O}D = 60°$, [1]

   (b) calculate the area of the segment BCD, [3]

   (c) calculate the number of litres of fuel in the cylinder. [2]

(b) [Volume of a sphere $= \frac{4}{3}\pi r^3$]

   A different fuel tank consists of a cylinder of diameter 1.5 m
   and a hemisphere of diameter 1.5 m at one end.
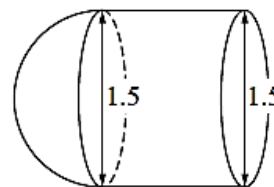
   The volume of the cylinder is 10 times the volume of the hemisphere.

   Calculate the length of the cylinder. [2]

Answers: **(a)(i)** 9.82 m **(ii)(a)** $\cos E\hat{O}D = \dfrac{0.45}{0.9}$ leading to $E\hat{O}D = 60°$ **(b)** 0.497 or 0.498 m²   **N09/2/Q7**

   (c) 4880 or 4890 litres **(b)** 5 m

37
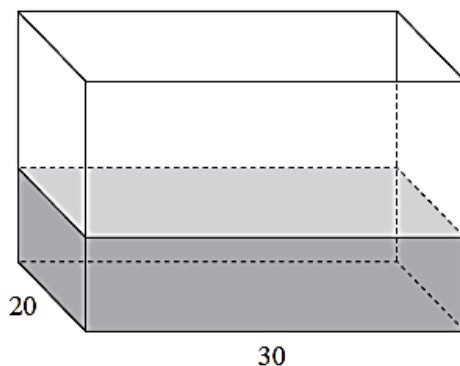
The diagram shows an open rectangular tank with base 20 cm by 30 cm.
The tank contains 9600 cm³ of water.

(a) (i) State the number of litres of water in the tank. [1]

    (ii) Calculate the depth of the water. [2]

    (iii) Calculate the **total** surface area of the tank that is in contact with the water. [2]

    (iv) The water had entered the tank through a circular pipe of radius 0.8 cm.
It flowed through the pipe at 25 centimetres per second.

How long did the 9600 cm³ of water take to enter the tank?
Give your answer correct to the nearest second. [3]

(b) [Volume of a sphere = $\frac{4}{3}\pi r^3$]

250 identical spheres are placed in the bottom of the tank.
Each sphere has a volume of 2.6 cm³.

    (i) Calculate by how much the water level in the tank will rise.
Give your answer in millimetres. [2]

    (ii) Calculate the radius of one of these spheres. [2]

Answers: **(a)(i)** 9.6 **(ii)** 16 cm **(iii)** 2 200 cm² **(iv)** 191 **(b)(i)** 11 **(ii)** 0.853 cm   **N10/21/Q7**

38 **(a)**
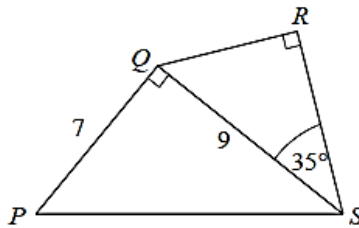
In the quadrilateral $PQRS$, $PQ = 7$ cm and $QS = 9$ cm.
$P\hat{Q}S = Q\hat{R}S = 90°$ and $Q\hat{S}R = 35°$.

Calculate

(i) $S\hat{P}Q$, [2]

(ii) RS. [2]

(b) [The area of the curved surface of a cone of radius r and slant height l is $\pi r l$]
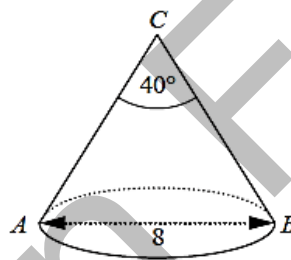
The diagram shows a cone ABC.
The diameter $AB = 8$ cm and $A\hat{C}B = 40°$.

Calculate the curved surface area of this cone. [3]

Answers: **(a)(i)** 52.1° **(ii)** 7.37 cm **(b)** 147 cm^2

39
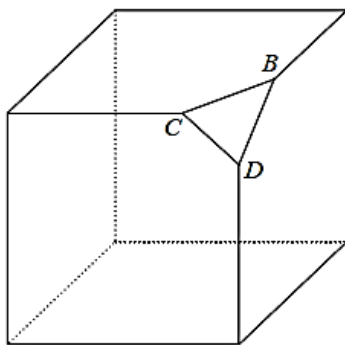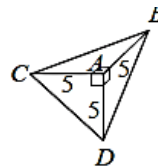
Diagram II

Diagram I

Diagram I shows a cube with a triangular pyramid removed from one vertex.
This triangular pyramid *ABCD* is shown in Diagram II.
$AB = AC = AD = 5$ cm.

(a) State the height of this pyramid when the base is triangle *ABD*. [1]

(b) [The volume of a pyramid = $\frac{1}{3}$ × area of base × height]

Calculate

   (i)   the volume of the pyramid, [2]

   (ii)   the area of triangle *BCD*, [3]

   (iii)   the height of the pyramid when the base is triangle *BCD*. [3]

(c) An identical triangular pyramid is removed from each of the other 7 vertices of the cube to form the new solid shown in Diagram III.
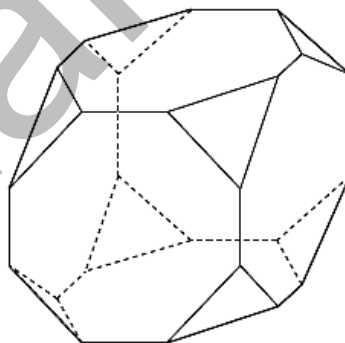
Diagram III

The original cube had 6 faces, 8 vertices and 12 edges.
For the new solid, write down the number of

   (i)   faces, [1]

   (ii)   vertices, [1]

   (iii)   edges. [1]

Answers: (a) 5 cm   (b)(i) 20.8 cm³   (ii) 21.6 cm² (iii) 2.89 cm (c)(i) 14 (ii) 24 (iii) 36 **N10/22/Q9**

40
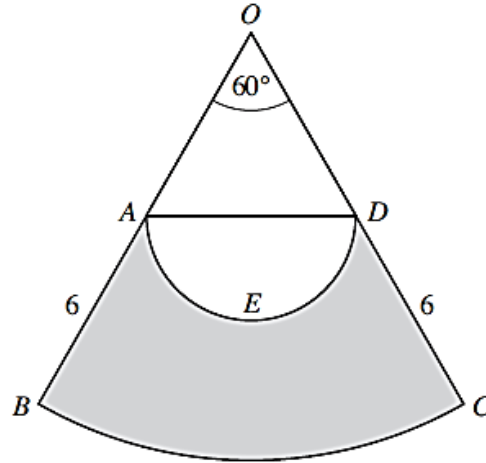
In the diagram, OBC is the sector of a circle, centre O, and $B\hat{O}C = 60°$.
A and D are the midpoints of OB and OC respectively, and $AB = DC = 6$ cm.
AED is a semicircle with AD as diameter.

(a) Show that $AD = 6$ cm.   [1]

(b) The length of the arc BC is $n\pi$ centimetres.   *Answer* .... [1]

    (i) Find n.

    (ii) Find $\dfrac{\text{the length of the arc } AED}{\text{the length of the arc } BC}$.   *Answer* .... [2]

(c) (i) Find the area of the sector BOC.   *Answer* .... cm^2 [2]

    (ii) Hence find the area of the shaded region.   *Answer* .... cm^2 [3]

Answers: (b)(i) 4 (ii) $\dfrac{3}{4}$ (c)(i) 75.4 (ii) 45.7   **N11/21/Q5**

393

41 **(a)**
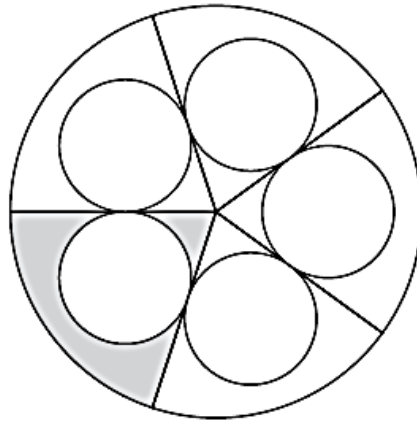
Diagram I

Diagram I shows one large circle and five identical small circles.
Each of the five radii shown is a tangent to two of the small circles.

(i) Describe the symmetry of the diagram.

Answer ....[1]

(ii) The radius of the large circle is R centimetres and the radius of each small circle is r centimetres.
Each small circle is equal in area to the shaded region.

Find $R^2 : r^2$.   *Answer* .... : ....[3]

(b)
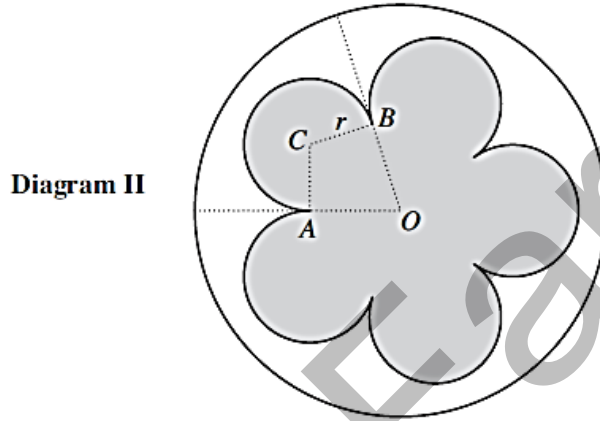
Diagram II

Diagram II shows the same large circle and arcs of the same small circles as in **Diagram I**.
C is the centre of one of the small circles.
This circle touches the adjacent circles at A and B.
O is the centre of the large circle.

(i) Show that reflex $A\hat{C}B = 252°$.   [2]

(ii) The perimeter of the shaded region is $k\pi r$ centimetres.

Calculate the value of k.   *Answer* .... [2]

Answers: (a)(i) One line of symmetry (ii) 10 : 1 (b)(ii) 7   N11/22/Q3

42
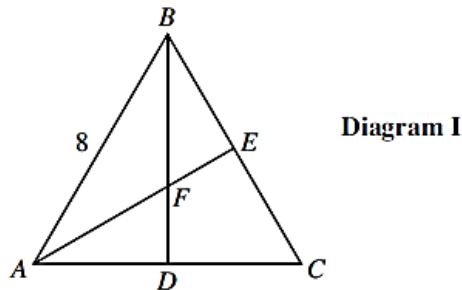
Diagram I

In **Diagram I**, ABC is an equilateral triangle of side 8 cm.
D and E are the midpoints of AC and BC respectively.
BD and AE intersect at F.

(a) (i) Find the area of triangle *ABC*.   *Answer* ....cm² [2]

(ii) Show that $A\hat{F}B = 120°$.

Answer ....

.... [1]

(iii) Calculate *AF*.   *Answer* .... cm [2]

<center>15</center>

(b) [The volume of a pyramid = $\frac{1}{3}$ × base area × height]
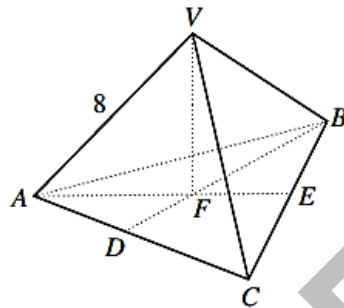
Diagram II

The equilateral triangle of side 8 cm in **Diagram I** forms the base of the triangular pyramid *VABC* in **Diagram II**.
The vertex *V* is vertically above *F*.
VA = VB = VC = 8 cm.

(i) Calculate the surface area of the pyramid.   *Answer* ....cm² [1]

(ii) Calculate the volume of the pyramid.   *Answer* ....cm³ [3]

(c) A pyramid *P* is geometrically similar to *VABC* and its volume is $\frac{1}{64}$ of the volume of *VABC*.

(i) Find the length of an edge of *P*.   *Answer* .... cm [2]

(ii) A pyramid that is identical to *P* is removed from each of the four vertices of *VABC*.

State the number of faces of the new solid.   *Answer* .... [1]

Answers: **(a)(i)** 27.7 **(iii)** 4.62 **(b)(i)** 111 **(ii)** 60.3 **(c)(i)** 2 **(ii)** 8   **N11/22/Q7**

43
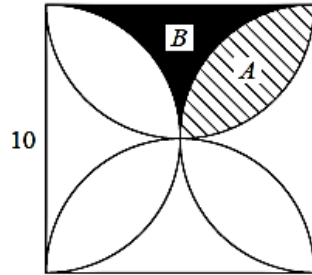
Diagram I

In **Diagram I**, four semicircles are drawn inside a square of side 10 cm.
Each semicircle has a side of the square as its diameter.
Two regions, A and B, are shown.

(a)   Calculate the perimeter of region A.

Answer ....cm [1

(b)   Calculate the perimeter of region B.

Answer ....cm [1]

(c)
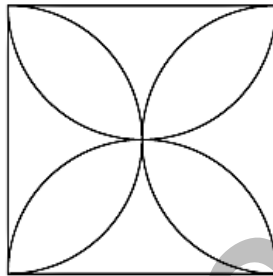
Diagram II

For **Diagram II**,

(i)   draw all the lines of symmetry,   [1]

(ii)   state the order of rotational symmetry.   *Answer* ....[1]

11

(d) **Diagram III** shows the combined regions A and B shaded.
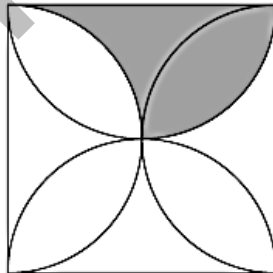
Diagram III

(i)   Calculate the area of this shaded region.   *Answer* ....cm² [1]

(ii)   Hence calculate the area of region A shown in **Diagram I**.

Answer ....cm² [3]

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44 (a) Tuna chunks are sold in cylindrical tins.
The 130 g tin costs \$1.00 and the 185 g tin costs \$1.50.

Which one is the better value for money?
Show all your working.   *Answer* ....[2]

(b) A closed cylindrical tin is 11 cm high and the base has a diameter of 7 cm.

(i) Calculate the volume of this tin.   *Answer* .... cm³ [2]

11

7

15
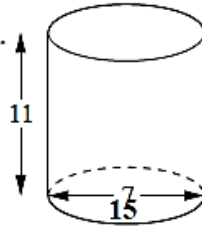
(ii) Calculate the total external surface area of this tin.   *Answer* ....cm² [3]

(iii) In addition to the surface area, a closed tin requires an extra 30 cm² of metal to allow the top, bottom and side to be joined together.

Calculate the area of metal required for 30 000 closed tins.
Give your answer in square metres.   *Answer* .... m² [2]

(c) Two geometrically similar jugs have volumes of 1000 cm³ and 512 cm³.
They have circular bases.
The diameter of the base of the larger jug is 9 cm.

Calculate the diameter of the base of the smaller jug.
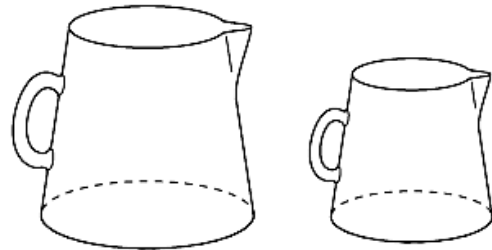
Answer ....cm [3]

45 A fuel tanker delivers fuel in a cylindrical container of length 9.5 m and radius 0.8 m.

(a) After several deliveries, the fuel remaining in the container is shown in the diagram.
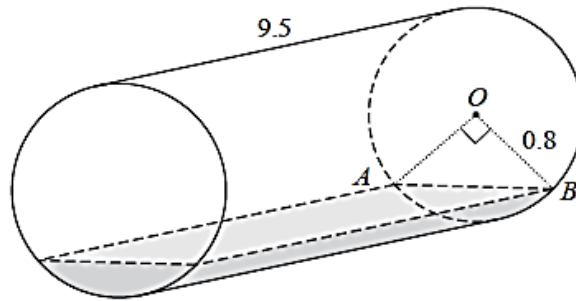
AB is horizontal, O is the centre of the circular cross-section and $A\hat{O}B = 90°$.

(i) Calculate the curved surface area of the container that is in contact with the fuel.

Answer .... m² [2]

(ii) Calculate the volume of fuel remaining in the container.

Answer .... m³ [4]

(iii) Calculate this volume remaining as a percentage of the volume of the whole container.

Answer ....% [2]

19

(b) The fuel is pumped through a cylindrical pipe of radius 4.5 cm at a rate of 300 cm/s.

(i) Calculate the volume pumped in 1 second.

Answer .... cm³ [1]

(ii) Calculate the time taken, in minutes, to pump 25 000 litres of fuel.
Give your answer correct to the nearest minute.

Answer .... minutes [3]

Answer: (a)(i) 11.9 (ii) 1.73 or 1.74 (iii) 9.1% (b)(i) 19 100 (ii) 22 **N13/21/Q10**

46 (b)
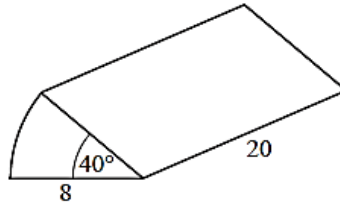
The cross-section of a prism is a sector of a circle, radius 8 cm and angle 40°.
The prism is 20 cm long.

Calculate

(i) the area of the cross-section,

Answer .... cm² [2]

(ii) the **total** surface area of the prism.

Answer .... cm² [4]

Answers: **(a)(i)(a)** $2x$; **(b)** $4x$; **(c)** $90 - 2x$; **(ii)** 19; **(b)(i)** 22.3; **(ii)** 476.

47 (a)
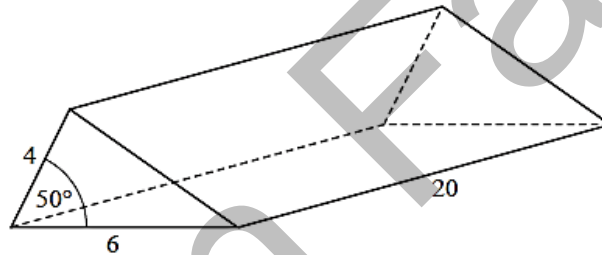
The diagram shows a solid triangular prism.
All lengths are given in centimetres.

(i) Calculate the area of the cross-section of the prism.

Answer .... cm² [2]

(ii) Calculate the volume of the prism.

Answer .... cm³ [1]

(iii) Calculate the **total** surface area of the prism.

Answer .... cm² [5]

19

(b) A cylinder has a height of 70 cm and a volume of 0.1 m³.

Calculate the radius of the cylinder, giving your answer in centimetres.

Answer .... cm [4]

48   [The volume of a sphere is $\frac{4}{3}\pi r^3$]

[The surface area of a sphere is $4\pi r^2$]
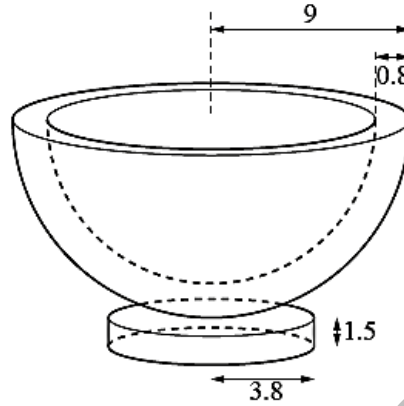
A hemispherical bowl is made of material that is 0.8 cm thick.
The outside rim of the bowl has radius 9 cm.
The bowl is attached to a base which is a solid cylinder, of radius 3.8 cm and height 1.5 cm.

(a) Calculate the surface area of the **inside** of the hemispherical bowl.

Answer .... cm² [2]

9

(b) Calculate the **total** volume of material used to make the bowl and the base.

Answer .... cm³ [5]

49
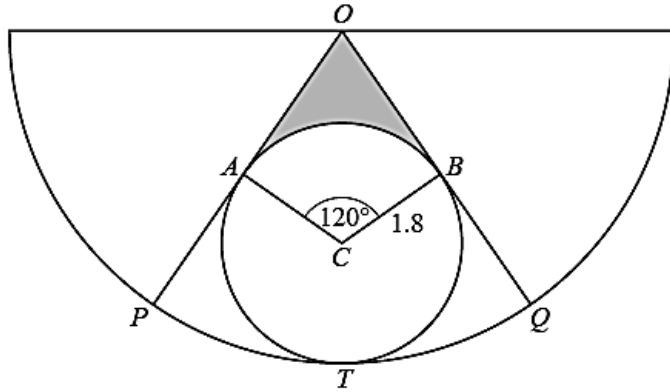
The diagram shows a semicircle with radii OP and OQ drawn.
The circle, centre C, touches the radii at A and B and the semicircle at T.
The radius of the circle is 1.8 cm.
$B\hat{C}A = 120°$.

(a) Calculate the length of the minor arc AB.

Answer .... cm [2]

(b) The shaded region lies between the circle and the radii OP and OQ.

Calculate the perimeter of this shaded region. Answer .... cm [3]

11

(c) (i) Show that the radius of the semicircle is 5.4 cm. [2]

(ii) Calculate the length of BQ.

Answer .... cm [1]

Answers: **(a)** 3.77 **(b)** 10.0 **(c)(i)** 5.4 correctly derived **(ii)** 2.28   **N16/21/Q5**

50
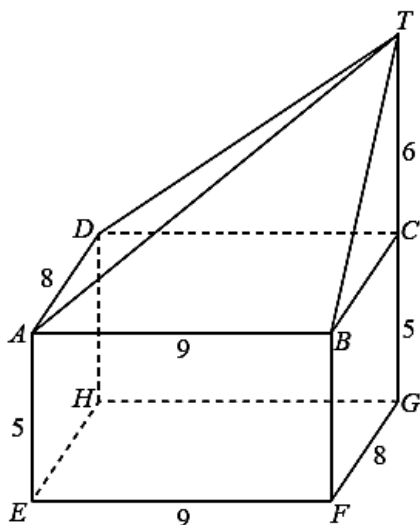
The four walls of a building are faces of a cuboid *ABCDEFGH*.
T is vertically above *C* and *G*, so $A\hat{B}T = A\hat{D}T = 90°$.

The cuboid has length 9 m, width 8 m and height 5 m.
TC = 6 m.

(a) Calculate the length of *DT*.

Answer .... m [2]

(b) The roof is formed by four triangles, *ABT*, *BCT*, *CDT* and *DAT*.

Calculate the **total** surface area of the roof.

Answer .... m² [3]

(c) [The volume of a pyramid is $\frac{1}{3}$ × area of base × perpendicular height]

Calculate the **total** volume of the building.

Answer .... m³ [2]

(d) Calculate the angle of elevation of *T* from *H*.

Answer .... [3]

Answers: **(a)** 10.8 **(b)** 139 **(c)** 504 **(d)** 50.7

N16/21/Q6

51

[Volume of a cone $= \frac{1}{3}\pi r^2 h$]

[Curved surface area of a cone $= \pi r l$]

[Volume of a sphere $= \frac{4}{3}\pi r^3$]

[Surface area of a sphere $= 4\pi r^2$]
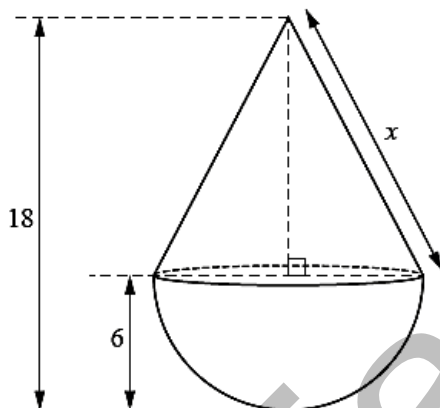
The diagram shows solid A which is made from a hemisphere joined to a cone of equal radius.
The hemisphere and the cone each have radius 6 cm.
The total height of the solid is 18 cm.

(a) Show that the slant height, x cm, of the cone is 13.4 cm, correct to 1 decimal place.　　　　[2]

(b) Calculate the total surface area of solid A.　　　　*Answer* .... cm² [3]

(c) Calculate the volume of solid A.　　　　*Answer* .... cm³ [3]

(d) Solid A is one of a set of three geometrically similar solids, A, B and C.
The ratio of the heights of solid A : solid B : solid C is 2 : 6 : 1.

　　(i) Calculate the surface area of solid B correct to 3 significant figures.

　　　　　　　　　　Answer .... cm² [2]

　　(ii) Calculate the volume of solid C correct to 3 significant figures.

　　　　　　　　　　Answer .... cm³ [2]

Answers: **(a)** 13.4 correctly derived **(b)** 479 **(c)** 905 **(d)(i)** 4310 **(ii)** 113　　　　**N17/21/Q8**

Volume 4

Loci Paper 1

1 In the diagram in the answer space, *TC*, *TD* and *AB* are straight lines.

(a) Construct the locus of the points which are equidistant from *TC* and *TD*.

(b) Construct the locus of the points which are equidistant from *A* and *B*.

(c) The two loci meet at *P*.

AB is a chord of a circle, centre *P*.
Draw the circle.

Answer
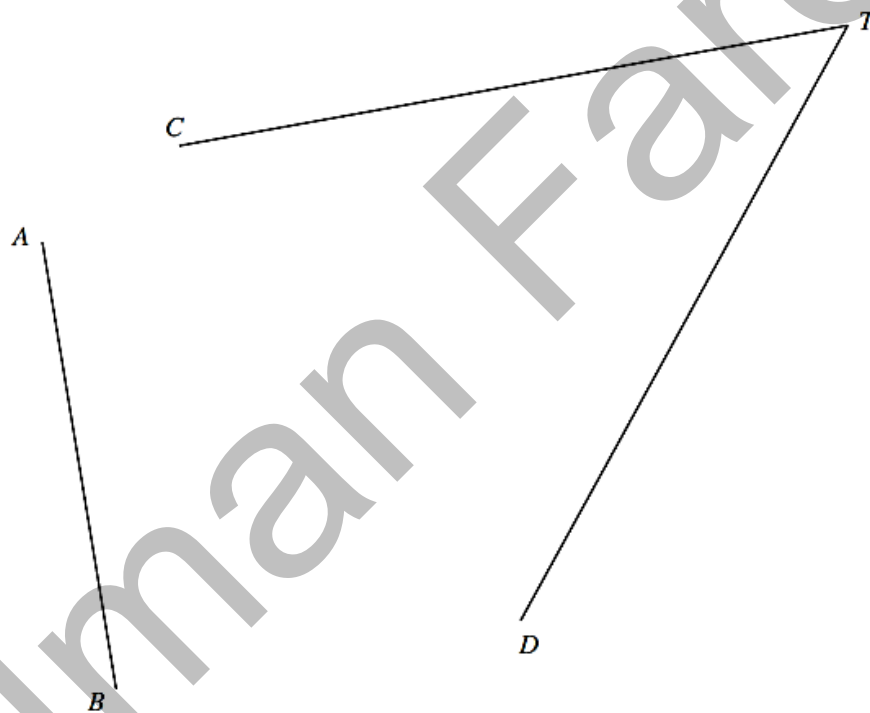
[4]

J02/1/Q15

2   The diagram in the answer space is a map showing a section of coastline and a beacon on land.
Fishing boats can only operate when they are
   I   not more than 6.5 km from the beacon,
   II   at least 2 km from the coastline.
The scale of the map is 1 cm to 1 km.

Construct the boundaries of the region where fishing can take place.
Label this region *F*.

Answer
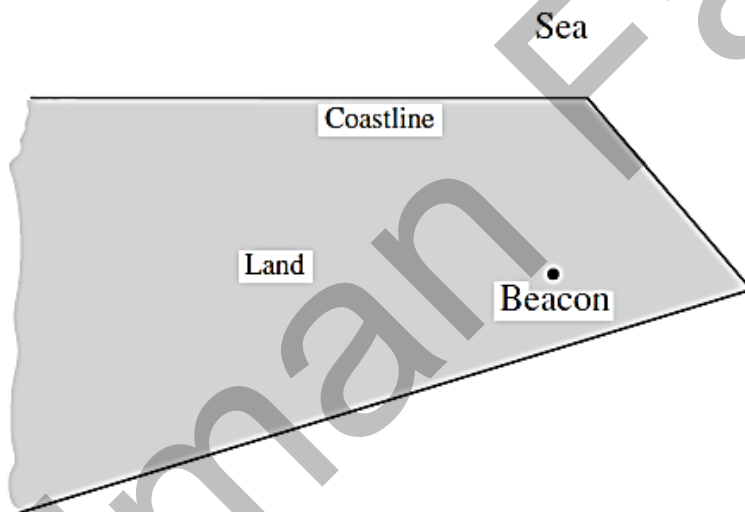
3 In triangle *PLQ*, *PL* = 14 cm, *PQ* = 10 cm and *LQ* = 7 cm.
The line *PL* is drawn in the answer space.

(a) Using ruler and compasses only, complete triangle *PLQ* where *Q* is above *PL*.

(b) Measure and write down *PQ̂L*.

(c) Draw a semicircle with *PL* as diameter.
The line *LQ* produced meets the semicircle at *M*.
Measure and write down the length of *QM*.

(d) (i) Explain why *PM* is perpendicular to *LM*.

(ii) Hence write down the value of cos *PQ̂L*.

Answer (a)

P‾_____‾L [1]
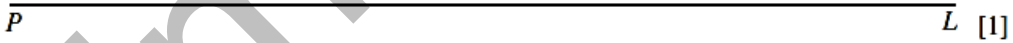
Answer (b) PQ̂L = ... [1]

(c) QM = ...cm [1]

(d)(i) .... [1]

(ii) cos *PQ̂L* = ... [1]

408

Answers: **(b)** 108° to 111°; **(c)** 3.2 to 3.5 cm; **(d)(ii)** –0.32 to –0.35.

J05/1/Q24

4   22   A map is drawn using a scale of 1 cm to 5 m.
        The position of *A* is shown in the answer space below.

(a)   The point *B* is 70 m due East of *A*.
      Draw the line representing *AB*.

(b)   The point *C* is North of *AB* and equidistant from *A* and *B*.

      Angle *BAC* = 40°.

      (i)   By drawing appropriate lines, find and label the point *C*.

      (ii) Find the actual distance *AC*.

      (iii) State the size of the reflex angle *BAC*.

Answer (a) and (b)(i)

N

A

[3]

Answer (*b*)(ii) .... m [1]

(iii) reflex $B\hat{A}C$ = ...[1]

5 The plan of a field has a scale of 1 cm to 5 metres.

(a) Express this scale in the form 1 : n.   *Answer (a)* ....[1]

(b) The plan was made by measuring angles from two points, A and B, 50 m apart.
The line AB is drawn to scale in the answer space below.

    (i) A tree is at the point T in the field.
$B\hat{A}T = 35°$ and $A\hat{B}T = 70°$.

        Locate and label T on the plan.

    (ii) Given that A is due west of B, state the bearing of T from B.

    (iii) By making an appropriate measurement, find the actual distance, in metres, of the tree from B.

Answer (b)(i)

N
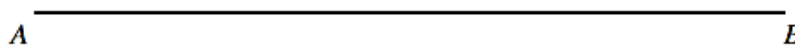
A ————————————————————————————————————— B   [1]

Answer (b)(ii) .... [1]

(iii) .... m [1]

Answer: (a) 1 : 500   (b)(ii) 340°   (iii) 28 to 31 m   **J07/1/Q20**

410

6   A map is drawn to a scale of 1 cm to 3 km.
The diagram below shows the positions of two villages A and B on the map.

(a)   (i)   Write the scale in the form $1 : n$.

(ii)   Find the actual distance, in kilometres, between the villages A and B.

Answer (*a*)(i) 1 : ....[1]

(ii) .... km   [1]

Answer (b), (c), (d)

North

B

A
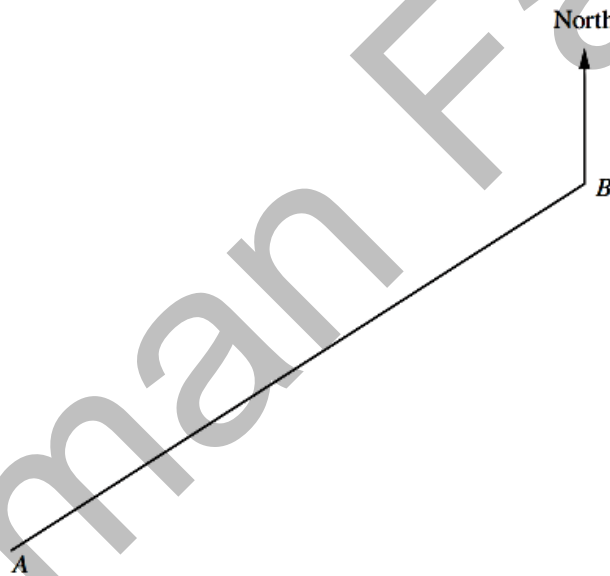
[4]

(b)   A third village, C, lies north of the line AB.
It is 21 km from A and 18 km from B.

Using ruler and compasses only, construct triangle ABC.

(c)   Construct the perpendicular bisector of AB.

(d)   A petrol station is to be built so that it is equidistant from A and B and 9 km from C.

Mark with letters F and G the two possible positions of the petrol station.

411

7   A sailing club has five moorings in the river at *A, B, C, D* and *E*.
     A and *B* are 12 metres apart.
     The positions of *A* and *B* are shown in the scale drawing below.

Answer (b), (c), (d)

North

A

North

B
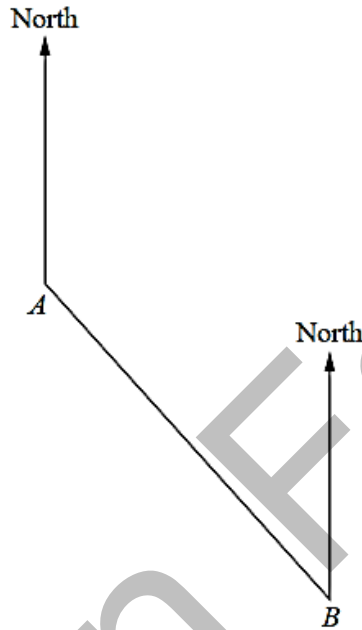
(a)   Write the scale in the form 1 : *n*.

Answer   *(a)* **1** : ....[1]

(b)   *C* is due west of *B* and on a bearing of 210° from *A*.

     Find and label the position of *C*.   [2]

(c)   *D* lies north of the line *AB* .
     The triangle *ABD* is equilateral.

     Using ruler and compasses only, construct triangle *ABD*.
     Show your construction arcs clearly.   [1]

(d)   The bearing of *E* from *A* is the same as the bearing of *B* from *A*.

     Given that *AB* : *AE* = 3 : 5, find and label the position of *E*.   [2]

8

A map is drawn to a scale of 1 cm to 5 km.
The diagram below shows the positions of two radio masts *A* and *B* on the map.

Answer (a), (b)

North

A   *B*

(a) A third radio mast, *C*, is north of the line *AB*.
It is 40 km from *A* and 50 km from *B*.

Using ruler and compasses, construct triangle *ABC*.   [2]

(b) A house *D*, inside the triangle, is more than 35 km from *B* and closer to *B* than to *A*.

Shade the region on your diagram that represents the possible positions
of the house *D*.   [3]

9 **(a)** Construct, using ruler and compasses only, triangle *ABC* with sides
AB = 9 cm, *BC* = 8 cm and *AC* = 6 cm.
The line *AB* has been drawn for you.

A ————————————————————— B

[2]

(b) Construct the locus of points, inside the triangle *ABC*, which are

    (i) equidistant from *AB* and *BC*, [1]

    (ii) 4 cm from *A*. [1]

(c) Shade the region, inside triangle *ABC*, containing the points that are
nearer to *BC* than *AB* and more than 4 cm from *A*. [1]

18 The diagram below shows the quadrilateral *PQRS*.

    (a) On the diagram, construct

        (i) the bisector of *SPQ*,   [1]

        (ii) the perpendicular bisector of *QR*.   [1]

    (b) On the diagram, shade the region inside the quadrilateral containing the points that are closer to *PQ* than to *PS* and nearer to *Q* than to *R*.   [1]
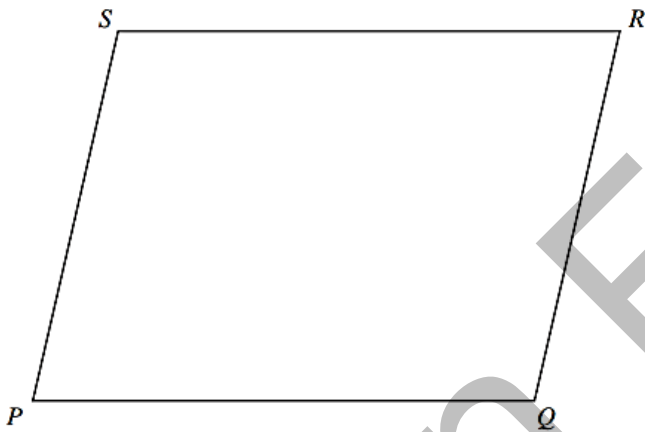
J11/12/Q18

11
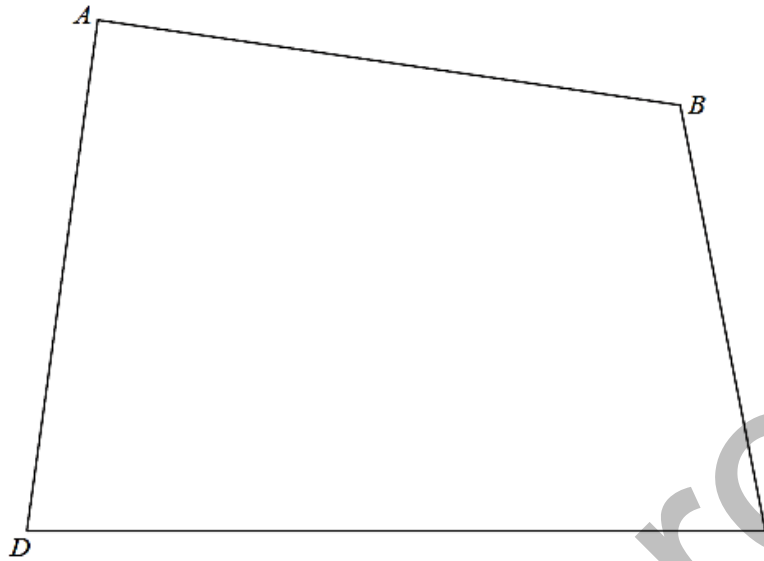
(a) Construct the locus of all points, **inside** the quadrilateral *ABCD*, which are

    (i)   equidistant from *DA* and *DC*,   [1]

    (ii) 5 cm from *B*.   [1]

(b) On the diagram, shade the region **inside** the quadrilateral containing the points that are

nearer to *DA* than *DC* and

more than 5 cm from *B*.   [1]

12 **(a)** Construct, using ruler and compasses only, an equilateral triangle *ABC*.
The side *AB* has been drawn for you.

A •

B •

[1]

(b) Construct the locus of points, inside triangle *ABC*, which are

    (i) equidistant from *A* and *C*,   [1]

    (ii) 4 cm from *A*.   [1]

(c) A point *X* lies within triangle *ABC*, is nearer to *A* than to *C* and is less than 4 cm from *A*.

    On your diagram shade the region in which *X* must lie.   [1]

13 The diagram shows the lines AB and BC.

(a) The point D is 11 cm from A and 9 cm from C.

On the diagram, construct the quadrilateral $ABCD$. [1]

(b) On the diagram, construct the locus of points, **inside** quadrilateral $ABCD$, that are

(i) equidistant from B and C, [1]

(ii) equidistant from AB and BC. [1]

(c) These two loci meet at the point P.

Label the point P on the diagram and measure DP.
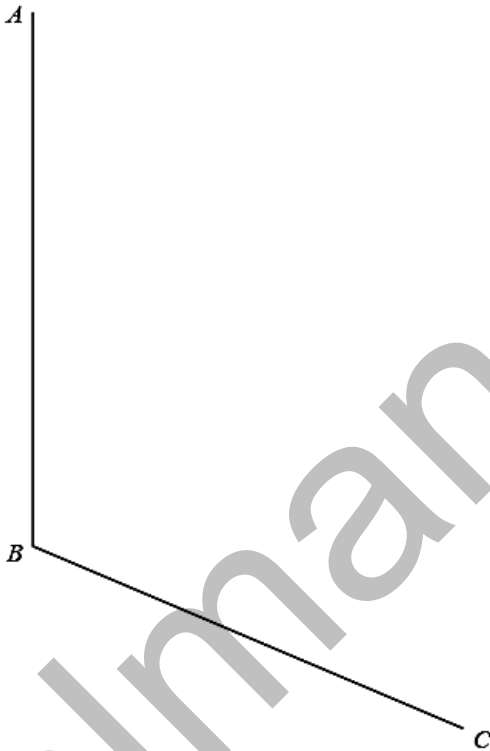
Answer DP = ... cm [1]

Answer: **(c)** 5.4 to 5.9 inclusive

J15/11/Q22

14 The diagram shows triangle *ABC*.
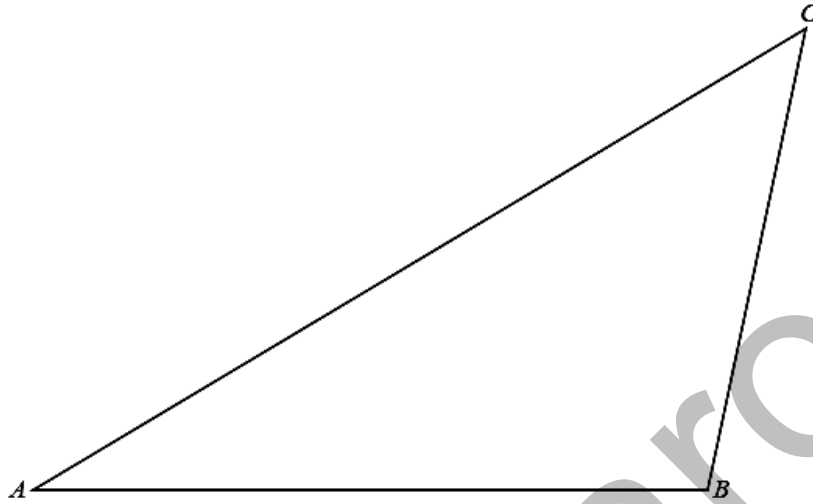
(a) Measure *AB̂C*.

Answer *AB̂C* = ... [1]

(b) On the diagram, construct the locus of points, **inside** triangle *ABC*, that are

   (i) 4 cm from *B*, [1]

   (ii) 2 cm from *AC*. [1]

(c) The point *P* is
      4 cm from *B*,
      2 cm from *AC*,
      and nearer to *A* than to *C*.

   Label the position of *P* on the diagram and find the length of *AP*.

Answer *AP* = ... cm [1]

Answers: **(a)** 101° to 103° **(b)(i)** Circular arc, centre *B*, radius 4 cm **(b)(ii)** Line parallel to *AC*, 2 cm away
      (b)(iii) 6.2 to 6.6

15 The diagram below is the scale drawing of a field.
In the drawing, 1 cm represents 10 m.

(a) Express the scale in the form 1 : *n*.

(b) Treasure is buried in the field.
It is known that the treasure is

    I   within 60 m of C,

    II   nearer to A than C,

    III   equidistant from AB and AC.

Use I, II and III to construct the locus of possible positions of the treasure.
Find the extreme positions of this locus and label them T_1 and T_2.

Answer (b)

A

C

B
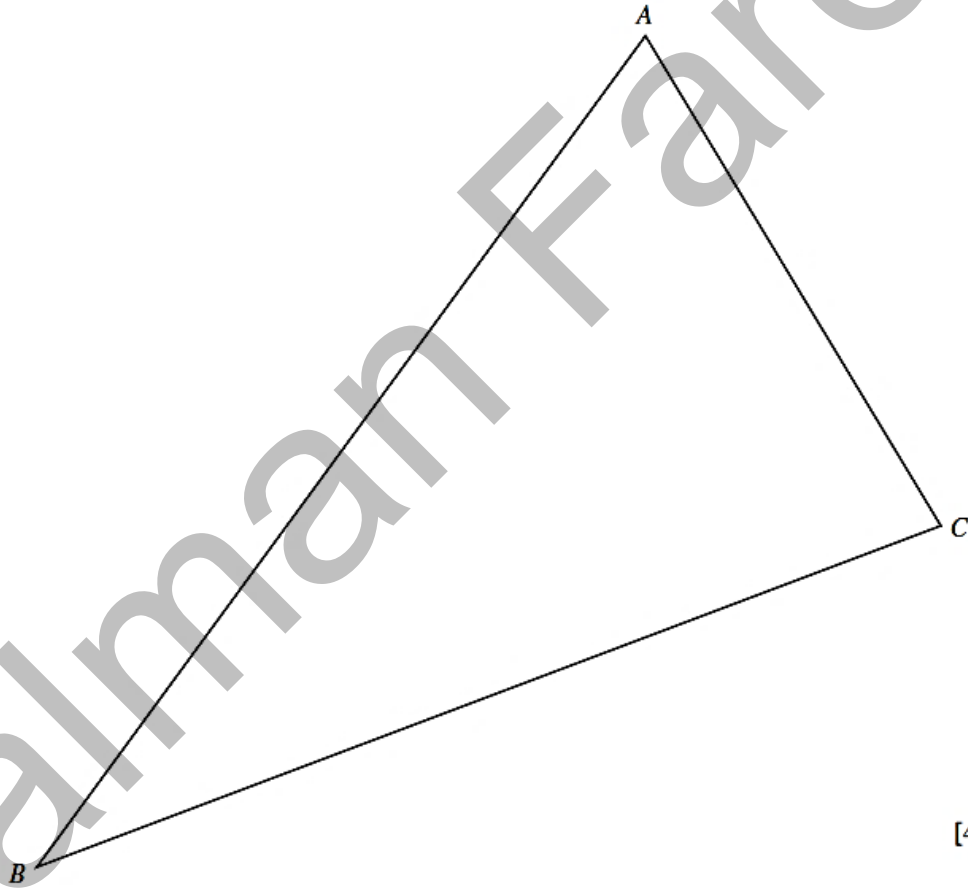
[4]

Answers: (a) 1000.

N01/Q23

16

(a) Measure angle *ABC*.

Answer (a) A$\hat{B}$C = ... [1]

(b) The point *D* is below *AC* where *AD* is 12 cm and *CD* is 9 cm.
Using ruler and compasses only, complete the construction of triangle *ADC* on the diagram in the answer space.

(c) The region, *S*, lies within the quadrilateral *ABCD*.
Points in *S* are

I   nearer to *C* than *A*,

II   more than 8 cm from *B*,

III nearer to *BA* than *BC*.

Use conditions **I**, **II** and **III** to construct appropriate loci.
Hence shade the region *S*.

Answer (b) and (*c*)

[5]
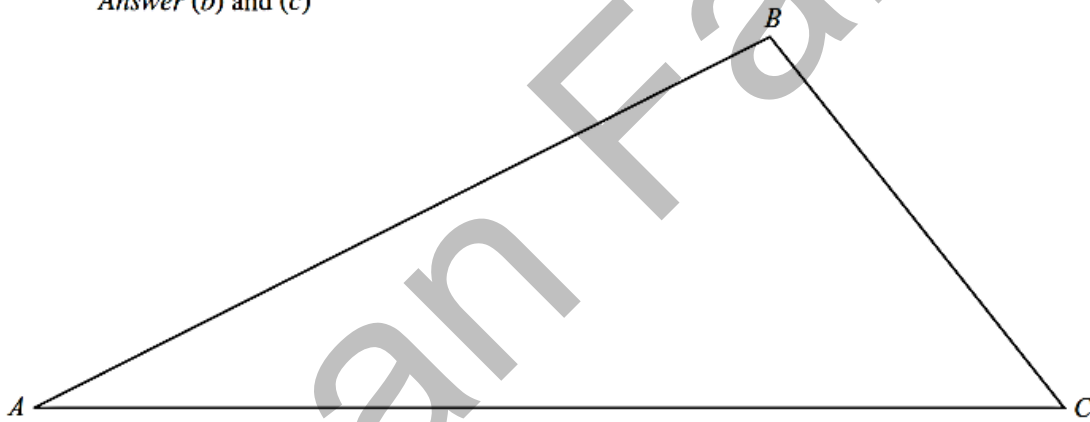
Answer: 101° - 103°.

17

24 In triangle ABC, $AB = 15$ cm, $BC = 8$ cm and $AC = 11$ cm.
The side AB is drawn in the answer space.

(a) Using ruler and compasses only, complete the triangle.

(b) Measure the largest angle of the triangle.

(c) Draw the locus of **all points within the triangle** that are

  (i) 5 cm from C,

  (ii) equidistant from BA and BC.

(d) The point P, within the triangle, is such that $PC = 5$ cm and P is equidistant from BA and BC.

  Label the point P and measure the distance PA.

Answer (a) and (c)

A —— B

[3]

Answers: **(b)** 103°; **(d)** 11.9 cm.   N04/1/Q24

422

18 The diagram below is a scale drawing representing three coastguard stations, *A*, *B* and *C*.
In the drawing, 1 cm represents 20 km.

(a) (i) Express the scale in the form 1 : *n*.

(ii) Find the distance between the coastguard stations *A* and *B*.

Answer (a)(i) 1 : .... [1]

(ii) ....km [1]

(b) It is known that a ship is

I equidistant from *CA* and *CB*,
II nearer to *C* than *A*,
III less than 200 km from *B*.

By constructing 3 loci corresponding to **I**, **II** and **III**, find the possible positions of the ship and label the extreme positions *S* and *P*.

Answer (b)

[4]

423
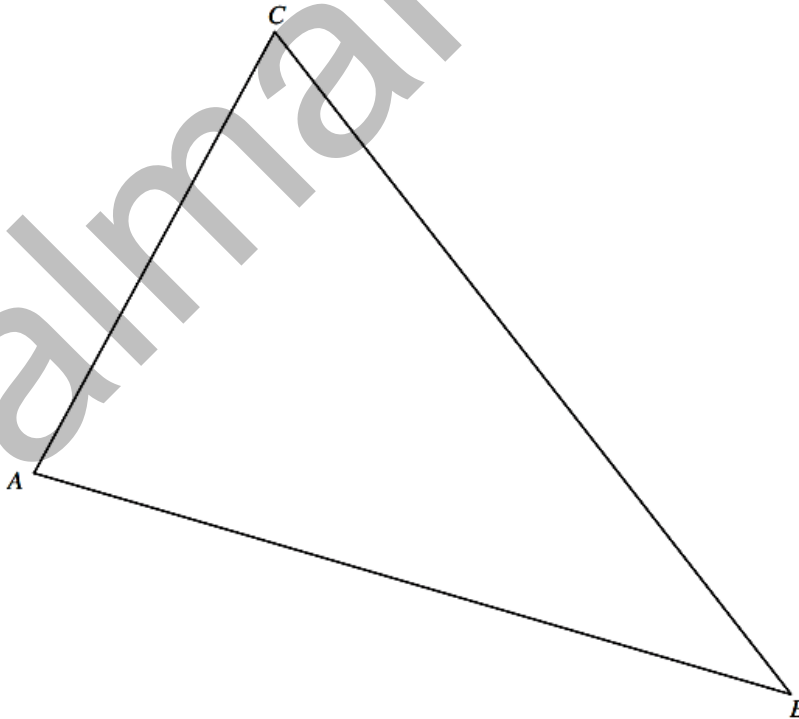
19

23 The diagram below is a map showing a coastline *ABC*, a lighthouse *L* and a point *P*.
The map is drawn to a scale of 1 cm to 100 m.
Ships must not sail within 200 m of the coastline nor within 200 m of the lighthouse.

(a) Construct the locus of points 200 m from the lighthouse *L*.

(b) Construct the locus of points 200 m from the coastline *ABC*.

Answer (a) (b)

C

Coastline

LAND

B

SEA

Coastline

A

North

SEA

• *L*

P
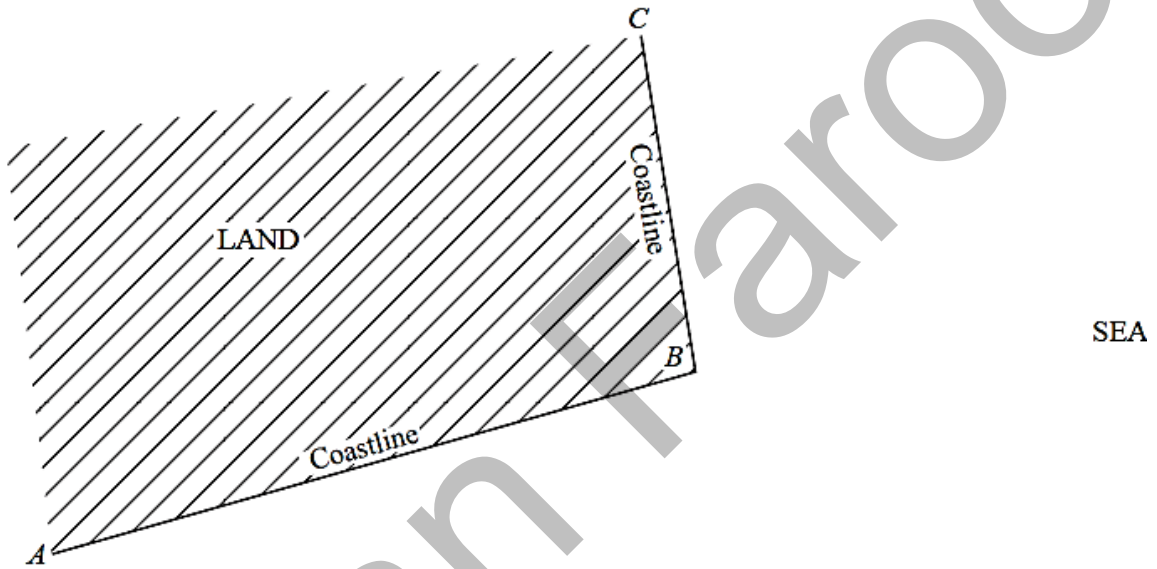
SEA

[3]

(c) A ship sailed from the point *P* on a bearing of θ°.
It passed between *B* and *L*.
Complete the statement in the answer space.

Answer (c) .... < θ < ....[2]

424

20

(a) By measuring an angle, find reflex angle *ACB*.

Answer (a) ....[1]

(b) The point *D* is on the opposite side of *AC* to *B*.
    AD = 6 cm and *CD* = 9 cm.
    Construct triangle *ACD*.

(c) On the diagram, construct the locus of points, **inside** the quadrilateral *ABCD*, which are

    I   equidistant from *A* and *C*,

    II   5 cm from the line *AB*.

(d) The point *P* is   inside quadrilateral *ABCD*,
                  equidistant from *A* and *C*,
                  5 cm from the line *AB*.

    (i)   Mark and label the position of *P*.

    (ii) Measure *CP*.

Answer (b)(c)(d)(i)
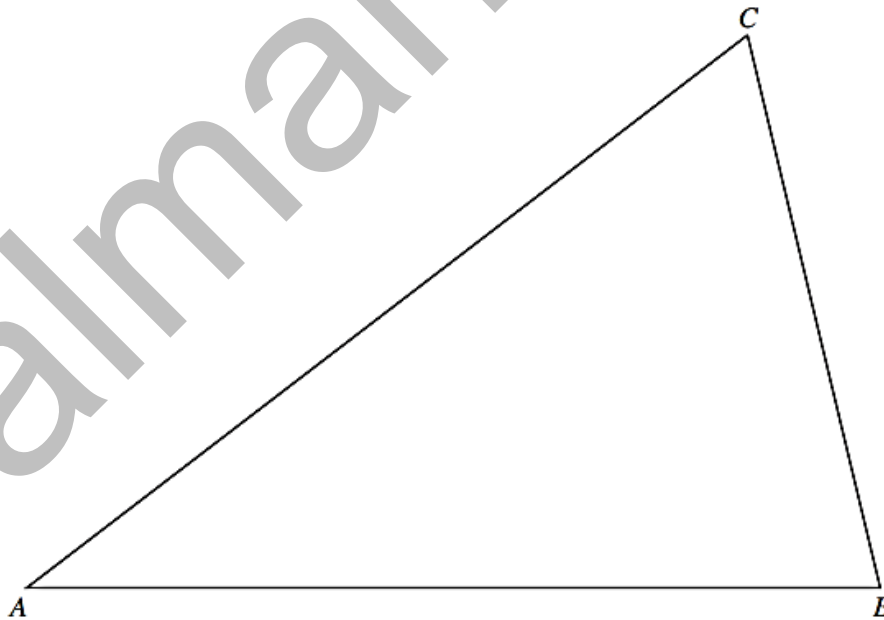
Answer: (a) 293° to 295° (b) completed ΔACD

(c)(i) perp. bisector of *AC* (ii) line parallel to *AB*, 5 cm above *AB* (d) 6.3 to 6.7 cm

21 A family wants to move to a new house.
The area where they are going to look depends on the positions of the children's school, *S*,
the father's place of work, *F*, and the market, *M*.
The diagram in the answer space is drawn to a scale of 1 cm to 1 km.
It shows the positions of *S*, *F* and *M*.

The house needs to be:
I   within 4 km of the children's school,
II nearer to the market than to the father's place of work.

(a) Use I and II to construct the appropriate loci.

(b) Shade the region of your diagram that represents the possible positions of the house.

(c) Find the greatest possible distance between the house and the market.

Answer (a) (b)

Scale 1 cm to 1 km

•*F*

M •

•*S*

[3]

22 The scale drawing below shows the positions of two towns, *F* and *G*.
It is drawn to a scale of 1 cm to 3 km.

Answer (b)
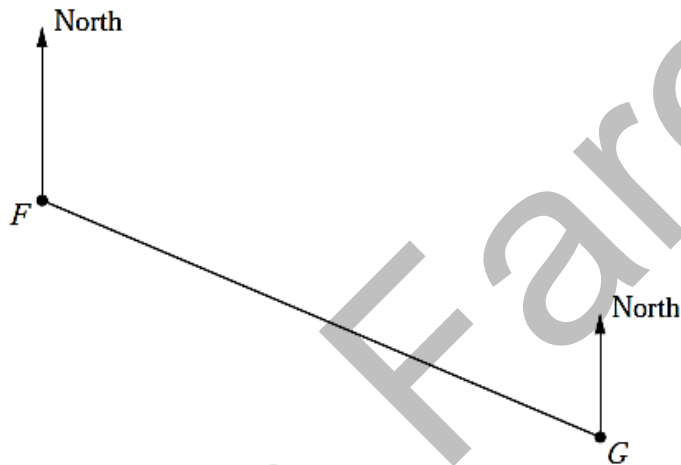
(a) Find

    (i) the distance, in kilometres, between towns *F* and *G*,

                                                 Answer (a)(i) .... km [1]

    (ii) the bearing of *G* from *F*.   *Answer (a)(ii)* ....[1]

(b) Town *H* is to the North of the line *FG*.
It is 19.5 km from *F* and 15 km from *G*.
On the diagram above, find and label the position of *H*.   [1]

Answers: **(a)(i)** 24.9 to 26.1 km inclusive   **(ii)** 111° to 115° inclusive   **N09/1/Q16**
       (b) *H* marked 6.5 cm from *F* and 5 cm from *G*

427

23

The diagram below shows triangle *ABC*.

(a) Measure *AB̂C*.   *Answer* (a) *AB̂C* = ...[1]

(b) On the diagram, construct the locus of points inside the triangle that are

   (i) 8 cm from *C*,   [1]

   (ii) equidistant from *AB* and *AC*.   [1]

(c) On the diagram, shade the region inside the triangle containing the points that are more than 8 cm from *C* and nearer to *AB* than to *AC*.   [1]

Answer (b),(c)

B

A ∠_____ C
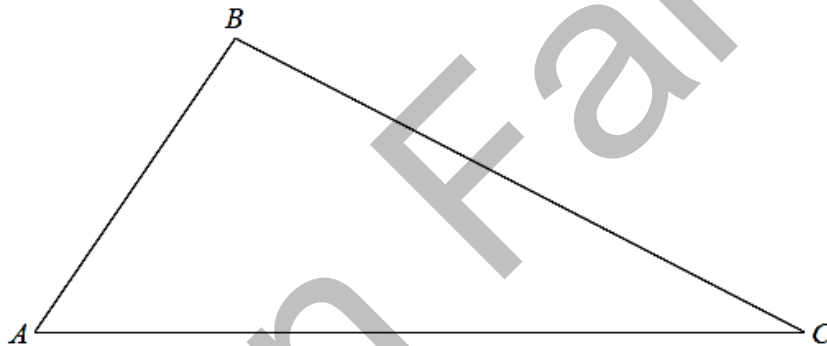
N10/11/Q27

24   The diagram below shows quadrilateral *ABCD*.

(a) Measure $A\hat{B}C$.   *Answer* $A\hat{B}C$ = ... [1]

(b) On the diagram, construct the locus of points, **inside** the quadrilateral, that are

   I   4 cm from *AD*,

   II   equidistant from *A* and *D*.   [2]

(c) On the diagram, shade the region **inside** the quadrilateral, containing the points that are more than 4 cm from *AD* and
nearer to *D* than to *A*.   [1]

(d) The point *P* is 4 cm from *AD* and as near as possible to *C*.
Mark, and label, the position of *P* on the diagram.   [1]
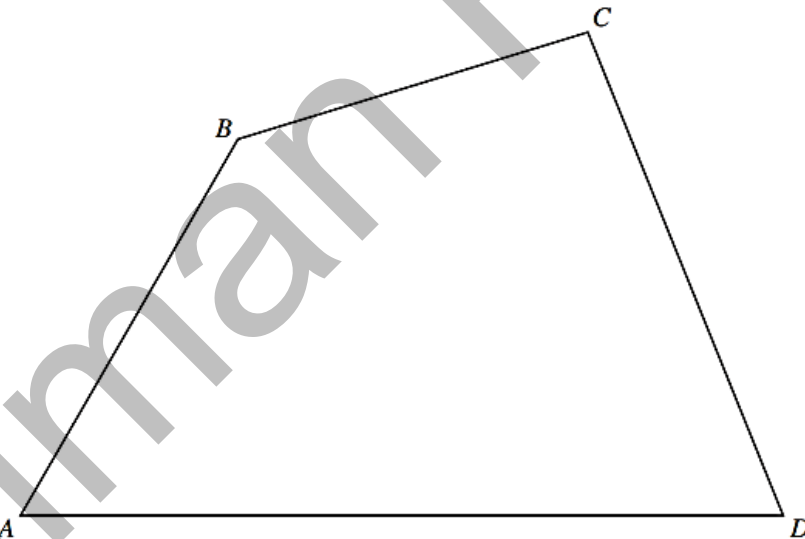
Answers: (a) 136° to 138°   **N11/11/Q25**

25 The diagram below shows triangle ABC.
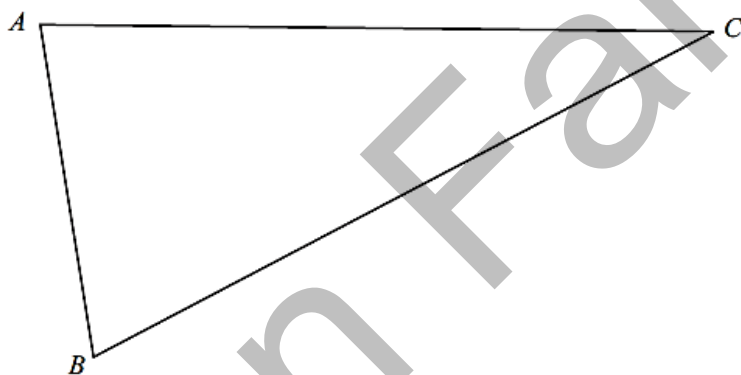
(a) The point D is on the opposite side of AC to B.
$AD = 6$ cm and $CD = 8$ cm.

Construct triangle ADC. [1]

(b) On the diagram, construct the locus of points **inside** the quadrilateral $ABCD$ that are

(i) 2.5 cm from AC, [1]

(ii) equidistant from AB and BC. [1]

(c) The points P and Q are 2.5 cm from AC and equidistant from AB and BC.

Mark and label P and Q.

Measure PQ.

Answer $PQ = $ .... cm [1]

Answer: (c) 5.4 to 5.7

26 The diagram at the bottom of the page shows the lines AB and BC.

(a) By measuring an angle, find reflex angle ABC.

Answer ....[1]

(b) The point D is on the opposite side of AC to B.
$CD = CB$ and $AD = 10$ cm.

On the diagram, construct quadrilateral $ABCD$. [1]

(c) On the diagram, construct the locus of points, **inside** the quadrilateral $ABCD$, that are

   (i) equidistant from A and B, [1]

   (ii) equidistant from BC and BA. [1]

(d) On the diagram, shade the region **inside** the quadrilateral $ABCD$ containing the points that are

                nearer to A than to B and
                nearer to BC than to BA. [1]

27 (a) Draw triangle ABC with $AB = 8\,cm$, $AC = 7\,cm$ and $C\hat{A}B = 130°$.
AB has been drawn for you.

A |

B |

[2]

(b) By making suitable measurements, find the area of triangle ABC.

Answer .... cm^2 [2]

N14/11/Q15

28
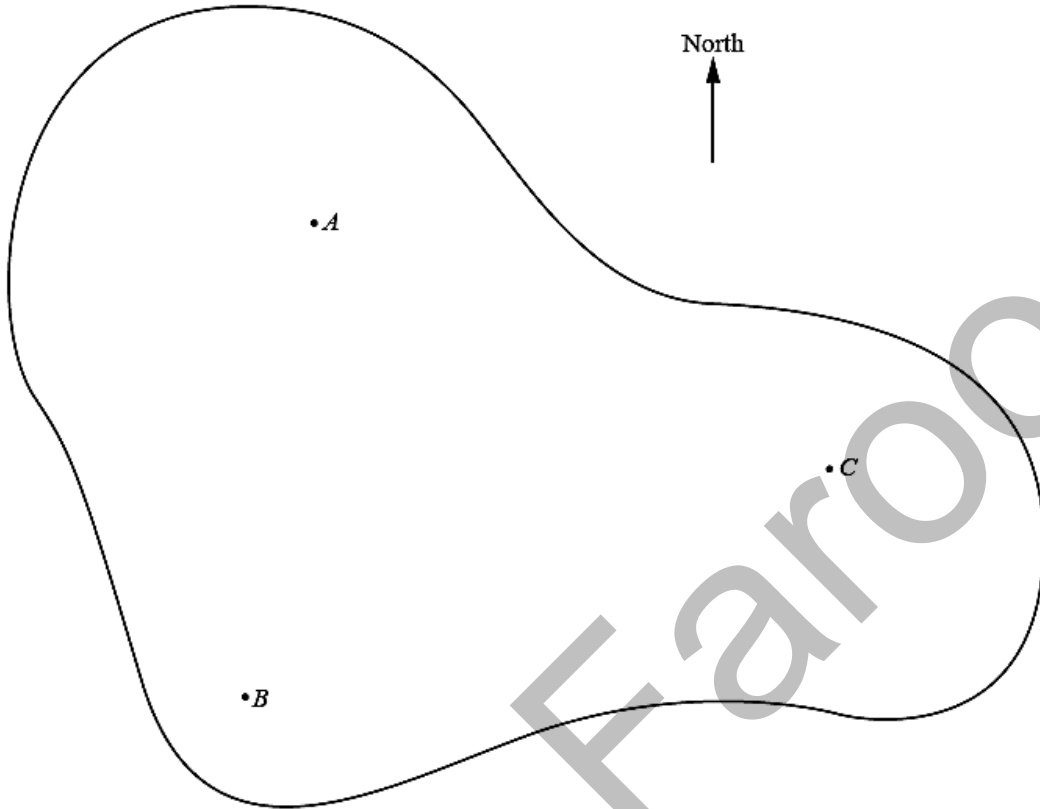
The land region shown has wheat storage depots at *A*, *B* and *C*.

(a) Given that the bearing of *C* from *A* is 115°, find the bearing of *A* from *C*.

Answer .... [1]

(b) Local farmers take their wheat to the nearest depot.

By drawing suitable accurate constructions, find and shade the region which is served by the depot at *B*.

[2]

N15/11/Q19

29 The diagram shows the quadrilateral *ABCD*.

(a) Measure $D\hat{C}B$.

Answer $D\hat{C}B =$ .... [1]

(b) (i) Construct the locus of points, **inside** the quadrilateral, that are 8 cm from *B*. [1]

   (ii) Construct the locus of points, **inside** the quadrilateral, that are 5 cm from *AB*. [1]
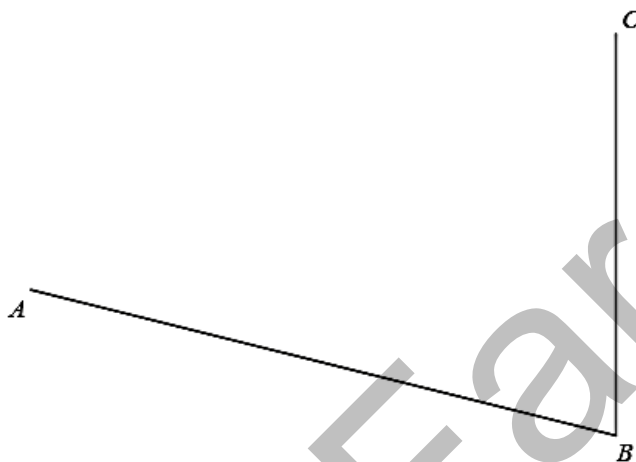
  (iii) These two loci meet at *P*.

      Mark, and label, the point *P* on the diagram and measure *PD*.

Answer   .... cm [1]
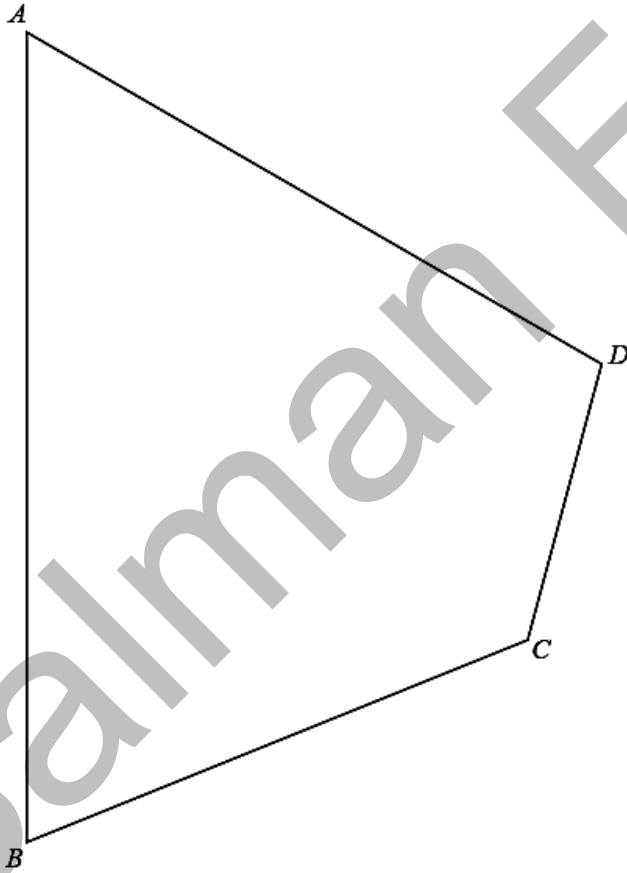
Answers: **(a)** 125° to 129° **(b)(i)** correct arc **(ii)** correct straight line **(iii)** *PD* = 3.4 cm to 3.8 cm   **N16/11/Q20**

The diagram shows the lines *AB* and *BC*.

The point *D* is on the opposite side of *AC* to *B*.
AD = 5 cm and *CD* = 6.5 cm.

(a) Construct quadrilateral *ABCD*. [1]

(b) On the diagram, construct the locus of points, **inside** the quadrilateral, that are

    (i) equidistant from *AB* and *BC*, [1]

    (ii) equidistant from *B* and *C*. [1]

(c) The line *PQ* consists of the points, inside the quadrilateral, which are

    equidistant from *AB* and *BC*,
   and nearer to *C* than to *B*.

    Mark and label the line *PQ* on the diagram. [1]

Answers: **(a)** Quadrilateral with visible arcs **(b)(i)** Bisector of angle ABC **(ii)** Perpendicular bisector of BC **N17/11/Q20**
        (c) PQ labelled

Loci Paper 2

1 The diagram below is a sketch of a rectangular field $WXYZ$.
$WX = 65\,\text{m}$ and $XY = 40\,\text{m}$.

(a) Using a scale of 1 cm to 5 m, make a scale drawing of the field. [1]

(b) (i) On your scale drawing, construct the locus of all the points in the field that are

    I equidistant from Z and Y,
    II 45 m from X. [2]

  (ii) Hence label the region, **R**, of points that are nearer to Z than Y and not more than 45 m from X. [1]

(c) A post P is placed in the region **R** so that it is at the maximum distance from Y.
A post Q is placed at the midpoint of the line ZY.

  (i) Label the posts P and Q on your drawing. [1]

  (ii) By measurement, find

    (a) the actual distance between the posts P and Q in the field, [1]

    (b) the obtuse angle PQ makes with QY. [1]

Answers: **(c)(ii)(a)** 41 to 43 m   **(b)** 105° to 109°   **J10/21/Q3**

2

(b) *JKLM* is a quadrilateral with *KL* = 7 cm and *KL̂M* = 72°.
N is the point on *LM* such that *KLN* is a sector of a circle, centre *L*.
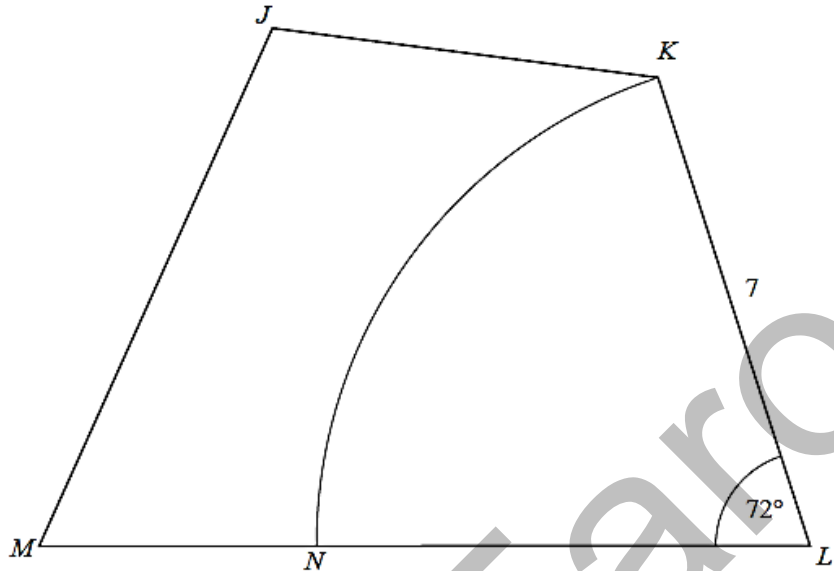
(i) Calculate the area of the sector *KLN*.   *Answer* ....cm² [2]

(ii) Calculate the perimeter of the sector *KLN*.   *Answer* .... cm [2]

(iii) On the diagram, construct the locus of points inside the quadrilateral *JKLM* which are

     I   5 cm from *JM*,
     II   equidistant from *JK* and *KL*.   [2]

(iv) The point *P* is   inside *JKLM*,
                  less than 5 cm from *JM*,
                  nearer to *KL* than *JK*,
                  less than 7 cm from *L*.

Shade the region containing the possible positions of *P*.   [1]

(b)(i)   30.8   (ii)   22.8   **J12/21/Q8b**

3   **(a)**   The scale diagram shows the positions, A and B, of two boats.

North

A

B

Scale: 1 cm to 50 m

(i)   Find the actual distance between the two boats.   *Answer*   ....m [1]

(ii)   A third boat is positioned at C, such that $AC = 350$ m and $BC = 300$ m.
C is east of the line AB.

Use ruler and compasses to find C.   [2]

(iii)   Measure the bearing of C from A.   *Answer*   .... [1]

(iv)   A fourth boat is positioned at D, such that AC is the line of symmetry of the
quadrilateral $ABCD$.

Complete the quadrilateral $ABCD$.   [2]

4   Answer this question on a new page.
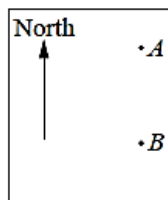
A and *B* are two coastguard stations with *A* due north of *B*.
On your new blank page, mark *A* in a position near the top right hand corner, as shown in the diagram.

(a) The distance between *A* and *B* is 140 km.

Using a scale of 1 cm to 10 km,

   (i)   mark the position of *B* and draw the line *AB*,   [1]

   (ii) construct the locus of the points west of *AB* that are

      (a) equidistant from *A* and *B*,   [1]

      (b) 90 km from *B*.   [1]

(b) A ship, *S*, lies to the west of *AB* and is

   I   nearer to *A* than *B*,

   II within 90 km of *B*.

On your diagram, shade the region in which the ship is situated.   [1]

(c) It is also known that the bearing of the ship from *A* is 204°.

   (i)   On your diagram, mark the two extreme positions, S_1 and S_2, of the ship.   [2]

   (ii)   Measure the angle S_1BS_2.   [1]

   (iii) The bearing of the ship from *B* is *x*°.

   Find the least possible value of *x*.   [1]

Answers: (c)(ii) 10° (iii) 336.   **N10/22/Q6**

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5 (a) Construct the triangle ABC in which $B\hat{A}C = 40°$ and $AC = 8\,\text{cm}$.

C is above the line AB, which is drawn for you.

A ——————————————————————————— B

[2]

(b) Construct the locus of all the points **outside** the triangle that are $2\,\text{cm}$ from the perimeter of the triangle. [2]

(c) Find and label the point P, **inside** the triangle, that is $6.5\,\text{cm}$ from A and equidistant from B and C. [2]

6 **(a)**
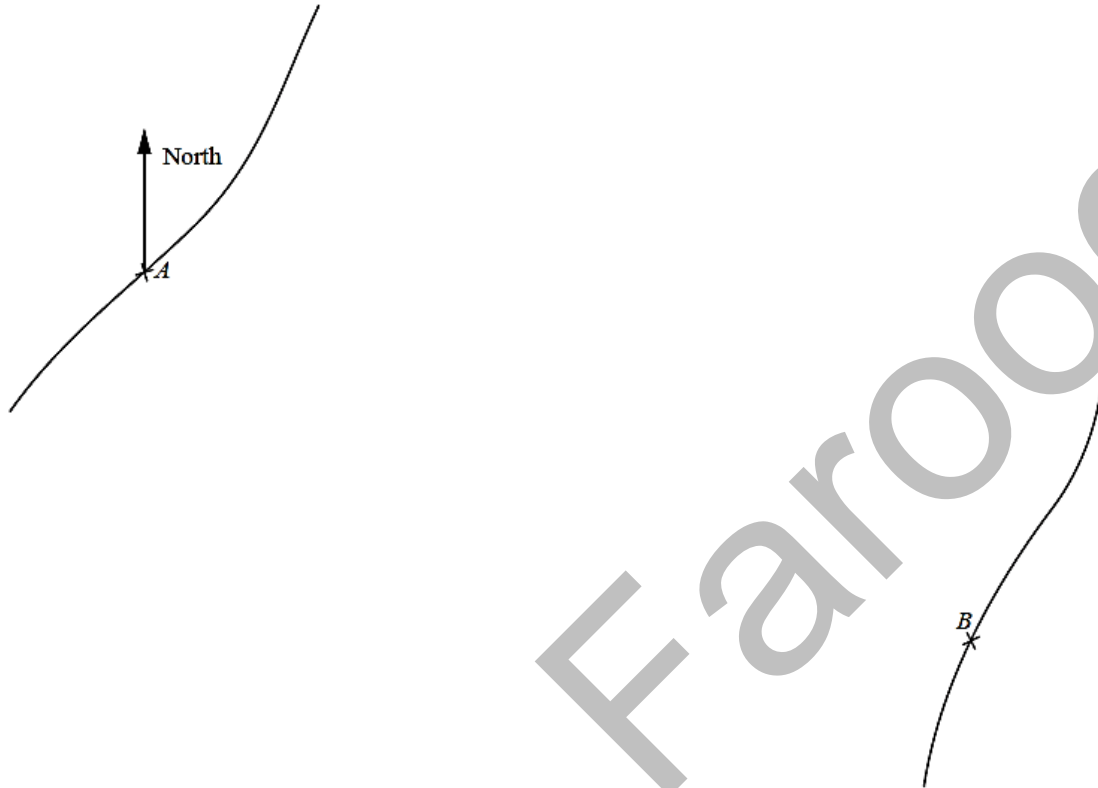
The scale drawing shows two coastal towns, *A* and *B*.
The scale of the drawing is 2 cm to 1 km.

(i) Measure the bearing of *B* from *A*.

Answer .... [1]

(ii) Draw the locus of points equidistant from *A* and *B*. [1]

(iii) A rock, *C*, is known to be less than 4 km from *B* and nearer to *A* than *B*.

(a) Construct and shade the region in which *C* must lie. [2]

(b) Find the shortest possible distance between *A* and *C*.

Answer .... km [1]

(iv) A boat, *D*, starts at the point 3.5 km due south of *A* and sails on a bearing of 075°.

Draw the path of *D* and state, with a reason, whether it is possible that *D* collides with *C*.

Answer .... [2]

Answers: **(a)(i)** 112 to 116; **(ii)** Perpendicular bisector of *AB*; **(iii)(a)** Correct region shaded; **(b)** 2.8 to 3.2; **(iv)** Yes as path of *D* passes through the shaded region; **(b)(i)** 9.43; **(ii)** 6.39.   **N14/21/Q8**

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Sets and Venn Diagrams Paper 1

1 There are 50 people on a tour.
One day, 26 people went on the morning cruise and 29 to the evening barbecue.
Using Venn diagrams, or otherwise, answer the following questions.

(a) It was thought that 4 people went to both events and 1 person to neither.
Explain why this was not possible.
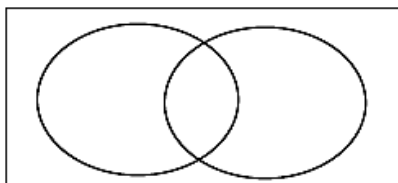
Answer (a) ....

....[1]

(b) Find the least number and the greatest number of people who could have gone to both events.

Answer (b) Least number .... [1]

Greatest number .... [1]

2 (a) In a group of language students,
24 studied Spanish, 23 studied French and 15 studied German,
12 studied Spanish and French,
10 studied German and French,
6 studied Spanish and German,
4 studied all three languages.
By drawing a Venn diagram, or otherwise, calculate the number of students who studied

(i) both Spanish and French, but not German,   *Answer* (a)(i) .... [1]

(ii) only one language.   (ii) .... [1]

(b) The set A consists of the points whose coordinates (x, y) are given by $A = \{(x, y) : y = 2x + 1\}$.
The points in set B are given by $B = \{(0, 0), (0, 1), (1, 2), (2, 5), (3, 6)\}$.

Find

(i) $n(B)$,   *Answer* (b)(i) $n(B) = $ .... [1]

(ii) $A \cap B$.   (ii) {...................................} [1]

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3

(a) The sets A and B are shown on the Venn Diagram in the answer space.
The element y is such that $y \in A$ and $y \notin B$.
On the diagram, write y in the correct region.

Answer (a)

[1]
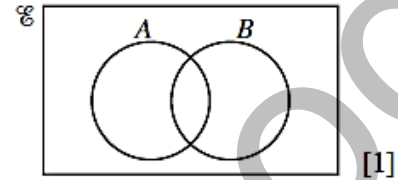
(b) $\mathscr{E} = \{x : x \text{ is an integer and } 1 \leqslant x \leqslant 8 \}$.   *Answer (b)(i)* .... [1]
$P = \{x : x > 5\}$.
$Q = \{x : x \leqslant 3\}$.   (ii) { .... } [1]

    (i)   Find the value of n$(P \cup Q)$.

    (ii) List the elements of $P' \cap Q'$.

4

3 (a) $\mathscr{E}$ = { 1, 2, 3, 4, 5 },
   A = { 1, 2, 3 },
   B = { 5 },
   C = { 3, 4 }.

List the elements of

    (i) $A \cup C$,   *Answer* (a)(i) .... [1]

    (ii) $B' \cap C'$.   *Answer* (a)(ii) .... [1]

) A group of 60 children attend an after school club.
Of these, 35 children play football and 29 play hockey.
3 children do not play either football or hockey.

By drawing a Venn diagram, or otherwise, find the number of children who play only hockey.

Answer (b) .... [2]

5 **(a)** Using set notation, describe the shaded region in the Venn diagram.
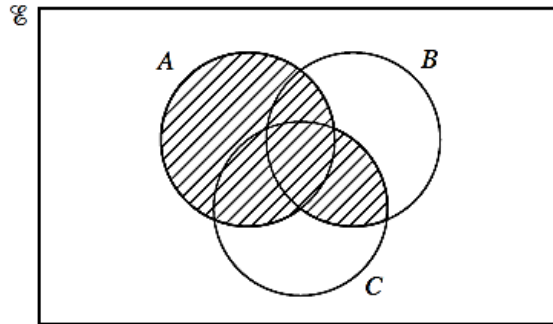
Answer ....[1]

(b) In the Venn diagram, shade the region represented by $P' \cap (Q \cup R)$.
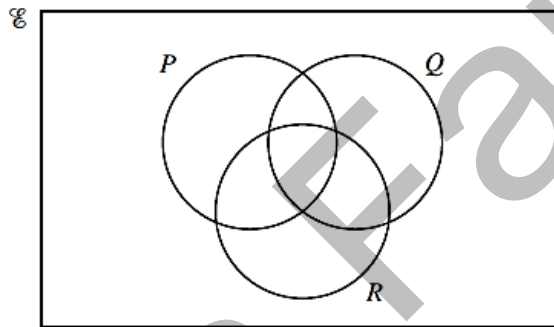
[1]

Answers: **(a)** A ∪ (B ∩ C)   **J11/11/Q7**

6 **(a)** On the Venn diagram, shade the set $A \cap B \cap C'$.

[1]
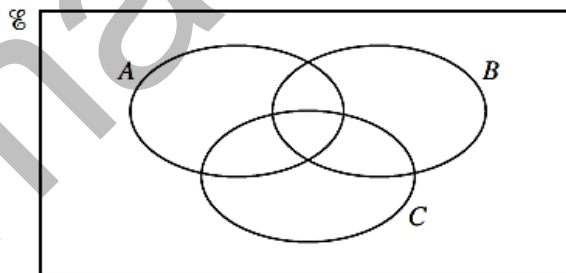
(b) $\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$P = \{x : x \text{ is a prime number}\}$
$Q = \{x : x \geqslant 5\}$

(i) Find the value of $n(P \cap Q)$.   Answer ....[1]

(ii) List the elements of $P \cup Q'$.   Answer ....[1]

Answer: **(b)(i)** 2   **(ii)** 2, 3, 4, 5, 7   **J11/12/Q12**

7   **(b)** $\mathscr{E} = \{$natural numbers$\}$
            $P = \{$factors of 8$\}$
            $Q = \{$factors of 12$\}$

            List the elements of the set $P \cup Q$.   *Answer*   ....[2]

   (c)
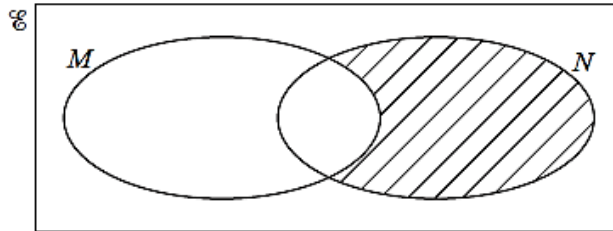
            Use set notation to describe the shaded subset in the Venn diagram.

                                                   Answer   ....[1]

   (b) 1, 2, 3, 4, 6, 8, 12   **(c)**   M′ ∩ N   **J13/11/Q24**

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8   **(a)** $\mathscr{E} = \{\, x : x$ is an integer, $40 \leqslant x \leqslant 50 \,\}$

            $P = \{\, x : x$ is a prime number $\}$

            $Q = \{\, x : x$ is a multiple of 6 $\}$

         (i)   Find $\text{n}(P)$.   *Answer*   .... [1]

         (ii)   List the members of Q.   *Answer*   .... [1]

   (b)   In a group of 25 people,
            11 people own both a bicycle and a skateboard,
            6 people own neither a bicycle nor a skateboard,
            n people own a bicycle.

         Find the smallest and the largest possible values of n.

                           Answer smallest   .... [1]

                                     largest   .... [1]

Answers: **(a)(i)** 3   **(a)(ii)** 42, 48   **(b)** 11, 19   **J15/11/Q18**

9 **(a)** In the Venn diagram, shade the region which represents the subset $(P \cup Q)' \cap R$.
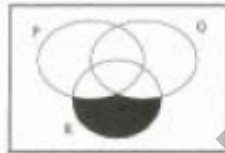

[1]
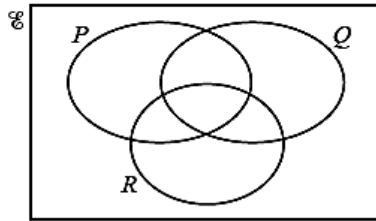
(b) $\mathscr{E} = \{\, x : x \text{ is an integer and } 22 \leqslant x \leqslant 33 \,\}$
$E = \{\, x : x \text{ is an even number} \,\}$
$T = \{\, x : x \text{ is a multiple of 3} \,\}$
$F = \{\, x : x \text{ is a multiple of 4} \,\}$

    (i) List the members of $T \cap F$.   *Answer* .... [1]

    (ii) Find $\mathrm{n}(E \cup T)$.   *Answer* .... [1]

    (iii) Given that $\in ' \cap$, find one possible value of y.

                                   Answer $y =$ .... [1]

Answers: **(a)**   **(b)(i)** 24 **(b)(ii)** 8 **(b)(iii)** 22 or 26 or 30   **J16/11/Q23**

10 **(a)** In a sports club   24 members play basketball (B),
                          28 play cricket (C),
                          16 play football (F),
                          9 play basketball and cricket,
                          11 play cricket and football and
                          6 play basketball and football.
Five members play all three games and **eight** members play none of these games.

    (i) Complete the Venn diagram to show this information.
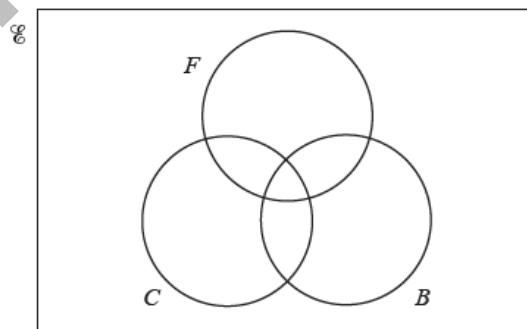
[2]

    (ii) Hence work out the **total** number of members in the club. *Answer* .... [1]

(b) In another sports club, the number of members playing basketball (*B*), cricket (*C*) and football (*F*) are shown in the Venn diagram below.
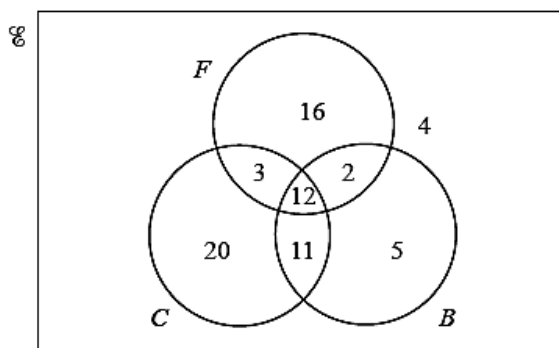
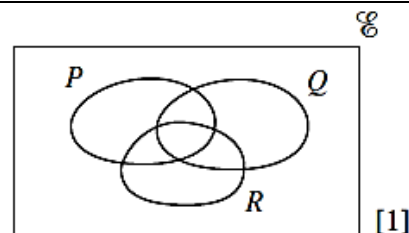**(i)** Find n(*F'*).   *Answer* .... [1]

(ii) Find n((*F* ∪ *C*) ∩ *B'*).   *Answer* .... [1]

Answers: **(a)(i)** Correct Venn diagram **(ii)** 55 **(b)(i)** 40 **(ii)** 39   **J17/11/Q21**

11 **(a)** On the Venn Diagram shown in the answer space, shade the set (*P* ∩ *Q*) ∪ *R*.

Answer (a)

[1]

(b) Express in set notation, as simply as possible, the set shaded in the Venn Diagram.
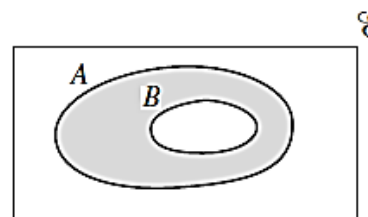
Answer (b) .... [1]

(c) There are 34 children in a class.
Of these, 22 take History, 19 take Geography and 5 take neither History nor Geography.

Using a Venn Diagram, or otherwise, find the number of children who take History but not Geography

Answer (c) .... [2]

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Answers: **(b)** A∩B'; **(c)** 10.

12

   (a) In the Venn diagram in the answer space, shade the region $(A \cup B) \cap C$.
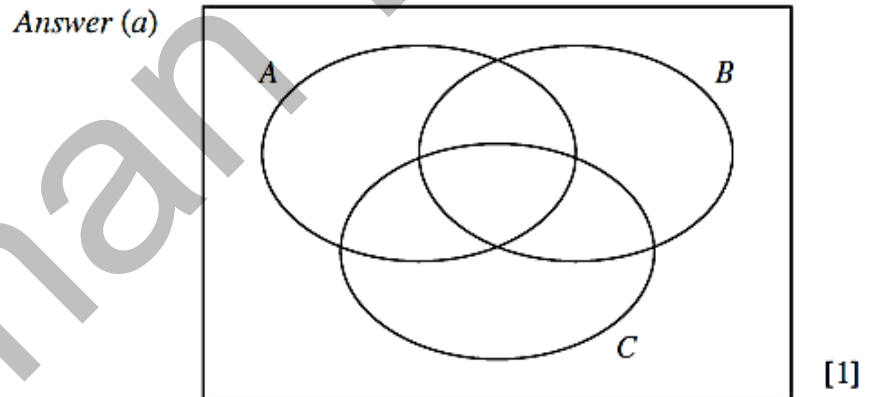
Answer (a)

[1]
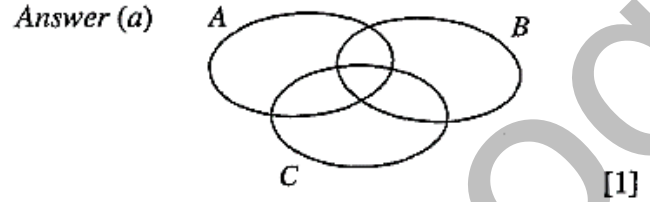
   (b) All 30 students in a class study **at least one** of the two subjects History and Geography.
Twice as many study History as Geography.
8 students study **only** Geography.
By drawing a Venn diagram, or otherwise, find the number of students who study both History and Geography.

Answer: **(b)** 3.

13

   (a) On the Venn Diagram in the answer space, shade the set $(A' \cup B') \cap C$.

Answer (a)

[1]

   (b) $\mathcal{E} = \{x : x \text{ is an integer and } 4 \leqslant x \leqslant 16\}$
     $P = \{x : x \text{ is a prime number}\}$
     $S = \{x : x \text{ is an odd number}\}$
     $T = \{x : x \text{ is a multiple of 3}\}$

     (i) List the members of the set $S \cap T$.

     (ii) Describe, **in words**, the set S'.

     (iii) Find n$(P \cup T)$.

Answers: **(b)(i)** 9 and 15, **(ii)** ...even..., **(iii)** 8.

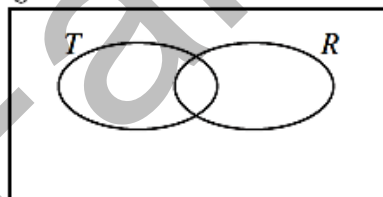**14** (a) Express in set notation, as simply as possible, the subset shaded in the Venn diagram.
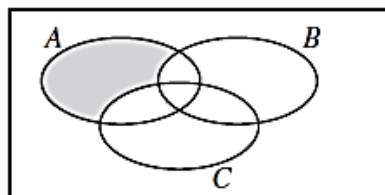
Answer (a) .... [1]

(b) $\mathscr{E}$ = {all polygons},
T = {all triangles},
R = {all regular polygons},
Q = {all quadrilaterals}.

Add the set Q to the Venn diagram in the answer space.
Answer (b) $\mathscr{E}$

[1]

(c) $\mathscr{E}$ = { $x : x$ is an integer and $3 \leqslant x \leqslant 11$},
F = { $x : x$ is a factor of 12},
O = { $x : x$ is an odd number}.

List the elements of the set $(F \cup O)'$.

Answer (c) .... [1]

(d) It is given that n($\mathscr{E}$) = 20.
P and S are sets such that n(P) = 7 and n(S) = 16.

Find the smallest possible value of n($P \cap S$) .   Answer (d) .... [1]

Answers: **(a)** $A \cap (B \cup C)'$ or $A \cap B' \cap C'$; **(c)** 8 and 10; **(d)** 3.   N04/1/Q18

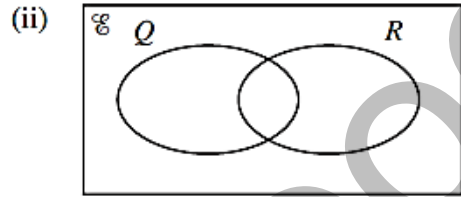**15** (a) On the Venn Diagram in the answer space, shade the region $A \cup B'$.
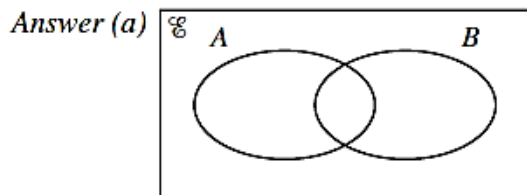
Answer (a)

[1]

(b) $\mathscr{E}$ = {all polygons}, Q = {all quadrilaterals}, R = {all regular polygons}.

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(i) What is the special name of the polygons which belong to $Q \cap R$?

(ii) On the Venn Diagram in the answer space, show the set
$T = \{$all equilateral triangles$\}$.

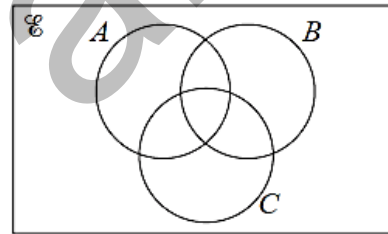*Answer (b)(i)*.. [1]

(ii)

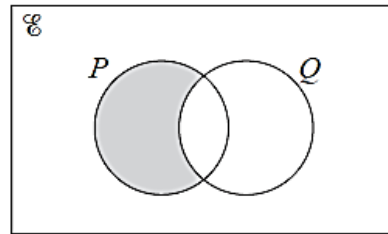[1]

Answer: **(b)(i)** squares.

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16 **(a)** On the Venn diagram in the answer space, shade the set $A \cup (B \cap C)$.

Answer (a)

[1]

(b) Express in set notation the subset shaded in the Venn diagram.

Answer (b) ....[1]

(c) In a class of 36 students, 25 study History, 20 study Geography and 4 study neither History nor Geography.
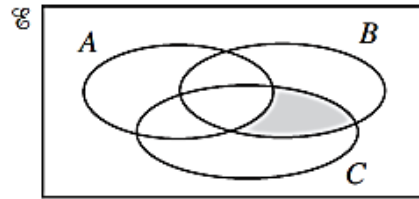Find how many students study both History and Geography.

Answer (c) ....[2]

Answer: **(b)** $P \cap Q'$; **(c)** 13.

17

(a) Express, in set notation, as simply as possible, the subset shaded in the Venn diagram.

Answer (a) ....[1]

(b) It is given that $n(\mathscr{E}) = 40$, $n(P) = 18$, $n(Q) = 20$ and $n(P \cap Q) = 7$.

Find

(i) $n(P \cup Q)$,

Answer (b)(i) ....[1]

(ii) $n(P' \cap Q')$.
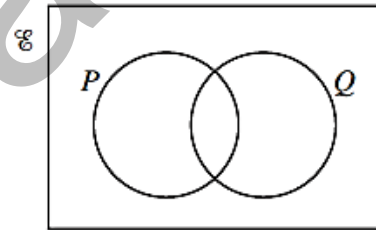
(ii) ....[1]

Answer: (a) $B \cap C \cap A'$　(b)(i) 31　(ii) 9

18

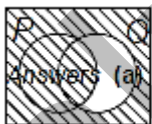(a) On the Venn diagram shown in the answer space, shade the set $P \cup Q'$.

Answer (a)

[1]

(b) There are 27 children in a class.

Answer (b) ....[2]

Of these children, 19 own a bicycle, 15 own a scooter and 3 own neither a bicycle nor a scooter.

Using a Venn diagram, or otherwise, find the number of children who own a bicycle but not a scooter.
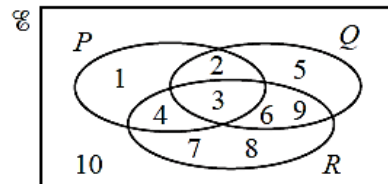
Answers (a)

(b) 9.

19 The Venn diagram shows the sets $\mathscr{E}$, P, Q and R.

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(a) Find the value of $n(Q \cup R)$.

Answer (a) ....[1]

(b) List the elements of the set $P' \cap (Q \cup R)$.   *Answer* (*b*) {.................................} [1]

(c) One element is chosen at random from $\mathscr{E}$.
Write down the probability that this element belongs to $(P \cap Q) \cup (P \cap R)$.

Answer (*c*) ....[1]

N10/12/Q11

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20 $\mathscr{E} = \{\, x : x$ is an integer and $x > 5\, \}$

$P = \{\, x : x$ is a prime number $\}$
$F = \{\, x : x$ is a multiple of 4 $\}$
$S = \{\, x : x$ is a multiple of 6 $\}$

The Venn diagram shows the Universal set and the set F.

[2]
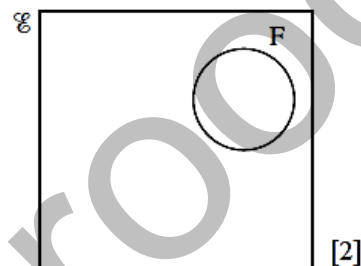
(a) Draw and label the two sets P and S to complete the Venn diagram.

(b) Write down a possible element y such that y is an even number and $y \in (F \cup S)'$.

Answer y = ... [1]

N11/11/Q15

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21   The Venn diagram shows the Universal set and the set B.
A and C are two sets such that

$$A \cup B = B, \qquad A \cap B \neq B, \qquad A \cap C = \varnothing \qquad \text{and} \qquad B \cap C \neq \varnothing.$$

Draw the sets A and C in the Venn diagram.
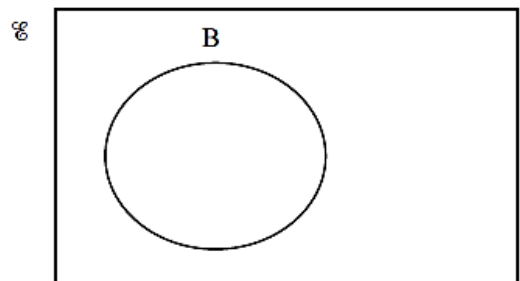
[2]

N11/12/Q10

22 **(a)** Given that $A = \{\,1, 2, 3, 4, 5\,\}$ and $B = \{\,3, 4, 5, 6, 7\,\}$, find n($A \cup B$).

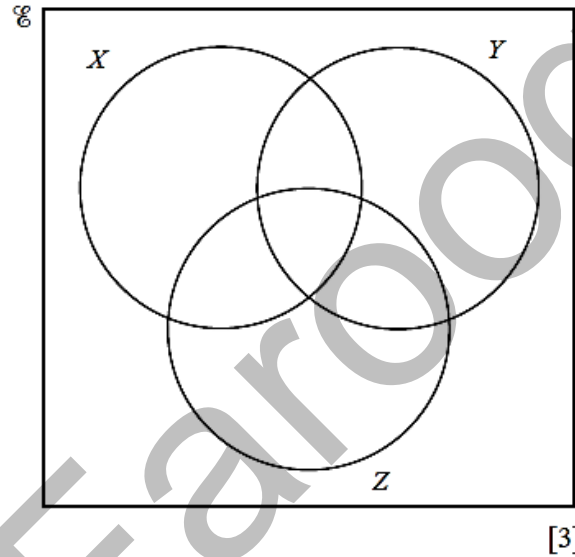*Answer* ....[1]

(b) $\mathscr{E} = \{\,p, q, r, ...\,\}$

On the Venn diagram, write each of the letters p, q, and r in its appropriate subset, given that

$p \in X \cap Y \cap Z$,

$q \in X' \cap Y' \cap Z'$,

$r \in (X \cup Y)' \cap Z$.

[3]

Answers: **(a)** 7 **(b)** Correct *p*, *q* and *r*

23 **(a)** Express, in set notation, the subset shaded in the diagram.
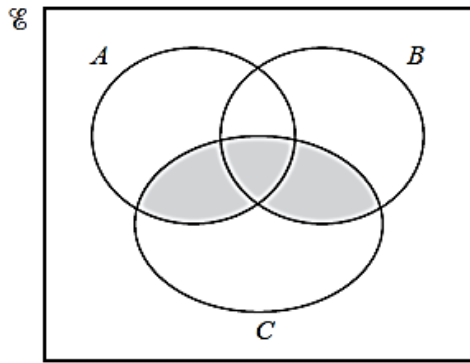
Answer ....[1]

(b) $\mathscr{E} = \{a, b, c, d, e, f, g, h\}$

$P = \{a, b, c\}$

$Q = \{b, c, d, e, f\}$

(i) Find n$(P \cup Q)$. *Answer* ....[1]

(ii) List the members of the subset $P' \cap Q$. *Answer* ....[1]

Answers: **(a)** $(A \cup B) \cap C$ **(b) (i)** 6 **(ii)** d, e, f

N13/11/Q14

24 **(a)** Use set notation to describe the shaded subset in the Venn diagram.
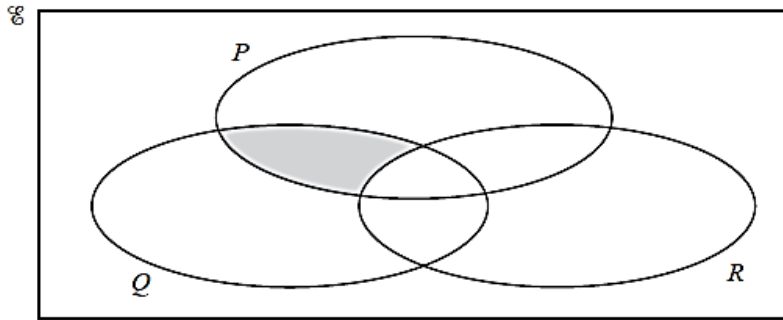
Answer .... [1]

(b) In a group of students

30 play cricket,
38 play football and
9 play neither cricket nor football.

Find the lowest possible number of students in the group.

Answer .... [2]

Answers: **(a)** $P \cap Q \cap R'$; **(b)** 47.   N14/11/Q10

25 **(a)** $\mathscr{E} = \{ 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 \}$

$P = \{ x : x$ is an even number $\}$

$Q = \{ x : x$ is a multiple of 3 $\}$

(i) Find $n(P \cup Q)$.   Answer .... [1]

(ii) Given that $y \in \mathscr{E}$ and that y is a prime number, write down the value of y.

Answer $y =$ .... [1]

(b) In the Venn diagram, shade the region represented by $A' \cap B$.
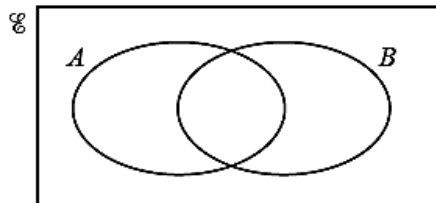
[1]

Answers: **(a)(i)** 9 **(ii)** 89 **(b)** region inside B but outside of A shaded   N16/11/Q12

26 **(a)** In the Venn diagram, shade the region which represents the subset $A' \cap B \cap C$.
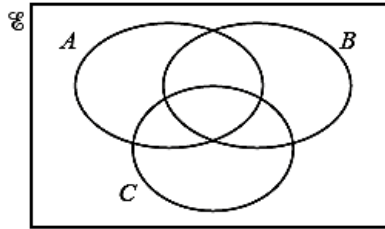
[1]

(b) $P = \{1, 4\}$

$Q = \{-1, 1, 2\}$

$R = \left\{ \dfrac{x}{y} : x \in P, y \in Q \right\}$

(i) Find $n(P \cup Q)$.

Answer .... [1]

(ii) List the members of R.

Answer .... [2]

Answers: (a) Correct diagram (b)(i) 4 (ii) $-1, 1, \dfrac{1}{2}, -4, 4, 2$

Sets Paper 2

1 In a group of 100 students, 80 study Spanish and 35 study French.
x students study Spanish and French.
y students study neither Spanish nor French.
The Venn diagram illustrates this information.

(a) Expressed in set notation, the value of x is $n(S \cap F)$.
Express the value of y in set notation.   [1]

(b) Find, in its simplest form, an expression for y in terms of x.   [2]
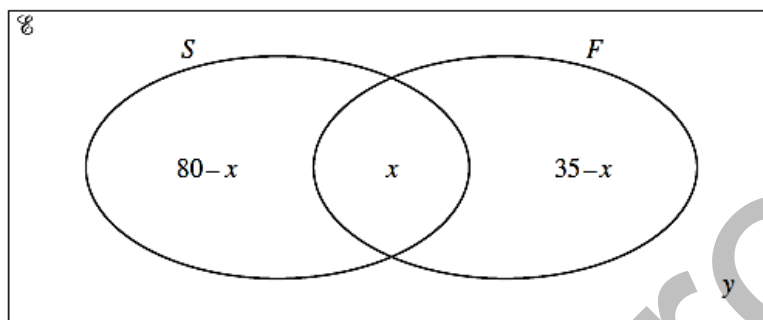
(c) Find

    (i) the least possible value of x,   [1]

    (ii) the greatest possible value of y.   [1]

Answers: **(a)** $n(S \cup F)'$ or $n(S' \cap F')$; **(b)** $x - 15$; **(c)(ii)** 15, **(ii)** 20.

2 (a) The results of a survey of 31 students are shown in the Venn diagram.

$\mathcal{E}$ = {students questioned in the survey}
M = {students who study Mathematics}
P = {students who study Physics}
S = {students who study Spanish}
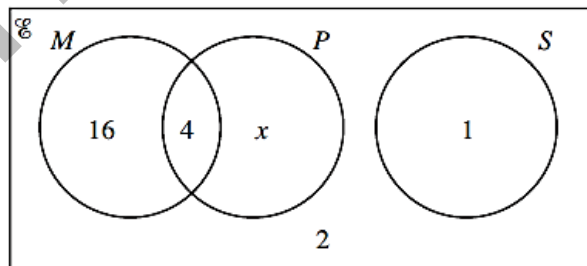
    (i) Write down the value of

        (a) x,   [1]

        (b) $n(M \cap P)$,   [1]

(c) n($M \cup S$), [1]

(d) n(P'). [1]

(ii) Write down a description, in words, of the set that has 16 members. [1]

3 (a) Mary has 50 counters.
Some of the counters are square, the remainder are round.
There are 11 square counters that are green.
There are 15 square counters that are not green.
Of the round counters, the number that are not green is double the number that are green.

By drawing a Venn diagram, or otherwise, find the number of counters that are

(i) round, [1]

(ii) round and green, [1]

(iii) not green. [1]

4 (b) The Venn diagram shows the three means of transport used by a group of workers during a week.
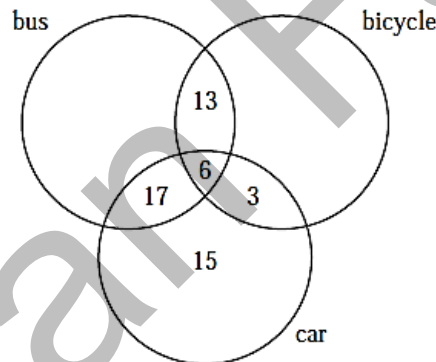
(i) How many used both a bus and a car but not a bicycle? [1]

(ii) Twice as many **only** used a bicycle as **only** used a bus.
There were 78 workers in the group.

How many used a bus **only**? [2]

5 (c) $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$
$A = \{x : x \text{ is a multiple of } 3\}$
$B = \{x : x \text{ is a factor of } 24\}$
$C = \{x : x \text{ is an odd number}\}$

(i) Find

(a) n(B), [1]

(b) ($A \cup B \cup C$)'. [1]

(ii) A number, k, is chosen at random from $\mathscr{E}$.

Find the probability that $k \in A \cap B$. [2]

6 **(a)** The sets A, B and C are shown in the Venn diagram.
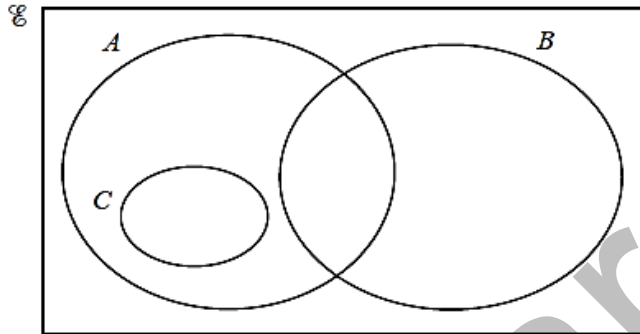
$\mathscr{E} = \{\, x : x \text{ is an integer, } 1 \leqslant x \leqslant 18 \,\}$
$A = \{\, x : x \text{ is an even number} \,\}$
$B = \{\, x : x \text{ is a multiple of 5} \,\}$

(i) Find $\mathrm{n}(A \cup B)$.   *Answer* ....[1]

(ii) **(a)** Given that $A \cap B' \cap C' = \{2, 6, 14, 18\}$, list the members of C.

Answer ....[1]

(b) Describe the set C in words.   *Answer* $C = \{\, x : x \text{ is } .....\}$ [1]

(b) A school offers piano lessons and flute lessons to a group of 50 children.

Of these children, 28 attend piano lessons
17 attend flute lessons
12 attend neither piano lessons nor flute lessons.

By drawing a Venn diagram, or otherwise, find the number of children who attend only the piano lessons.

Answer .... [2]

7 **(a)** $\mathscr{E} = \{x : x \text{ is an integer and } 5 \leqslant x \leqslant 15\}$

$A = \{x : x \text{ is a multiple of 3}\}$

$B = \{x : x \text{ is a factor of 60}\}$

$C = \{x : x \text{ is a prime number}\}$

461

(i) Find n(∩ ∩). *Answer* .... [1]

(ii) Find $(A \cup B)'$. *Answer* .... [1]

(iii) A number, *r*, is chosen at random from $\mathscr{E}$.

Find the probability that $r \in A \cap B$. *Answer* .... [1]

(iv) Given that $D \subset B$ and $D \subset C$, find *D*. *Answer* .... [1]

(b) An activity camp offers 3 sports: tennis, cricket and volleyball.
One day, 50 children took part in these sports.

19 children played tennis, 34 children played cricket and 23 children played volleyball.
2 children played all three sports.
5 children played tennis and cricket.
10 children played tennis and volleyball.

By drawing a Venn diagram, or otherwise, find the number of children who played

(i) tennis and cricket but not volleyball,

Answer .... [1]

(ii) cricket and volleyball but not tennis, *Answer* .... [1]

(iii) cricket only. *Answer* .... [1]

Answers: **(a)(i)** 0 or None **(ii)** 7, 8, 11, 13, 14 **(iii)** 3/11 or 0.27 **(iv)** 5 **(b)(i)** 3 **(ii)** 11 **(iii)** 18   **J14/21/Q2**

8 **(a)** $\mathscr{E} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$

$L = \{x : x \text{ is an odd number}\}$

$M = \{x : x \text{ is a multiple of 3}\}$

(i) Write down

(a) $L \cap M$, [1]

(b) $L' \cap M$. [1]

(ii) A number *n* is chosen at random from $\mathscr{E}$.
Find the probability that $n \in L \cup M$. [1]

(b) In a survey, a number of people were asked "Do you own a car?" and "Do you own a bicycle?".
The Venn diagram shows the set *C* of car owners and the set *B* of bicycle owners.
The letters *p*, *q* and *x* are the numbers of people in each subset.
11 people owned neither a car nor a bicycle.

462
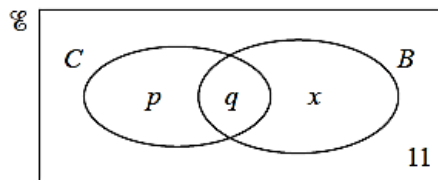
A total of 66 people owned a car.
4 times as many people owned a car **only** as owned a bicycle **only**.

(i) Write down expressions, in terms of x, for

    (a) p,   [1]

    (b) q.   [1]

(ii) A total of 27 people owned a bicycle.

    Calculate

    (a) x,   [2]

    (b) the total number of people who were in the survey.   [1]

Answers: (a)(i)(a) {3, 9, 15} (b) {6, 12} (ii) $\dfrac{10}{15}$ (b)(i)(a) $4x$ (b) $66 - 4x$ (ii)(a) 13 (b) 90   **N09/2/Q4**

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9   (a) $\mathscr{E} = \{x : x \text{ is an integer and } 10 \leqslant x \leqslant 21\}$

       $E = \{x : x \text{ is an even number}\}$

       $M = \{x : x \text{ is a multiple of 5}\}$

       $P = \{x : x \text{ is a prime number}\}$
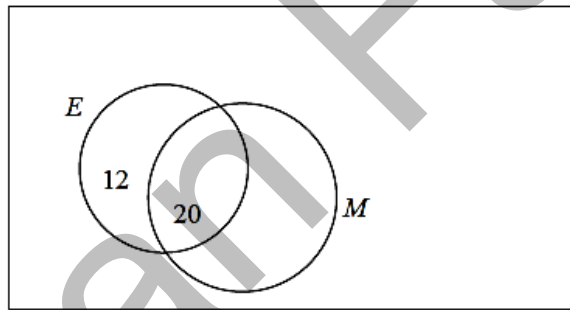
The Venn diagram shows the Universal set $\mathscr{E}$ and the subsets E and M.
Two elements of $\mathscr{E}$ are shown in their appropriate subsets.

(i) Copy the Venn diagram shown above and draw the subset P.   [1]

(ii) Write the remaining 10 elements of $\mathscr{E}$ in the appropriate subsets of your Venn diagram.   [3]

(b) $\mathscr{E} = \{\text{letters of the alphabet}\}$

     $J = \{a, b, c, d, e, f, g\}$

     $K = \{a, e, i, o, u\}$

(i) Find $n(J \cup K)$.   [1]

(ii) Find $J \cap K'$.   [1]

(iii) Given that $L \subset J$ and $L \subset K$, state the maximum value of $n(L)$.   [1]

(iv) A letter is picked at random from K.

     Find the probability that it is **not** an element of J.   [1]

(c) In a school, 24 students are members of the Athletics Club, 30 students are members of the Basketball Club and 36 students are members of the Drama Club.
5 students are members of all three clubs.
12 students are members of both the Athletics Club and the Drama Club.
13 students are members of both the Basketball Club and the Athletics Club.
19 students are members of both the Drama Club and the Basketball Club.

(i) How many students are members of the Basketball Club, but are not members of the other two Clubs? [1]

(ii) How many students are members of one or more of the Clubs? [1]

Answers: **(b)(i)** 10 **(ii)** { b, c, d, f, g } **(iii)** 2 **(iv)** 3/5 **(c)(i)** 3 **(ii)** 51

10 (a) Shade the subset $(A \cap B) \cup C$.

Answer
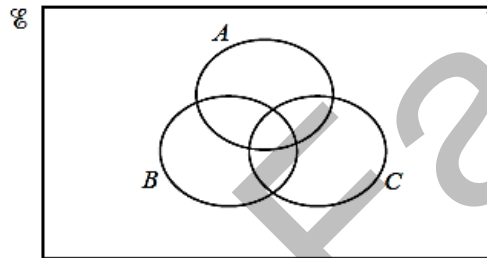
[1]

(b) Use set notation to describe the subset shaded in the diagram.
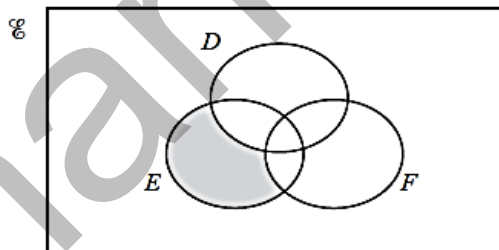
Answer ....[1]

(c) $\mathcal{E} = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$
$P = \{ x : x$ is an odd number $\}$
$Q = \{ x : x$ is a square number $\}$

(i) Write the members of $\mathcal{E}$ in the correct regions on the Venn diagram.

Answer

464
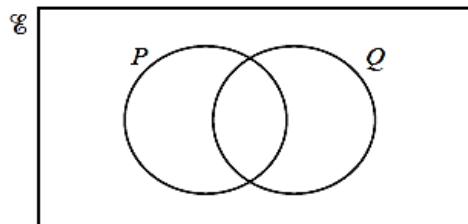
(ii) State n(Q').

Answer .... [1]

(iii) A number, m, is chosen at random from $\mathscr{E}$.

Find the probability that m is a member of $P \cap Q'$.

Answer .... [2]

Answers: **(a)** Correct shading; **(b)** $E \cap (D \cup F)'$; **(c)(i)** 10 numbers positioned correctly; **(ii)** 7; **(iii)** $\frac{3}{10}$ **N15/21/Q4**

Matrices Paper 1

1

Given that $A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix}$,

find

(a) the determinant of A,

Answer (a) .... [1]

(b) A^{-1}.

(b) .... [1]

Answers: (a) 10; (b) $\dfrac{1}{10}\begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix}$.

2

(a) Evaluate $\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$.

Answer (a) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(b) Write down the inverse of $\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$.

(b) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

Answers: (a) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$; (b) $\dfrac{1}{2}\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$.

3

(a) Evaluate $\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$.

Answer (a) $\begin{pmatrix} \\ \\ \end{pmatrix}$ [1]

(b) A business makes toy buses and toy lorries.
The following table is used in calculating the cost of making each toy.

	Labour (hours)	Wood (blocks)	Paint (tins)
Bus	2	3	1
Lorry	1	w	2

Labour costs \$10 per hour, wood costs \$1 per block and paint costs \$$p$ per tin.

The information above can be summarised in the matrices A and B,

where $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & w & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 10 \\ 1 \\ p \end{pmatrix}$.

466

(i) Given that $AB = \begin{pmatrix} 28 \\ 24 \end{pmatrix}$, find

    (a) p,

    (b) w.

(ii) Evaluate $(100 \quad 200)\begin{pmatrix} 28 \\ 24 \end{pmatrix}$.

(iii) Explain what your answer to (ii) represents.

Answer $(b)(i)(a)\ p =$ ....[1]

$(b)\ w =$ ....[1]

(ii) ....[1]

Answer: $(a) \begin{pmatrix} 3 \\ 7 \\ 0 \end{pmatrix}$ $(b)(i)(a)\ p = 5$ $(b)\ w = 4$ $(ii)\ (7600)$ (iii) Total cost

---

4   It is given that

$$A = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -2 \\ 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}.$$

Find

(a) $A - 2B$,   Answer (a) $\begin{pmatrix} \\ \end{pmatrix}$ [2]

(b) C^{-1}. (b) $\begin{pmatrix} \\ \end{pmatrix}$ [2]

Answers: $(a) \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$ $(b)\ \dfrac{1}{5}\begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$

5 Two families ordered three basic food items from their local shop.
The Jones family ordered 1 bag of sugar, 4 cartons of milk and 3 loaves of bread.
The Singh family ordered no sugar, 3 cartons of milk and 5 loaves of bread.
Their orders can be represented by the matrix **A** where

$$A = \begin{pmatrix} 1 & 0 \\ 4 & 3 \\ 3 & 5 \end{pmatrix}.$$

The cost of a bag of sugar is 80 cents, the cost of a carton of milk is 50 cents and the cost of a loaf of bread is 40 cents.
This information can be represented by the matrix **B** where

$$B = (80 \quad 50 \quad 40).$$

(a) Work out **BA**. *Answer* (a) .... [2]

(b) What does the matrix **BA** represent? ....[1]

Answer. (a) (400 350) (b) Total cost (of each family's order) **J10/11/Q13**

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6 $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}.$

Find

(a) $A - B$, *Answer* (a) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(b) B^{-1}. *Answer* (b) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

J10/12/Q19

Answer. (a) $\begin{pmatrix} 3 & -1 \\ 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{3}{2} & -1 \\ \frac{1}{2} & 0 \end{pmatrix}$

7
(a) $A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$

(i) Find $2A - B$. *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(ii) Find B^{-1}. *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

J13/11/Q24a

Answer. (a)(i) $\begin{pmatrix} 6 & 9 \\ 1 & 3 \end{pmatrix}$ (ii) $\frac{1}{5}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$

8

$$A = \begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 3 \\ 0 & -2 \end{pmatrix}$$

(a) Find $3A - B$. *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(b) Find A^2. *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(c) Find the 2×2 matrix X, where $AX = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

J14/11/Q25

Answers: (a) $\begin{pmatrix} 4 & -6 \\ -6 & 14 \end{pmatrix}$ (b) $\begin{pmatrix} 11 & -7 \\ -14 & 18 \end{pmatrix}$ (c) $\dfrac{1}{10}\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

9

$$P = (3 \quad 4 \quad -5) \qquad Q = \begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Evaluate PQ. *Answer* [2]

Answer: (1 8) **J15/11/Q10**

10

(a) Express as a single matrix $\begin{pmatrix} -1 & -3 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & -2 \\ 2 & -5 \end{pmatrix}$. *Answer* .... [1]

(b) Find the inverse of $\begin{pmatrix} 1 & -1 \\ 5 & 3 \end{pmatrix}$. *Answer* .... [2]

J15/11/Q16

Answers: (a) $\begin{pmatrix} -2 & -1 \\ -1 & 5 \end{pmatrix}$ (b) $\dfrac{1}{8}\begin{pmatrix} 3 & 1 \\ -5 & 1 \end{pmatrix}$

11

Given that $A = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix}$, find

(a) $A - 2B$, *Answer (a)* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(b) the determinant of A, *(b)* .... [1]

(c) A^{-1}. *(c)* $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

N01/Q19

Answers: (a) $\begin{pmatrix} 0 & 3 \\ 6 & 3 \end{pmatrix}$; (b) -2; (c) $\begin{pmatrix} -1/2 & 3/2 \\ 1 & -2 \end{pmatrix}$.

12

$$A = \begin{pmatrix} 5 & 3 \\ -2 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 3 \\ 2 & -3 \end{pmatrix}$$

Find

(a) $A - 2B$,

(b) A^{-1}.

Answers: (a) $\begin{pmatrix} 3 & -3 \\ -6 & 6 \end{pmatrix}$; (b) $\frac{1}{6}\begin{pmatrix} 0 & -3 \\ 2 & 5 \end{pmatrix}$.

13

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 1 \\ -4 & -3 \end{pmatrix}, \quad C = (1 \quad 2 \quad 3), \quad D = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Find

(a) $A - B$,   *Answer (a)* .... [1]

(b) A^2,   *(b)* .... [1]

(c) $AA^{-1}\begin{pmatrix} 7 \\ 8 \end{pmatrix}$,   *(c)* .... [1]

(d) CD.   *(d)* .... [1]

Answers: (a) $\begin{pmatrix} -1 & 2 \\ 3 & 3 \end{pmatrix}$; (b) $\begin{pmatrix} 1 & 6 \\ -2 & -3 \end{pmatrix}$; (c) $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$; (d) (1).

14

(a) The determinant of the matrix $\begin{pmatrix} k & 5 \\ -1 & 2 \end{pmatrix}$ is 14.

Find k.   *Answer (a) k =* .... [1]

(b) Find the inverse of the matrix $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$.   *(b)* $\begin{pmatrix} \\ \end{pmatrix}$ [2]

Answers: (a) $4\frac{1}{2}$; (b) $\frac{1}{2}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

15

$$A = (1 \quad 2 \quad 3) \qquad B = \begin{pmatrix} 2 & 0 \\ 1 & 4 \\ -1 & -3 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & -1 \\ 2 & 2 \\ -1 & 0 \end{pmatrix}$$

Find

(a) B – C, *Answer (a)* .... [2]

(b) AB. *(b)* .... [2]

Answers: (a) $\begin{pmatrix} 0 & 1 \\ -1 & 2 \\ 0 & -3 \end{pmatrix}$; (b) $(1, -1)$.

16 (a) The matrix **M** satisfies the equation

$$3M + 4\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} = M.$$

Answer (a) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

Find **M**, expressing it in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

(b) Find the inverse of the matrix $\begin{pmatrix} 5 & -3 \\ -4 & 2 \end{pmatrix}$. *(b)* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

Answer: (a) $\begin{pmatrix} -4 & 2 \\ -6 & 0 \end{pmatrix}$; (b) $-\dfrac{1}{2}\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

17

$$A = \begin{pmatrix} 4 & -1 \\ 3 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ -3 & 4 \end{pmatrix}$$

Answer (a) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

Find

(a) 2A – B, *(b)* $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(b) AB,

(c) A⁻¹. *(c)* $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

Answer : (a) $\begin{pmatrix} 8 & -3 \\ 9 & -4 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & \frac{1}{3} \\ -1 & 1\frac{1}{3} \end{pmatrix}$

18

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4 & -3 \\ 1 & 0 \end{pmatrix}.$$

Find

(a) **AB**,

(b) **B**$^{-1}$.

Answer (*a*) AB = $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(*b*) B^{-1} = $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

Answers: (a) $\begin{pmatrix} 7 & -6 \\ 7 & -3 \end{pmatrix}$, (b) $\begin{pmatrix} 0 & 1 \\ -\frac{1}{3} & 1\frac{1}{3} \end{pmatrix}$.

19

$$A = \begin{pmatrix} 4 & -3 & 0 \\ 0 & 6 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 5 & -4 & -1 \\ 0 & 6 & 2 \end{pmatrix} \quad C = (2 \quad 1)$$

(a) Find $2A - B$.

(b) Find **CA**.

Answer (*a*) [2]

Answer (*b*) [1]

Answers: (a) $\begin{pmatrix} 3 & -2 & 1 \\ 0 & 6 & -6 \end{pmatrix}$ (b) (8 0 -2)

20

$$A = \begin{pmatrix} 2 & -3 \\ -1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 5 & -4 \\ -2 & 2 \end{pmatrix}$$

Find

(a) $2A - B$,

(b) **A**$^{-1}$.

Answer (*a*) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

Answer (*b*) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

Answer: (a) $\begin{pmatrix} -1 & -2 \\ 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & -1 \\ -\frac{1}{3} & -\frac{2}{3} \end{pmatrix}$

21 A shop sells bunches of flowers.
One bunch contains 3 roses, 4 carnations and 5 freesias.
Another bunch contains 6 roses and 4 carnations.
Each rose costs 60 cents, each carnation costs 40 cents and each freesia costs 30 cents.

This information can be represented by the matrices P and Q below.

$$P = \begin{pmatrix} 3 & 4 & 5 \\ 6 & 4 & 0 \end{pmatrix} \qquad Q = \begin{pmatrix} 60 \\ 40 \\ 30 \end{pmatrix}$$

(a) Find PQ.

Answer (a) .... [2]

(b) Explain what the numbers in your answer represent.

....[1]

Answer: a) $\begin{pmatrix} 490 \\ 520 \end{pmatrix}$ **(b)** The cost, in cents, of each bunch.

22 Find the values of x and y, where

$$2\begin{pmatrix} x \\ 7 \end{pmatrix} = 3\begin{pmatrix} -2 \\ y \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \end{pmatrix}.$$

Answer $x =$ ....

$y =$ .... [2]

Answer: $x = -5$, $y = 4$

23

$$A = \begin{pmatrix} 5 & 2 \\ -1 & 1 \end{pmatrix}$$

(a) Find the determinant of A. *Answer* .... [1]

(b) Write down A^{-1}. *Answer* [1]

(c) Find the matrix X, where $AX = \begin{pmatrix} 11 \\ -5 \end{pmatrix}$. *Answer* [2]

Answers: **(a)** 7 **(b)** $\frac{1}{7}\begin{pmatrix} 1 & -2 \\ 1 & 5 \end{pmatrix}$ **(c)** $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

24

$$A = \begin{pmatrix} 4 & -2 \\ -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & 2 \\ -1 & 4 \end{pmatrix}$$

(a) Find $2A - B$.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(b) Find A^{-1}.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

Answer: (a)(i) $\begin{pmatrix} 11 & -6 \\ -1 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 2 \end{pmatrix}$

25

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Find A^{-1}.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

Answers: (a) $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

26

(a) Express as a single matrix $2\begin{pmatrix} -1 & -2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -3 \\ 2 & 3 \end{pmatrix}$.

Answer   [2]

(b) The matrix X satisfies the equation $X\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = (8 \quad 5)$.

(i) Complete the following statement.

"The matrix X has .... row(s) and .... column(s)."   [1]

(ii) Find X.   Answer   [2]

Answers: (a) $\begin{pmatrix} -3 & -1 \\ -2 & -1 \end{pmatrix}$ (b)(i) 1 row, 2 columns (ii) (4 3)

27

$$A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$$

(a) Find $\begin{pmatrix} 5 & -3 \\ 2 & 6 \end{pmatrix} - 2A$.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(b) Find $A \times A$.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(c) Write down, as a 2×2 matrix, the answer to $3 \times A \times A^{-1}$.

Answers: (a) $\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -18 \\ 6 & 13 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

---

28 John works in a shop.
The matrix below shows the number of hours he worked on Monday to Friday, Saturday, and Sunday during two different weeks.

$$\begin{array}{c} \\ \text{Week 1} \\ \text{Week 2} \end{array} \begin{array}{ccc} \text{Monday} & & \\ \text{to Friday} & \text{Saturday} & \text{Sunday} \\ \left(\begin{array}{ccc} 30 & 5 & 0 \\ 35 & 6 & 2 \end{array} \right) \end{array}$$

The matrix below shows the pay that he received per hour on Monday to Friday, Saturday, and Sunday.

$$\begin{array}{l} \$/\text{hr} \\ \begin{pmatrix} 9 \\ 12 \\ 15 \end{pmatrix} \begin{array}{l} \text{Monday to Friday} \\ \text{Saturday} \\ \text{Sunday} \end{array} \end{array}$$

(a) $\mathbf{P} = \begin{pmatrix} 30 & 5 & 0 \\ 35 & 6 & 2 \end{pmatrix} \begin{pmatrix} 9 \\ 12 \\ 15 \end{pmatrix}$

Answer $\mathbf{P} =$ .... [2]

Find P.

(b) Explain the meaning of the information given by matrix P.

.... [1]

Answers: (a) $\begin{pmatrix} 330 \\ 417 \end{pmatrix}$; (b) P shows the amount earned in Week 1 and Week 2.

29
$$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

(a) Evaluate $3A - 2\begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix}$.

Answer .... [2]

(b) Find A^{-1}.

Answer .... [2]

(c) Write down the single matrix that is equivalent to $A^{-1}A$.

Answer .... [1]

Answers: (a) $\begin{pmatrix} 0 & -5 \\ 7 & 9 \end{pmatrix}$ (b) $\frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

30

$$A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$$

(a) Express $A - 2B$ as a single matrix.

Answer .... [2]

(b) Find A^2.

Answer .... [2]

(c) $B^{-1} = kA$ where k is a rational number.

Find k.

Answer $k =$ .... [1]

Answers: **(a)** $\begin{pmatrix} -6 & -6 \\ 3 & 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} -2 & -6 \\ 3 & 7 \end{pmatrix}$ **(c)** $\dfrac{1}{2}$

Matrices Paper 2

1 **(d)** The matrix **Y** satisfies the equation

$$4\mathbf{Y} - 2\begin{pmatrix} 12 & 6 \\ -9 & 0 \end{pmatrix} = \mathbf{Y}.$$

Find **Y**, expressing it in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. [2]

(d) $\begin{pmatrix} 8 & 4 \\ -6 & 0 \end{pmatrix}$

---

2 **(a)** In a swimming match between two schools, C and D, two students from each school took part in each event.
The number of places each school gained in each position is shown in the table.

	First	Second	Third	Fourth
School C	6	3	5	6
School D	4	7	5	4

The points awarded for First, Second, Third and Fourth places were 5, 3, 1 and 0 respectively.

Matrices related to this information are defined below.

$$A = \begin{pmatrix} 6 & 3 & 5 & 6 \\ 4 & 7 & 5 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 \\ 3 \\ 1 \\ 0 \end{pmatrix}$$

(i) What does the sum of the elements in each column of **A** represent? [1]

(ii) **(a)** Find **AB**. [2]

(b) What information is shown by **AB**? [1]

(iii) It was suggested that the points awarded for First, Second, Third and Fourth places should have been 5, 3, 2 and 1 respectively.

Would this suggestion have made any difference to which school won this match?
Show clear working to justify your answer. [1]

Answers: (a)(i) Number of events, (ii)(a) $\begin{pmatrix} 44 \\ 46 \end{pmatrix}$, (b) School scores, (iii) $\begin{pmatrix} 55 \\ 55 \end{pmatrix}$ ⇒ Yes, now a tie;

(a) $A = \begin{pmatrix} 1 & -3 \\ 3 & -2 \end{pmatrix}$ $B = \begin{pmatrix} -2p & 3p \\ -3p & p \end{pmatrix}$ $C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

    (i)   Evaluate 4C – 2A.   [2]

    (ii) Given that $B = A^{-1}$, find the value of p.   [2]

    (iii) Find the 2×2 matrix X, where AX = C.   [2]

    (iv) The matrix C represents the **single transformation T.**

        Describe, fully, the transformation T.   [2]

Answers: (a)(i) $\begin{pmatrix} -6 & 6 \\ -6 & 8 \end{pmatrix}$, (ii) $\frac{1}{7}$, , (iii) $\frac{1}{7}\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$, (iv) Reflection in the y axis (or x = 0);   **J06/2/Q11**

3

(c) $M = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$

    (i)   Find the determinant of M.   [1]

    (ii) Write down the inverse of M.   [1]

    (iii) Find the matrix X, where $MX = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$.   [2]

(c)(i) 2, (ii) $\begin{pmatrix} 2 & -1\frac{1}{2} \\ 1 & -\frac{1}{2} \end{pmatrix}$;   **J07/2/Q11c**

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4   (c) Ann went on a car journey that was split into three stages.
Two relevant matrices are shown below.
The first matrix shows the average speed, in kilometres per hour, of the car during each stage.
The second matrix shows the time, in hours, taken for each stage.

	First stage	Second stage	Third stage
Average speed	40	30	50

Time
$$\begin{pmatrix} 1\frac{1}{2} \\ 1 \\ 2\frac{1}{2} \end{pmatrix} \begin{matrix} \text{First stage} \\ \text{Second stage} \\ \text{Third stage} \end{matrix}$$

    (i)   Find $\begin{pmatrix} 40 & 30 & 50 \end{pmatrix} \begin{pmatrix} 1\frac{1}{2} \\ 1 \\ 2\frac{1}{2} \end{pmatrix}$.   [1]

    (ii) What information is given by the matrix obtained in part (i)?   [1]

    (iii) Calculate the average speed for the whole journey.   [1]

(c) (i) (215), (ii) total distance travelled,   (iii) $4\overline{3}$ km/h.   **J08/2/Q5c**

5 (a) $A = \begin{pmatrix} 0 & 3 \\ -1 & x \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}$

(i) Express $2A - 3B$ in terms of x. [2]

(ii) Given that $A = B^{-1}$, find the value of x. [2]

Answers: (a)(i) $\begin{pmatrix} -3 & 9 \\ -3 & 2x \end{pmatrix}$, (ii) 3;

6 (a) Bertie goes shopping and buys three different types of fruit.
The first matrix below shows the number of kilograms of each fruit bought during two different weeks.
The second matrix shows the price per kilogram, in cents, of each fruit.

	bananas	apples	grapes
Week 1	1	2	0.5
Week 2	1.5	1	1

price/kg	
290	bananas
160	apples
640	grapes

(i) $F = \begin{pmatrix} 1 & 2 & 0.5 \\ 1.5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 290 \\ 160 \\ 640 \end{pmatrix}$.

Find F. [2]

(ii) Explain the meaning of the information given by the matrix F. [1]

(iii) Find the total amount of money, in dollars, that Bertie spent on fruit during the two weeks. [1]

(b) The matrix M satisfies the equation

$$8 \begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} + 5M = M.$$

Find M. [2]

Answers: (a) (i) $\begin{pmatrix} 930 \\ 1235 \end{pmatrix}$, (ii) cost of fruit in each week; (iii) \$21.65; (b) $\begin{pmatrix} -6 & 0 \\ 2 & -4 \end{pmatrix}$;

7 (a) $A = \begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$.

Find

(i) $2A - B$, Answer [2]

(ii) B^{-1}. Answer [2]

Answers: (a) (i) $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} -1 & -2 \\ 1.5 & 2.5 \end{pmatrix}$

8

(a) $A = \begin{pmatrix} -3 & 6 \\ -2 & 2 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix}$

Find

(i) $A + 2B$,

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(ii) A^{-1}.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(b) Mark and Luke spend three days training for a cycling event.
Mark cycles at an average speed of 24 km/h on the first two days and 26 km/h on the third day.
Luke cycles at an average speed of 25 km/h on the first day, 24 km/h on the second day and 27 km/h on the third day.
They each cycle for 1 hour on the first day and increase their cycling time by $\frac{1}{2}$ hour each day.

This information is represented by the matrices P and Q below.

$$P = \begin{pmatrix} 24 & 24 & 26 \\ 25 & 24 & 27 \end{pmatrix} \qquad Q = \begin{pmatrix} 1 \\ m \\ n \end{pmatrix}$$

(i) Find m and n.   *Answer* $m = $ .... $n = $ ....[1]

(ii) Find PQ.

Answer   [2]

(iii) Calculate the difference between the numbers in the matrix PQ and explain what this number represents.

Answer Difference is .... and this number represents ....

....[2]

Answers: (a)(i) $\begin{pmatrix} -5 & 6 \\ 0 & -2 \end{pmatrix}$ (ii) $\frac{1}{6}\begin{pmatrix} 2 & -6 \\ 2 & -3 \end{pmatrix}$ (b)(i) 1.5, 2   (ii) $\begin{pmatrix} 112 \\ 115 \end{pmatrix}$ (iii) 3   **J12/21/Q12**

9

(a) Evaluate

(i) $3\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$,   [2]

(ii) $(1 \ 3 \ 4)\begin{pmatrix} 0 & 4 \\ 3 & 1 \\ 5 & 0 \end{pmatrix}$.   [2]

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(b) $A = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$

(i) Find A^{-1}. [2]

(ii) The transformation represented by the matrix A maps (h,k) onto $(10,2)$.
Find the value of h and the value of k. [2]

N09/2/Q5

Answers: (a)(i) $\begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$ (ii) $(29 \; 7)$ (b)(i) $\frac{1}{2}\begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$ (ii) $h = 8, k = 2$

10

(a) $A = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}$ $\qquad B = \begin{pmatrix} -1 & 2 \\ -3 & 2 \end{pmatrix}$

Find

(i) $2A - B$, *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) B^{-1}. *Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(b) The matrix C satisfies the following equation.

$$3C + 4\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix} = C$$

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

Find C.

481

(c) Theresa sells raspberries and blackcurrants.
The first matrix shows the number of kilograms of each fruit she sells during three different weeks.
The second matrix shows the price per kilogram, in cents, of the fruit Theresa sells.

	raspberries	blackcurrants
Week 1	3	2
Week 2	1.5	3
Week 3	2	2.5

price/kg

$$\begin{pmatrix} 650 \\ 580 \end{pmatrix} \begin{matrix} \text{raspberries} \\ \text{blackcurrants} \end{matrix}$$

(i) $D = \begin{pmatrix} 3 & 2 \\ 1.5 & 3 \\ 2 & 2.5 \end{pmatrix} \begin{pmatrix} 650 \\ 580 \end{pmatrix}$

Answer .... [2]

Find **D**.

(ii) Explain the meaning of the information given by matrix **D**.

Answer .... [1]

(iii) Find the total amount, in dollars, that Theresa gets for the fruit she sells.

N15/21/Q3

Coordinate Geometry Paper 1

1 (a) P is the point $(-3, 3)$ and Q is the point $(13, -2)$.
Find the coordinates of the midpoint of PQ.

Answer (a) (.................. , ...) [1]

(b) The line $x - 3y = 2$ is shown on the diagram in the answer space.
The line $x - 3y = k$ cuts the y-axis at the point $(0, -4)$.

(i) Draw the line $x - 3y = k$ on the diagram.

(ii) Calculate the value of k.

Answer (b)(i)

[1]

(ii) $k =$ .... [1]

J05/1/Q13

483

2   The diagram shows the points A (1, 2), B (4, 6) and D (–5, 2).

(a)   Find the coordinates of the midpoint of AB.

(b)   Calculate the length of AB.

(c)   Calculate the gradient of the line AB.

(d)   Find the equation of the line AB.

(e)   The triangle ABC has line of symmetry $x = 4$.
      Find the coordinates of C.

(f)   Find the value of cosine $D\hat{A}B$.

Answer (a)   (.................... , ...)

(b) .... [1]

(c) ....[1]

(d) ....[2]

(e)   (.................... , ...)   [1]

(f) $\cos D\hat{A}B =$ ....[1]

3   A straight line passes through the points P (1, 2) and Q (5, −14).

Find

(a) the coordinates of the midpoint of PQ,   *Answer (a)*   (............. , ...)   [1]

(b) the gradient of PQ,   (b) ....[1]

(c) the equation of PQ.   (c) ....[2]

Answers: (a) (3, −6)   (b) − 4   (c) $y = 6 − 4x$ **J08/1/Q17**

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4   A straight line passes through the points P (−8, 10) and Q (4, 1).

Find

(a) the coordinates of the midpoint of PQ,   *Answer (a)* (............. , ...) [1]

(b) the equation of PQ.   *Answer (b)* .... [2]

Answer: (a) (− 2, 5.5)   (b) $y = − 0.75x + 4$ **J10/12/Q14**

---

5 P is the point (−2, 1) and Q is the point (3, 7).

(a) M is the midpoint of PQ.

Find the coordinates of M.   *Answer*   (............, ...)   [1]

(b) Find the gradient of the line PQ.   *Answer* .... [1]

(c) The line with equation $2y + 3x + k = 0$ passes through the point P.

(i) Find k.   *Answer* $k =$ .... [2]

(ii) Find the gradient of this line.   *Answer* .... [1]

Answer: (a) (0.5,4)   (b) 1.2   (c)(i) 4   (ii) − 1.5   **J11/12/Q24**

---

6   20   Here are the equations of four straight lines.

Line 1: $y = 2x + 4$
Line 2: $y = 2 − x$
Line 3: $y = 2x − 1$
Line 4: $2y − 8 = 3x$

(a) Which two lines are parallel?   *Answer* Line .... and Line .... [1]

(b) Which two lines intersect the y-axis at the same point?   *Answer* Line .... and Line .... [1]

(c) Which line passes through the points (1, 1) and (−3, 5)?   *Answer* Line ....[1]

(d) Find the midpoint of the line segment joining (1, 1) and (−3, 5).

Answer (..............., ...) [1]

485

J12/11/Q20

7   (a) The line $2y = 6 - 3x$ meets the y-axis at A and the x-axis at B.

Write down   *Answer* $A = ($ .... , ... $)$

   (i)   the coordinates of A and B, $B = ($ .... , ... $)$ [2]

   (ii) the gradient of the line.   *Answer*   .... [1]

(b) Another straight line cuts the x-axis at $P(-4, 0)$ and passes through $Q(2, 18)$.

Find the coordinates of the midpoint of PQ.   *Answer* $($ .... , ... $)$ [1]

J14/11/Q21

8

The diagram shows three points $A(-2, 7)$, $B(-2, 2)$ and $C(6, -4)$.

Find

(a) the length BC,

(b) the area of triangle ABC,

(c) the value of $\sin A\hat{B}C$.

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Answer (a) ....units [2]

(b) ....units² [2]

(c)... [1]

9   The points $A(-5, 5)$, $B(1, -3)$ and $C(4, -3)$ are shown in the diagram.

Find

(a) the coordinates of the midpoint of AC,

(b) the gradient of the line AB,

(c) the equation of the line which passes through $(0, 3)$ and is parallel to AB,

(d) the length of AB,

(e) the value of cosine $A\hat{B}C$.

10

PQRS is a parallelogram.
P is (–4, 0), Q is (1, 0) and R is (9, 4).
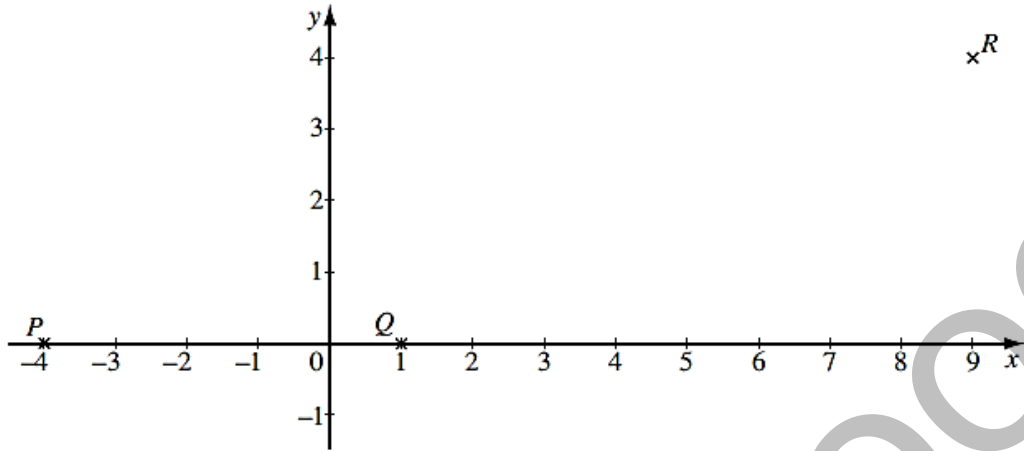
(a) Find the coordinates of S.

Answer (a) (............ , ...) [1]

(b) Find the coordinates of the midpoint of PR.

(b) (............ , ...) [1]

(c) Find the equation of the line RS.

(c) .... [1]

(d) Find the equation of the line QR.

(d) .... [2]

(e) Calculate the area of the parallelogram PQRS.

(e) ....unit^2 [1]

Answers: (a) (4, 4); (b) $(2\frac{1}{2}, 2)$; (c) $y = 4$; (d) $y = \frac{1}{2}x - \frac{1}{2}$; (e) 20 unit^2.

11

The triangle with vertices $A(4, 4)$, $B(-2, -6)$ and $C(4, -1)$ is shown in the diagram. Find

(a) **(i)** the area of $\triangle ABC$,   *Answer (a)* (i) ....unit^2 [1]

     (ii) the coordinates of the point P such that $ABCP$ is a parallelogram,

     (iii) the area of the parallelogram $ABCP$,

     (iv) $\tan B\hat{A}C$.

(b) It is given that the length of $BC = k$ units.
Write down $\cos B\hat{C}A$, giving your answer in terms of k.

                                                   (ii)   (............ , ...)   [1]

                                                   (iii) ....unit^2 [1]

                                                   (iv) $\tan B\hat{A}C =$ ....[1]

                                                   (b) $\cos B\hat{C}A =$ ....[1]

Answer: (a)(i) 15,   (ii) (10, 9),   (iii) 30,   (iv) $\dfrac{3}{5}$;   (b) $-\dfrac{5}{k}$ or $\dfrac{k^2 - 111}{10k}$.   **N06/1/Q22**

12 (a) Find the coordinates of the point where the line $2y = 3x + 15$ crosses the y-axis.

Answer (a) (..........,..........) [1]

(b) The coordinates of the points P and Q are $(-1, 10)$ and $(3, 4)$ respectively.

Find

(i) the gradient of PQ,

Answer (b)(i) ....[1]

(ii) the midpoint of PQ.

Answer (b)(ii) (..........,..........) [1]

Answers: (a) (0, 7.5)   (b)(i) -1.5   (ii) (1, 7)

13 M is the midpoint of the line joining P and Q.

(a) R lies on PQ produced, such that $PR = 3PQ$.
Find $PM : PR$.

Answer (a) .... : ....[1]

(b) P is $(1, -2)$ and Q is $(5, 6)$.

(i) Find the coordinates of M.

Answer (b)(i) (..........,..........) [1]

(ii) The line $4x + ky + 10 = 0$ passes through $Q(5, 6)$.
Find the value of k.

Answer (b)(ii) $k =$ ....[2]

Answers: (a) 1:6 (b)(i) (3,2) (ii) $k = -5$

14 The coordinates of P and Q are $(0, 7)$ and $(10, -1)$.

(a) Find the coordinates of the midpoint of PQ.

Answer (................. , ...) [1]

(b) The length of PQ is $\sqrt{N}$ units, where N is an integer.

Find N.

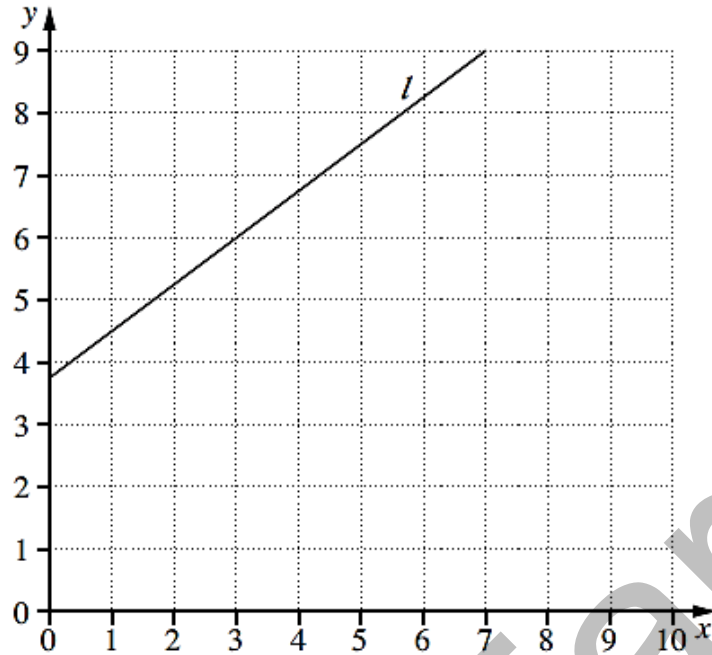Answer $N =$ .... [2]

Answers: (a) (5, 3) (b) 164

Coordinate Geometry Paper 2

1

(a) Express as a single fraction in its simplest form $\frac{2a}{3} + \frac{3}{2a}$. [1]

(b) Factorise completely $5b^2 - 10b$. [1]

(c) The points P and Q are $(4, 7)$ and $(8, -3)$ respectively.

Find

   (i) the midpoint of PQ, [1]

   (ii) the length of PQ. [2]

(d) Solve the equation $3x^2 + 11x - 7 = 0$, giving each answer correct to 2 decimal places. [4]

Answers: (a) $\dfrac{4a^2 + 9}{6a}$; (b) $5b\,(b-2)$; (c) (i) $(6, 2)$, (ii) 10.8; (d) 0.55 and -4.22. **J09/2/Q1**

2

(a) A is the point $(3, 6)$ and B is the point $(11, 12)$.

Find the coordinates of the midpoint of AB.   *Answer*   (........... , ...) [1]

(b) C and D have coordinates $(10, 15)$ and $(-8, -21)$.

   (i) Find the equation of the line CD in the form $y = mx + c$.

                                               Answer $y = $ ....[2]

   (ii) Does the point $(-2, -9)$ lie on the line CD?
       Show your working to justify your answer. [1]

(c) The line l has equation $4y = 3x + 15$.

   (i) (a) Find the coordinates of the point where l crosses the x axis.

                                       Answer   (........... , ...) [1]

      (b) Find the coordinates of the point where l intersects the line $y = p$.
      Express each coordinate in terms of p.
                        Answer (................... , ...) [2]
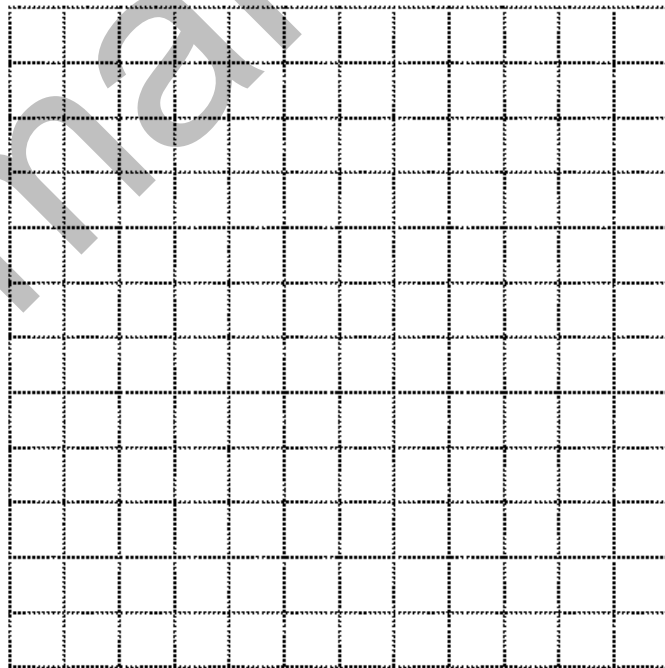
   (ii) The line l is drawn on the grid below.

By drawing the line $3x + 2y = 30$ on the grid, find the coordinates of the point where these two lines intersect.

Answer   (.........., ...)   [2]

3

Q is the point $(-1, 2)$, R is the point $(3, 10)$ and S is the point $(-4, 2)$.

(a) Calculate the length of QR.

Answer ....units [2]

(b) Calculate the value of $\cos S\hat{Q}R$.   *Answer* ....[2]

(c) A point $P(x, y)$ is such that $PQ = PR$.

   (i) Show that $x + 2y = 13$.   [2]

   (ii) P is on the line $y = 7$.

       Find the coordinates of P.   *Answer* (................, ...) [1]

4

(a) The points A and B are $(-2, 1)$ and $(6, -5)$ respectively.

Calculate

   (i) the gradient of the line AB,   [1]

   (ii) the equation of the line through A and B.   [2]

(b) The points C and D are $(4, 5)$ and (p, q) respectively.

   (i) Write down, in terms of p and q, the coordinates of the midpoint of CD.   [1]

   (ii) Given that the midpoint of CD is $(7, 1)$, find the coordinates of D.   [2]

Answers: **(a)(i)** - ¾, **(ii)** $y = -¾\,x - ½$; **(b)(i)** (½ (4 + p), ½(5 + q)), **(ii)** (10, - 3).

5

The points A, B and C are $(9, 8)$, $(12, 4)$ and $(4, -2)$ respectively.

(a) Find

   (i) the gradient of the line through A and B,   [1]

   (ii) the equation of the line through C which is parallel to AB.   [2]

(b) Calculate the length of the line segment

   (i) AB,   [1]

   (ii) BC.   [1]

(c) Show that AB is perpendicular to BC.   [1]

(d) Calculate the area of triangle ABC.   [1]

Answers: **(a)(i)** $-\dfrac{4}{3}$, **(ii)** $4x + 3y = 10$; **(b)(i)** 5 units, **(ii)** 10 units; **(d)** 25 units².

6 The line AB joins the point A $(-2, 1)$ to the point B $(6, 5)$.

(a) Find the coordinates of the midpoint of AB.

Answer (.............. , ...) [1]

(b) Find the gradient of AB.

Answer ....[1]

(c) AB intersects the y-axis at the point $(0, c)$.

Find c.

Answer ....[2]

(d) Express $\overrightarrow{AB}$ as a column vector.

Answer [1]

(e) C is the point $(5, 2)$ and D is the point (h, k).
The lines AB and CD are equal in length and parallel.

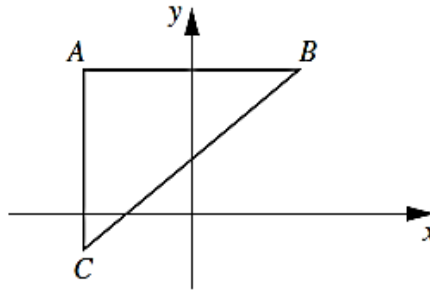Find the coordinates of each of the possible points D.

Answer (.............. , ...) and (.............. , ...) [3]

Answer: (a) (2,3) (b) $\dfrac{4}{8}$ (c) 2 (d) $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ (e) (-3,-2) and (13,6)

Inequalities and Graphs Paper 1

1

The triangle ABC has vertices $A(-3,5)$, $B(3,5)$ and $C(-3,-1)$.

The equation of the line BC is $y = x + 2$.

Write down the three inequalities which define the region **inside** the triangle ABC.

This region **does not include** points on the boundaries. [3]

2

(a) Solve the equation $\dfrac{5}{x+1} = 4$. Answer (a) $x = $ .... [1]

(b) Solve the inequality $7 - y < 9$. (b) y .... [1]

(c) Write down the least integer value of z for which $z > -4$. (c) .... [1]

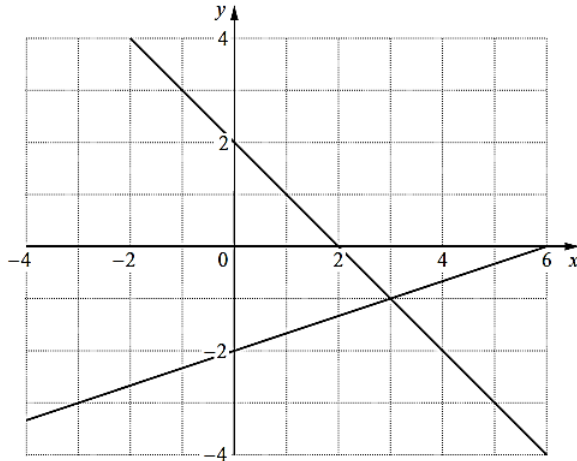*Answers:* (a) $\dfrac{1}{4}$; (b) $y > -2$; (c) -3.

3

The lines $x + y = 2$ and $x - 3y = 6$ are shown on the diagram in the answer space.

(a) Find the gradient of the line $x - 3y = 6$. Answer (a) .... [1]

(b) On the diagram in the answer space, shade the region defined by the inequalities
$x + y \leqslant 2$, $x - 3y \leqslant 6$ and $x + 1 \geqslant 0$.

Answer (b)

[2]

4

$A\,(-4,-1)$ C

In the diagram, A is the point $(-4\,,\,-1)$ and AC is parallel to the x-axis.

(a)   The equation of BC is $y + 2x = 4$.
Find the x-coordinate of C.

(b)   The equation of AB is $y = x + 3$.
Write down the inequalities which describe the region **inside** the triangle ABC.

Answer (a) .... [1]

) .... [2]

5 **(a)**

The set $A = \{x : 1 \leqslant x < 3\}$ is shown on the number line above.

(i) Set B is shown on the number line below.

Complete the description in the answer space.

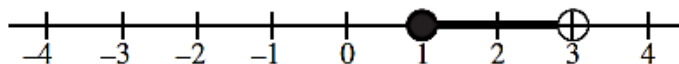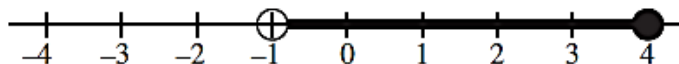*Answer (a)*(i) $B = \{x : \text{..............} x \text{..............}\}$ [1]

(ii) The set $C = \{x : x \leqslant -3\}$.
Illustrate the set C' using the number line in the answer space.

Answer (a)(ii)

[1]

(b) $X = \{1, 3, 5\}$, $Y = \{3, 5\}$, $Z = \{(x, y) : x \in X, y \in Y, x \neq y\}$.
List the members of Z.

Answer (b) $Z = \{$ ....$\}$ [2]

Answers: **(a)(i)** $-1 < x \leqslant 4$; **(b)** $\{(1, 3), (1, 5), (3, 5) \text{ and } (5, 3)\}$.   **J05/1/Q20**

---

6   **(a)**   **(i)**   Find the smallest integer k which satisfies $7k \geqslant 36$.

(ii)   Find the largest integer n which satisfies $3n - 1 < 26$.

Answer (a)(i) smallest $k = $.............................[1]

(ii) largest $n = $.............................[1]

(b)

The diagram shows the graphs of $x = 1$, $y = 3$ and $y = x - 1$.
The region, **R**, is defined by the inequalities $x > 1$, $y < 3$ and $y > x - 1$.
Given that the point (x, y) is in the region **R**, find the integer values of x and y.

Answer (b) $x = $ .... $y = $ ....[1]

7

In the diagram, A is the point $(6, 3)$ and C is the point $(-8, -4)$.
The equation of AB is $y = 3$ and the equation of CB is $y = x + 4$.

(a) Find the coordinates of B.

(b) The unshaded region **R** inside triangle ABC is defined by three inequalities.
     One of these is $y < x + 4$.

Answer (a)   (.................. , ...)   [1]

     Write down the other two inequalities.

(b)...

....[2]

8   The diagram below shows the line $2y = 4 - 3x$.

Answer (a), (b)

On this diagram,

(a) draw the line $y = \frac{1}{2}x - 2$,   [1]

(b) shade and label the region, R, defined by the following inequalities.

$$x \geqslant 0 \qquad\qquad 2y \leqslant 4 - 3x \qquad\qquad y \geqslant \tfrac{1}{2}x - 2$$

[2]

<div style="text-align: right;">J08/1/Q16</div>

9   (a) Solve $8 - 3t > 14 + t$.

   (b) Evaluate $x^2 - 6xy + 2y^2$ when $x = 2$ and $y = -3$.

<div style="text-align: right;">Answer (a) t ....[2]</div>

<div style="text-align: right;">(b) ....[2]</div>

Answers: **(a)** $t < -1.5$ **(b)** 58   J08/1/Q21

10

17 (a) Solve $\dfrac{3x-2}{5}=\dfrac{x}{3}$.

Answer $(a)\, x =$ .... [2]

(b) Given that y is an integer and $-3 < 2y - 6 < 4$, list the possible values of y.

Answer (b) .... [2]

Answer. **(a)** 1.5   **(b)** 2, 3, 4

11

(a) Solve $6x - 5 < 9 + 2x$.

Answer (a) .... [1]

(b) Write down the largest integer which satisfies the inequality
$6x - 5 < 9 + 2x$. *Answer* (b) .... [1]

Answer. **(a)** $x < 3.5$ **(b)** 3

12

In the diagram, the region, **R**, is bounded by the lines AB, BC, CD and DA.

(a) Write down the coordinates of the midpoint of AB.

Answer   [1] (a) (............ , ...)

(b) Region **R** is defined by four inequalities.   *Answer* (b) ....
One of these is $y \leqslant x + 5$.

....

Write down the other three inequalities.

.... [2]

Answer. **(a)** (1.5, 6.5)   **(b)** $x \geq 0, y \geq 4, x \leq 3$

13 In the diagram below, the equation of the line *AB* is $2y = 9 - 3x$.

(a) What is the gradient of *AB* ?

Answer .... [1]

(b) The shaded region **R** is defined by five inequalities. Two of these are $x \leqslant 5$ and $y \leqslant 5$.

Write down the other three inequalities.

Answer ....

....

.... [2]

J11/11/Q11

14 Solve

(a) $3x \geqslant 2 - 5x$,

Answer x .... [1]

(b) $\dfrac{5x}{x+1} - \dfrac{2}{x-1} = 8$.

Answer x = ... or .... [3]

J11/11/Q16

15 (a) Solve $5y - 3 > 3y + 12$.

Answer y .... [1]

(b) Write down all the integers that satisfy the inequality $-6 \leqslant 3x < 6$.

Answer .... [1]

16

The shaded region on the diagram is represented by three inequalities.

One of these is $y \geqslant 3x - 2$.

Write down the other two inequalities.

Answer ....

.... [2]

17

The diagram shows a triangle *RST*.

(a) Write down

(i) the gradient of the line *ST*,   *Answer* ....[1]

(ii) the equation of a line that is parallel to *ST*,   *Answer* ....[1]

(iii) the equation of the line with gradient 3 that passes through *S*.

                                          Answer ....[1]

(b) One of the inequalities that defines the shaded region *RST* is $x \leqslant 6$.

Write down the other two inequalities that define this region.

                                     Answer ....

                                                       ....[2]

Answer: (a)(i) $\dfrac{4}{6}$ (ii) e.g. $y = \dfrac{4}{6}x + 3$ (iii) $y = 3x + 2$ (b) $y \geqslant 2$ and $y \leqslant \dfrac{4}{6}x + 2$ **J13/11/Q23**

18

(a) Solve $4 \leqslant 3y - 11$.   *Answer* y ....[1]

Write down all the integers that satisfy the inequality $-4 \leqslant 2x < 4$.

                                Answer ....[1]

Answers: (a) $\geqslant 5$ (b) $-2, -1, 0, 1$ **J14/11/Q5**

19

Find the integers *n* that satisfy $20 < 4n - 3 < 30$.   *Answer* ....[2]

Answer: 6, 7, 8   **J15/11/Q8**

20

503

(a) On the grid, shade the region, R, given by these inequalities.

$$1 \leqslant x \leqslant 5$$
$$2 \leqslant y \leqslant 4$$

[2]

(b) The line L, with equation $y = \frac{1}{3}x$, is drawn on the grid.

(i) Draw the line $y = \frac{1}{3}x + k$ so that it passes through a point belonging to R such that k is as large as possible. [1]

(ii) Write down this largest value of k. *Answer* $k=$ .... [1]

21

The shaded region in the diagram is defined by three inequalities.

One of these is $y \geqslant \frac{1}{2}x + 2$. *Answer* ....

Write down the other two inequalities. .... [2]

22 **(a)** Solve the inequality $-5 < 2x + 3 < 1$.

(b) Write down the largest integer, x, which satisfies $-5 < 2x + 3 < 1$.

Answer (a) ....$< x <$ .... [2]

(b) .... [1]

23 The region R is defined by 4 inequalities.
Two of these are $x \leqslant 4$ and $x + y \geqslant 5$.
Write down the other two inequalities.

N02/1/Q13

24 Three lines, l_1, l_2 and l_3, are drawn on the diagram in the answer space.
The equation of the line l_1 is $y = x + 5$.
The equation of the line l_2 is $3y + x = -3$.

(a) Use the diagram to solve the simultaneous equations

$$y = x + 5 \text{ and } 3y + x = -3 .$$

Answer (a) $x =$ ...., $y =$ .... [2]

(b) Write down the equation of the line l_3.

Answer (b) .... [1]

(c) The equation of another line, l_4, is $y = -1$.
Draw this line on the diagram in the answer space.

Answer (c)

[1]

(d) The region enclosed by these four lines is defined by four inequalities.
One of these is $3y + x \geqslant -3$.
Write down the other three inequalities.   *Answer (d)* ....

.... [2]

Answers: (a) $x = -4\frac{1}{2}$, $y = \frac{1}{2}$; (b) $x + y = 4$; (d) y $x + 5, y$ $-1, x + y$ 4.

---

25   (a) During one week the temperatures at midnight were

$$3\,°C,\ 4.5\,°C,\ 1\,°C,\ -2\,°C,\ 0\,°C,\ -6.5\,°C,\ -3.5\,°C.$$

Find the difference between the highest and lowest temperatures.

(b) Find all the integers which satisfy both

$$2x + 7 < 3 \quad \text{and} \quad x \geqslant -4.$$

Answer (a) ....°C [1]

(b) .... [1]

Answers: (a) 11°C; (b) −3 and −4.

26 The unshaded region **R** is defined by 3 inequalities.
One of these is $x \geqslant 1$.
Write down the other two inequalities.

Answer ....

.... [3]

Answer: $y \geqslant -3$ and $x + y \leqslant 5$.

27 (a) In the diagram, the unshaded region, **R**, is defined by three inequalities.

Two of these are

$y \leqslant 2x + 2$ and $y \leqslant 5 - x$.

Write down the third inequality.

(b) Find the integer values of x which satisfy the following.

$$4 \leqslant 2x + 13 < 9$$

Answer (b).. [2]

Answers: $y \geqslant \dfrac{1}{2}x$; **(b)** −4 and −3.

28

(a) Solve $-7 \leqslant 3x - 4 < 2.$ *Answer (a)* .... $\leqslant x <$ ....[2]

(b) Write down all the integers which satisfy $-7 \leqslant 3x - 4 < 2.$

(b) ....[1]

Answer: **(a)** −1 ($\leqslant x <$) 2; **(b)** −1, 0, 1.

29 The line ℓ is drawn on the grid in the answer space.

(a) Write down the equation of the line ℓ.

(b) On the grid,

    (i) draw and label the lines $x = 1$, $y = 3$ and $x + y = 2$,

    (ii) shade the region which satisfies the three inequalities

$$x \geqslant 1, \ y \leqslant 3 \ \text{and} \ x + y \geqslant 2.$$

Answer (a) ....[1]

Answer (b)

[3]

Answer: (a) $y = 2x + 3$.

30 The shaded region inside the triangle ABC is defined by three inequalities.

One of these is $x + y < 5\frac{1}{2}$.

(a) Write down the other two inequalities. *Answer (a)* ....

....[2]

(b) How many points, with integer coordinates, lie in the shaded region?

Answer (b) ....[1]

Answer: (a) $y > 1$, $y < 2x$ (b) 3   **N07/1/Q8**

31

(a) Solve the inequality $3 - 2x < 5$.   *Answer (a) x* ....[2]

(b) Solve the equation $3(y + 2) = 2(2y - 7) + y$.

Answer (b) y = ...[2]

Answers: (a) $x > -1$,   (b) $y = 10$.   **N08/1/Q16**

32

The equation of a line ℓ is $x + 2y = -1$.

(a) Write down the gradient of the line ℓ.   *Answer (a)* ....[1]

(b) Find the equation of the line parallel to ℓ that passes through the point $(0,5)$.

   (b) ....[2]

(c) The diagram in the answer space shows the line ℓ and the line $y = 2x + 1$.
   On this diagram,

   (i) draw the line $y = -2$,

   (ii) shade and label the region, R, defined by the three inequalities

   $$y \geqslant -2 \qquad x + 2y \leqslant -1 \qquad y \geqslant 2x + 1.$$

   Answer (c)(i)(ii)

[2]

33

(a) Express $\dfrac{2m}{5} + \dfrac{m}{4}$ as a single fraction in its simplest terms.

   Answer (a) ....[1]

(b) Solve the inequality $5(x + 4) < 7x$.   *Answer (b)* x.......................................[2]

34   The diagram below shows triangle *LMN*.

The equations of the lines *LM* and *LN* are $2y = 3x + 5$ and $x + 4y = 24$ respectively.

(a)   Solve the simultaneous equations $x + 4y = 24$,
                                              $2y = 3x + 5$.

Hence write down the coordinates of *L*.   *Answer* (a)   (............ , ...)   [3]

(b)   *M* is $(-3, -2)$ and *MN* is parallel to the *x*-axis.
The shaded region, *R*, inside triangle *LMN*, is defined by three inequalities.
One of these is $2y < 3x + 5$.
Write down the other two inequalities.   *Answer* (b) ....

...................................[2]

35
(a) Solve the inequality $18 - 3x < x$.   *Answer* (a) x ....[1]

(b) Given that *n* is an integer, where $-10 \leqslant 3n < -3$, find the possible values of *n*.

Answer (b) $n =$ ....[1]

36 The lines $3y + x = 3$ and $y = x + 3$ are shown in the diagram below.

(a) Find the gradient of the line $3y + x = 3$.   *Answer* (a) ....[1]

(b) On the diagram shade, and label with the letter R, the region defined by the inequalities

$$3y + x \geqslant 3, \quad y \leqslant x + 3, \quad x \geqslant 0.$$

Answer (b)

[2]

Answers: (a) -⅓ (b) Correct region shaded   **N10/11/Q16**

37
The three lines
$3x = 7$,
$2y = 5$ and
$4x + 4y = 35$
intersect to form the triangle ABC,
as shown in the diagram.

The region **inside** the triangle ABC
is defined by three inequalities.
One of these is $2y > 5$.

(a) Write down the other two inequalities.   ....[2]

(b) Find the point, with integer coordinates, that lies **inside** the triangle ABC and is closest to B.

Answer (b)   (..........., ...) [1]

Answer: (a) $3x > 7$, $4x + 4y < 35$ (b) (5, 3)   **N10/12/Q20**

513

38   It is given that $13 < 7 - 2x < 18$ has the solution $a < x < b$.

Find the values of a and b.

Answer $a =$ ....

$b =$ .... [2]

N11/11/Q9

39

In the diagram, B is the point $(8, 2)$.
The equation of the line AB is $y = 2$ and the equation of the line AC is $2x - y = 3$.
BC produced passes through the origin.

(a) AC produced intersects the y-axis at D.

Find the coordinates of D.   *Answer*   (.......... , ...) [1]

(b) The region inside triangle ABC is defined by three inequalities.
One of these is $y < 2$.

Answer   ....

Find the other two inequalities.

.... [2]

N11/11/Q17

40   **(a)** Solve the inequality $2(4 - x) < x - 10$.   *Answer* x .... [1]

(b) Find the smallest integer n such that $3n > -17$.   *Answer* $n =$ ....[1]

N11/12/Q6

41 The diagram shows the graphs of

$x + y = 12\frac{1}{2}$,

$y = \frac{x}{4}$ and

$x = 2$.

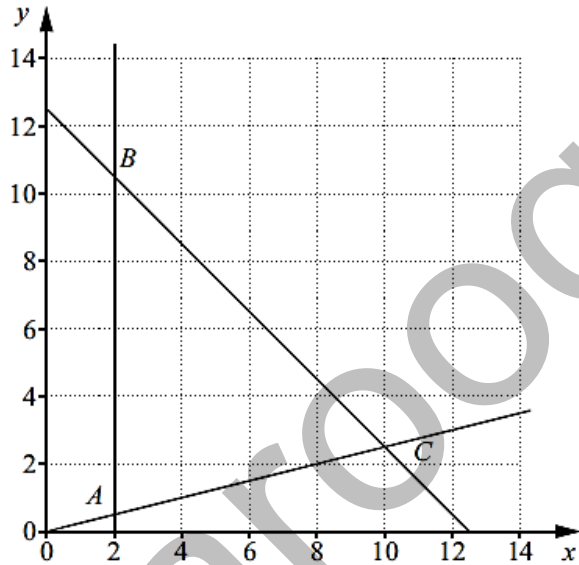These graphs intersect to form triangle ABC.

The region inside triangle ABC is defined by three inequalities.

One of these is $y > \frac{x}{4}$.

(a) Write down the other two inequalities.

Answer ....

.... [2]

(b) $P = \{(x, y) : x \text{ and } y \text{ are integers, } (x, y) \text{ lies inside triangle } ABC\}$
$Q = \{(7, y) : y \text{ is an integer}\}$

(i) Find the member of the set P that is closest to the point C.

Answer .... [1]

(ii) Find $n(P \quad Q)$.

Answer .... [1]

Answer: (a) $x > 2$, $x + y < 12\frac{1}{2}$ (b)(i) (9, 3) (ii) 4   **N11/12/Q25**

---

42 The quadrilateral $ABCD$ is bounded by the lines
$x = 1$, $y = 2$, $2y = x$ and $x + y = 9$.

The region inside the quadrilateral is defined by four inequalities.
Two of these are $y > 2$ and $2y > x$.

(a) Write down the other two inequalities.

Answer ....

.... [2]

(b) How many points, with **integer** coordinates, lie inside the quadrilateral $ABCD$?

Answer .... [1]

Answers: (a) $x > 1$, $x + y < 9$ (b) 10   **N12/11/Q17**

515

43 Find two solutions of the inequality $3x + 4 < 11$ that lie **between** 2 and 3.

Answer $x =$ .... and .... [2]

44 The diagram shows the points $A(1, 4)$, $B(3, 12)$ and $C(15, 4)$.
The equation of the line through B and C is $2x + 3y = 42$.

The region **inside** triangle ABC is defined by three inequalities.
One of these is $2x + 3y < 42$.

(a) Write down the other two inequalities.   *Answer*   ....

....[2]

(b) How many points, with coordinates $(10, k)$, where k is an **integer**, lie inside the
triangle ABC?

Answer   ....[1]

(a) On the grid above, draw the graph of $x + y = 6$. [1]

(b) On the grid above, draw the graph of $2y + x = 4$. [1]

(c) On the grid above, shade and label the region **R**, defined by the following inequalities.

$x + y \leqslant 6$ $2y + x \geqslant 4$ $y \geqslant 2$ $x \geqslant -1$ [2]

Answers: **(a)** $x + y = 6$ drawn correctly; **(b)** $2y + x = 4$ drawn correctly; **(c)** Correct region shaded.   **N14/11/Q16**

46 Here is a list of numbers.

$$-8 \quad -5 \quad -3 \quad -2 \quad 0 \quad 2 \quad 4 \quad 9$$

(a) Write down two numbers from the list that have a difference of 10.

Answer .... and .... [1]

(b) Find the sum of the numbers in the list. *Answer* .... [1]

(c) It is given that $-4 \leqslant 2x \leqslant 7$.

Write down all the numbers from the list which satisfy this inequality.

Answer .... [1]

Answers: **(a)** –8 and 2 **(b)** –3 **(c)** –2, 0, 2   **N15/11/Q10**

47

A is the point $(5, 2)$ and B is the point $(9, 6)$.
AC is parallel to the x-axis. CB is parallel to the y-axis.
The equation of the line AB is $x - y = 3$.

(a) Find the coordinates of C.   *Answer* (.................. , ...) [1]

(b) The region **inside** triangle ABC is defined by three inequalities.

Write down these inequalities.   .... [3]

(c) The point (a, b), where a and b are integers, lies **inside** triangle ABC.

It also lies on the line $y = \frac{1}{2}x$.   *Answer* $a = $ ....

Find the value of a and the value of b. $b = $ .... [2]

Answers: **(a)** (9, 2) **(b)** $x < 9$, $y > 2$, $x - y > 3$ **(c)** $a = 8$, $b = 4$ **N16/11/Q24**

518

48 (a) Solve the inequality $12 - 2x < x$.

Answer x .... [1]

(b) Find the integer values of n that satisfy $-8 < 2n \leqslant -4$. Answer .... [1]

Answers: (a) $x > 4$ (b) -3 and -2 **N17/11/Q9**

49

The diagram shows the triangle ABC formed by the lines

$$y = 6, \quad x = 23 \quad \text{and} \quad y = \frac{x}{3}.$$

(a) Find the coordinates of A.

Answer (................. , ...) [1]

(b) The region **inside** the triangle is defined by three inequalities.
One of these is $x < 23$.

Answer ....

Write down the other two inequalities.

.... [1]

(c) The point $P(h, k)$, where h and k are integers, lies **inside** triangle ABC.

Find the values of h and k.

Answer $h =$ ....

$k =$ .... [2]

Answers: (a) $(18, 6)$ (b) $y > 6$ and $y < \dfrac{x}{3}$ (c) $h = 22$ and $k = 7$ **N17/11/Q21**

Inequalities Paper 2

1

The shaded region, **R**, contained **inside** the dotted boundary lines, is defined by three inequalities.

(a) One of these inequalities is $x > 2$.

Write down the other two inequalities. [3]

(b) The points (c, d), where c and d are integers, lie in the shaded region **R**.

Find

   (i) the maximum value of $c + d$, [1]

   (ii) the value of d given that $d = 3c$. [1]

Answers: (a) $y < 12$ and $y > 2x$,   (b) (i) 16,   (ii) 9.   **J10/22/Q4**

2

Triangle R has vertices $(-2, -1)$, $(2, 1)$ and $(4, -1)$.

(a) The gradients of the sides of triangle R are 0, -1 and k.

Find k. Answer ....[1]

(b) One of the inequalities that defines the shaded region is $x + y \leqslant 3$.

Write down the other two inequalities that define this region.

Answer ....

....[2]

(c) Triangle R is mapped onto triangle P by a reflection in the line $y = -2$.

Draw and label triangle P. [2]

Answers: **(a)** $\frac{1}{2}$ **(b)** $y \leqslant \frac{x}{2}$, $y \geqslant -1$ **(d)(i)** 2 **(ii)** $(8, -1)$ **(iii)** 12   **J12/21/Q6**

521

3

(a) Factorise completely $6x^2y^3 - 15x^3y$.

    Answer ....[2]

(b) Solve $\dfrac{4}{x} + \dfrac{2}{x+2} = 3$.

    Answer $x = $ .... or .... [3]

(c) (i) Shade and **label** the region R defined by these four inequalities.

        $x \geqslant 1$ $y \leqslant 4$ $x + y \leqslant 6$ $y \geqslant x$

    Answer

                                                                                                         [3]

(ii) The point M is the intersection of $x = 1$ and $y = 4$.
     The point N is the intersection of $x + y = 6$ and $y = x$.

    Find the gradient of MN.

    Answer ....[2]

Answers: (a) $3x^2y(2y^2 - 5x)$; (b) $x = \pm 1.63$; (c)(i) Correct region identified; (ii) $-\dfrac{1}{2}$ **N15/21/Q5**

Functions Paper 1

1 It is given that $f : x \mapsto m + nx$, where m and n are constants.
Given also that $f(0) = 1$ and $f(4) = 21$, find the value of

(a) m,

Answer (a) $m = $ .... [1]

(b) n,

(b) $n = $ .... [1]

(c) $f^{-1}(21)$.

(c) $f^{-1}(21) = $ .... [1]

2 A function is defined by $f(x) = 3x + 4$.

(a) Given that $f(k) = k$, find k.

Answer (a) $k = $ .... [2]

(b) Find the inverse of f.

(b) $f^{-1}(x) = $ .... [2]

Answers: (a) -2; (b) $\dfrac{x-4}{3}$.

3 A function f is defined by $f : x \mapsto \dfrac{x+5}{3}$.

(a) Given that $f : 1 \mapsto k$, find the value of k.

(b) Given also that $f^{-1} : x \mapsto cx + d$, find the value of c and the value of d.

Answer (a) $k = $ .... [1]

(b) $c = $ .... $d = $ .... [2]

Answers: (a) 2; (b) $c = 3$, $d = -5$.

4 $f(x) = \dfrac{2x - 1}{3}$.

Find an expression for $f^{-1}(x)$.

Answer: $\dfrac{3x + 1}{2}$.

5

(a) f(x) = (x + 2)(2x − 1).
Evaluate f(5.5).

Answer (a) f(5.5) = ...[1]

(b) g(x) = $\frac{1}{3}$ (2x − 1).
Find g^{-1} (5).

(b) g^{-1}(5) = ...[2]

Answer: (a) 75   (b) 8   J06/1/Q14

6

Given that f(x) = $\frac{5x - 4}{3}$, find

(a) f($1\frac{1}{5}$),

Answer (a) ....[1]

(b) f^{-1}(x).

(b) ....[2]

Answer: (a) $\frac{2}{3}$ (b) $\frac{3x + 4}{5}$ J07/1/Q16

7

It is given that f(x) = 5x + 2.

Find

(a) f(−2),

Answer (a) ....[1]

(b) f^{-1}(x).

(b) f^{-1}(x) = ...[1]

Answer: (a) 12   (b) 26   J07/1/Q3

8

It is given that f(x) = 12 − 5x.

Find

(a) f(4),

Answer (a) f(4) = ...[1]

(b) the value of x for which f(x) = 17,

Answer (b) x = ...[1]

(c) f^{-1}(x).

Answer (c) f^{-1}(x) = ...[2]

Answer: (a) − 8   (b) − 1   (c) $\frac{12 - x}{5}$ J09/1/Q16

9

Given that f(x) = $\frac{5 - 2x}{3x}$, find

(a) f(−2),

Answer (a) f(−2) = ...[1]

(b) f^{-1}(x).

Answer (b) f^{-1}(x) = ...[2]

J10/12/Q11

Answer: (a) − 1.5   (b) $\frac{5}{3x + 2}$

10 It is given that $h(x) = 2x - 5$ and $g(x) = \dfrac{3}{x-2}$.

Find

(a) h(4), *Answer* .... [1]

(b) $g^{-1}(x)$, *Answer* .... [2]

(c) the value of *t* such that $h(t) = g(3)$. *Answer* $t = $ .... [2]

Answers: (a) 3 (b) $\dfrac{3+2x}{x}$ (c) 4

11 $$f(x) = 6x^2 - x + 3$$

(a) Find

(i) f(2), *Answer* f(2) = ... [1]

(ii) f(–1), *Answer* f(–1) = ... [1]

(iii) the values of *x* for which $f(x) = 5$. *Answer* $x = $ .... or ....[2]

(b) Write down and simplify an expression for $f(a + 1)$. *Answer* f(a + 1) = ...[2]

Answer. (a) (i) 25 (ii) 10 (iii) $\dfrac{2}{3}, -\dfrac{1}{2}$ (b) $6a^2 + 11a + 8$

12 $$f(x) = 2x - 9$$

(a) Find $f\left(-\dfrac{3}{4}\right)$. *Answer* .... [1]

(b) Find $f^{-1}(3)$. *Answer* .... [2]

Answers: (a) $-10\dfrac{1}{2}$ (b) 6

13 Given that $f(x) = \dfrac{2x+5}{x}$, find

(a) $f(2\tfrac{1}{2})$, *Answer (a)* $f(2\tfrac{1}{2})$ = ...[1]

(b) $f^{-1}(x)$. *(b)* $f^{-1}(x)$ = ... [2]

Answers: (a) 4; (b) $\dfrac{5}{x-2}$

14

Given that $f(x) = 3x + 7$, find

(a) $f(-2)$,

(b) $f^{-1}(x)$,

(c) The value of x for which

$$f(x) = f^{-1}(x).$$

Answers: (a) 1; (b) $\frac{1}{3}(x - 7)$; (c) -3 ½.

15

It is given that $f(x) = \dfrac{3}{2x - 5}$.

Find

(a) $f(2)$,

Answer (a) $f(2)$ = ... [1]

(b) $f^{-1}(\frac{1}{2})$.

(b) $f^{-1}(\frac{1}{2})$ = ... [2]

Answers: (a) -3; (b) $5\frac{1}{2}$.

16

(a) Given that $f(x) = 3x + 5$, find $f(3)$.

(b) The function g is defined by $g(x) = (2x - 3)(x + k)$.

Given that $g(0) = -15$, find

(i) k,

(ii) x such that $g(x) = 0$.

Answer (a) $f(3)$ = ... [1]

(b) (i) k = ... [1]

(ii) x = ... [2]

Answers: (a) 14; (b)(i) 5, (ii) $1\frac{1}{2}$, -5.

17 It is given that $f(x) = 3x - 5$.

Find

(a) $f(-4)$,

(b)   the value of t, given that $f(t) = 10$,

(c) $f^{-1}(x)$,

(d) $f^{-1}(4)$.

Answer (a) $f(-4)$ = ...[1]

(b) t = ...[1]

(c) $f^{-1}(x)$ = ...[1]

(d) $f^{-1}(4)$ = ... [1]

Answers: **(a)** −17; **(b)** 5; **(c)** $\frac{1}{3}(x+5)$; **(d)** 3.

N05/1/Q16

---

18 (a) Given that $f(x) = x^2 - 2px + 3$, find

  (i) $f(-2)$, giving your answer in terms of p,

  (ii) the value of p when $f(-2) = f(0)$.

(b) Given that $g(y) = y^2 - 1$, find $g(a - 1)$. Give your answer in its simplest form.

Answer (a) (i) $f(-2)$ = ...[1]

(ii) p = ...[1]

(b) $g(a-1)$ = ...[2]

Answer: **(a)(i)** $4p + 7$,   **(ii)** −1;   **(b)** $a^2 - 2a$ or equivalent.

N06/1/Q16

---

19 It is given that $f(x) = \dfrac{3 - x}{2}$.

Find

(a) $f(-9)$,

(b) $f^{-1}(x)$.

Answer (a) ....[1]

(b) $f^{-1}(x)$ = ...[1]

Answer: **(a)** 6   **(b)** $3 - 2x$

N07/1/Q6

---

20 Given that $f(x) = \dfrac{4x + 3}{2x}$, find

(a) $f(3)$,

(b) $f^{-1}(x)$.

Answer (a) $f(3)$ = ...[1]

(b) $f^{-1}(x)$ = ...[2]

Answers: **(a)** $2\frac{1}{2}$,   **(b)** $\dfrac{3}{2x - 4}$.

N08/1/Q12

21 Given that f(x) = 4x − 7, find

(a) $f\left(\frac{1}{2}\right)$,

Answer (a) $f\left(\frac{1}{2}\right)$ = ...[1]

(b) the value of p when f(p) = p.

Answer (b) p = ...[2]

Answers: (a) -5　(b) $2\frac{1}{3}$

22 $f(x) = 6 - \frac{x}{2}$

(a) Find f(5).

Answer (a) ....[1]

(b) Find $f^{-1}(x)$.

Answer (b) $f^{-1}(x)$ = ...[2]

Answer. (a) $3\frac{1}{2}$　(b) 12 − 2x

23 Given that $f(x) = \frac{2x+3}{5x}$, find $f^{-1}(x)$.

Answer $f^{-1}(x)$ = ... [2]

Answer. $\frac{3}{5x-2}$

24 It is given that $f(x) = \frac{3+x}{2}$.

(a) Find f(−3).

Answer .... [1]

(b) Find $f^{-1}(x)$.

Answer $f^{-1}(x)$ = ... [1]

Answer. (a) 0　(b) 2x - 3

25 $f(x) = \frac{x+3}{2}$

(a) Find $f^{-1}(x)$.

Answer $f^{-1}(x)$ = ... [1]

(b) Given that f(−9) + f(t) = A + Bt , find the values of A and B.

Answer A = ...

B = ...[2]

Answers: (a) 2x − 3 (b) A = $-\frac{3}{2}$　B = $\frac{1}{2}$

26

$$f(x) = \frac{7 - 3x}{2x}$$

(a) Find f(4).

Answer ....[1]

(b) Find $f^{-1}(x)$.

Answer $f^{-1}(x) =$ ....[2]

Answers: (a) $-\dfrac{5}{8}$ (b) $\dfrac{7}{2x+3}$

27

$$f(x) = 5 + x^2$$

Find t given that $f(3 - t) = 9$.

Answer $t =$ .... or .... [3]

Answer: 1 and 5

28

$$f(x) = \frac{3 - x}{10}$$

(a) Evaluate $f(-\frac{1}{2})$.

Answer .... [1]

(b) Find $f^{-1}(x)$.

Answer $f^{-1}(x) =$ .... [2]

Answers: (a) 0.35 (b) $3 - 10x$

29

$$f(x) = 3x + 7$$

(a) Find f(3.2).

Answer .... [1]

(b) Find $f^{-1}(x)$.

Answer $f^{-1}(x) =$ .... [1]

Answers: (a) 16.6 (b) $\dfrac{x - 7}{3}$

Functions Paper 2

1 A function is defined by $f(x) = \dfrac{x-2}{5}$.

(a) Find f(7). [1]

(b) Given that f(t) = t, find t. [2]

(c) Find $f^{-1}(x)$. [2]

Answers: (a) 1 (b) $t = -\dfrac{1}{2}$ (c) 5x + 2

---

2 (b) A function is defined by $f(x) = \dfrac{2x-3}{4}$.

(i) Find f(2). *Answer* ....[1]

(ii) Given that $f^{-1}(x) = cx + d$, find the values of c and d.

Answer c = ... d = ... [2]

(iii) Given that f(g) = −g, find the value of g. *Answer* ....[2]

(b)(i) 0.25 (ii) c = 2, d = 1.5 (iii) 0.5

---

3 (a) $$f(x) = \dfrac{4x-3}{2}$$

Find

(i) f(−2), *Answer* f(−2) = ... [1]

(ii) $f^{-1}(x)$, *Answer* $f^{-1}(x)$ = ... [2]

(iii) the value of g such that f(2g) = g. *Answer* g = ... [2]

Answer: (a)(i) −5.5 (ii) $\dfrac{2x+3}{4}$ (iii) 0.5

---

4 $$f(x) = 2x - 7$$

(a) Calculate f(3). *Answer* f(3) = ... [1]

(b) Find $f^{-1}(x)$. *Answer* $f^{-1}(x)$ = ... [2]

(c) Find the value of g given that f(3g) = g + 4 . *Answer* g = ... [3]

Answers: (a) −1; (b) $\dfrac{x+7}{2}$; (c) 2.2.

Transformations Paper 1

1   *P* is the point (1, 1) and *Q* is the point (5, –2).

(a) A translation maps *P* onto *Q*.
Write down the column vector which represents this translation. $\begin{pmatrix} \\ \end{pmatrix}$ [1]
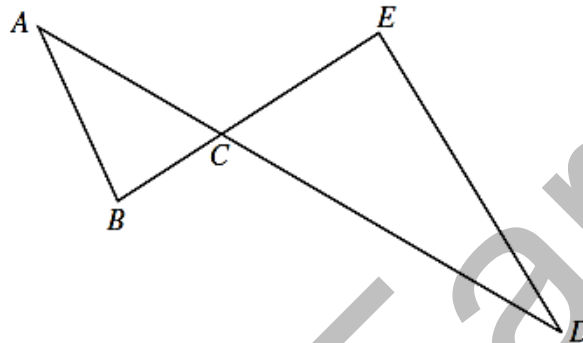
(b) Find the coordinates of the midpoint of *PQ*.   *(b)* (..............., ...) [1]

2

In the diagram, *ACD* and *BCE* are straight lines.

$$\frac{CB}{CE} = \frac{CA}{CD} = \frac{1}{2}.$$

(a) Describe fully the **single** transformation that maps $\triangle CAB$ onto $\triangle CDE$.

(b) It is given that $\overrightarrow{ED} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.
Calculate $\overrightarrow{AE}$.

Answer (a) .... [2]

(b) $\overrightarrow{AE} =$ $\begin{pmatrix} \\ \end{pmatrix}$ [2]

Answers: **(a)** Enlargement, centre *C*, scale factor –2; **(b)** $\begin{pmatrix} 12 \\ -1 \end{pmatrix}$.

3   (a) Under the transformation T, the origin is invariant.
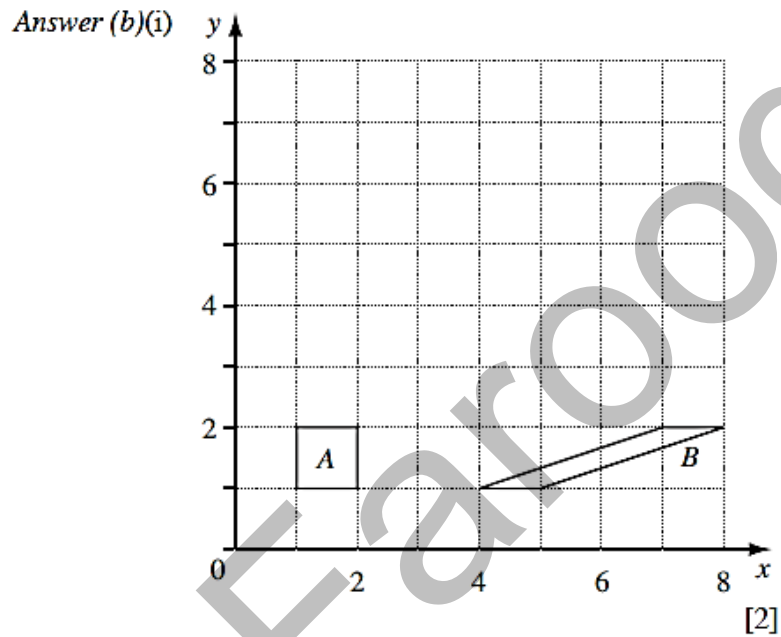T maps (1, 0) onto (2, 0) and (0, 1) onto (0, 2).

(i) Find the matrix that represents T.

(ii) Describe, fully, the **single** transformation T.

Answer (a)(i) $\begin{pmatrix} \\ \end{pmatrix}$ [1]

Answer (a)(ii) ....[1]

(b)   The diagram shows shapes *A* and *B*.

Answer (b)(i)

[2]

(i)   Shape *B* is mapped onto shape *C* by a rotation, centre (8, 3), through 90° clockwise.
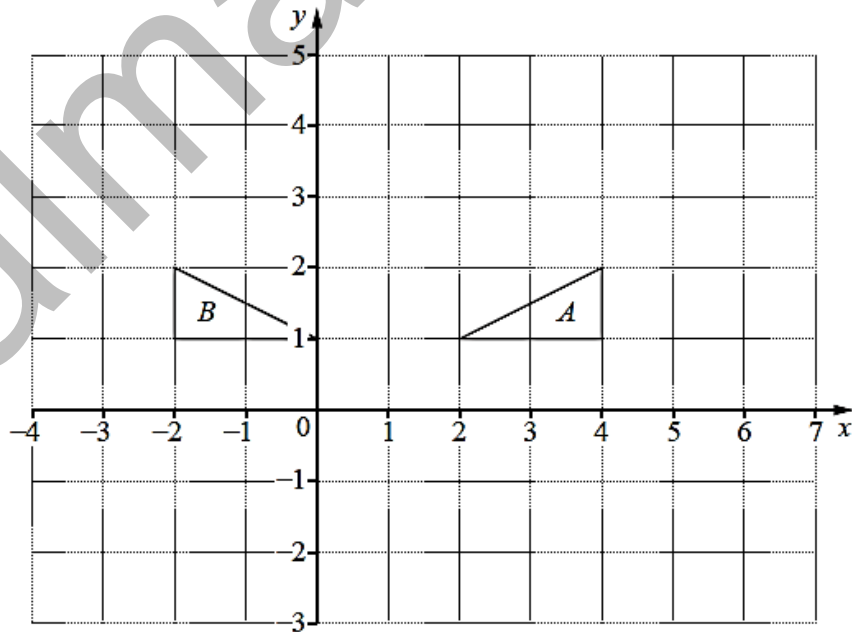   Draw shape *C* on the diagram.

Answer: **(a)(i)** $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ **(ii)** Enlargement, centre (0 , 0), scale factor 2   **J06/1/Q24**

(b)(i)   Vertices at (6 , 6), (6 , 7), (7 , 4) and (7 , 3)   **(ii)** Shear, factor 3, *x*-axis invariant

4

The diagram shows triangles A and B.

(a) The translation $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ maps $\triangle A$ onto $\triangle C$.

On the diagram, draw and label $\triangle C$. [1]

(b) The rotation 90° clockwise, centre (2, 0), maps $\triangle A$ onto $\triangle D$.

On the diagram, draw and label $\triangle D$. [2]

(c) Describe **fully** the **single** transformation which maps $\triangle A$ onto $\triangle B$. .[2]
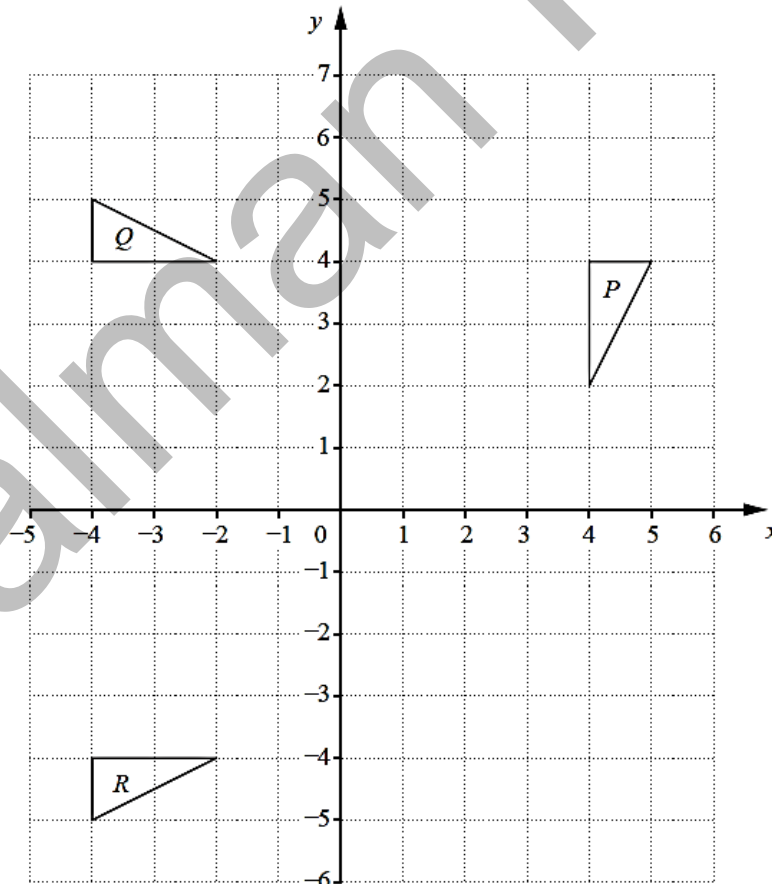

Answer. (a) Triangle C with vertices (–1,3), (1,3) and (1,4)   (b) Triangle D with vertices (3,0), (3,–2)   **J09/1/Q20**
and (4,–2)   (c) Reflection in the line $x = 1$

---

5   The diagram below shows three triangles, P, Q and R.

(a) Triangle T is the image of triangle P under an enlargement with centre (5, 2) and scale factor 2.

Draw and label triangle T on the diagram.

Answer (a)

534

(b) Describe fully the **single** transformation that maps triangle P onto triangle Q.
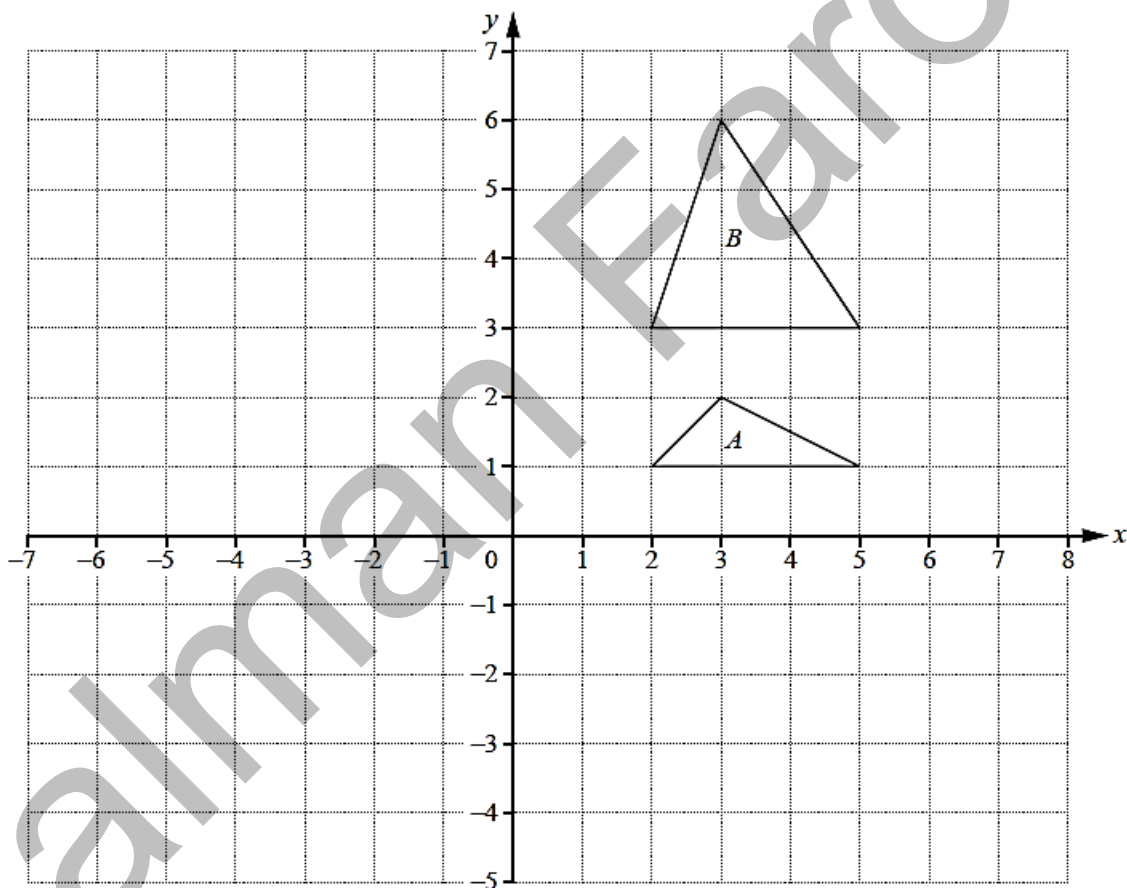
Answer (b) ....

.... [2]

(c) Find the matrix representing the transformation that maps triangle Q onto triangle R.

Answer *(c)* $\left(\quad\quad\right)$ [1]

6

The diagram shows triangles A and B.

(a) The translation $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ maps triangle A onto triangle C.

On the diagram, draw and label triangle C. [1]

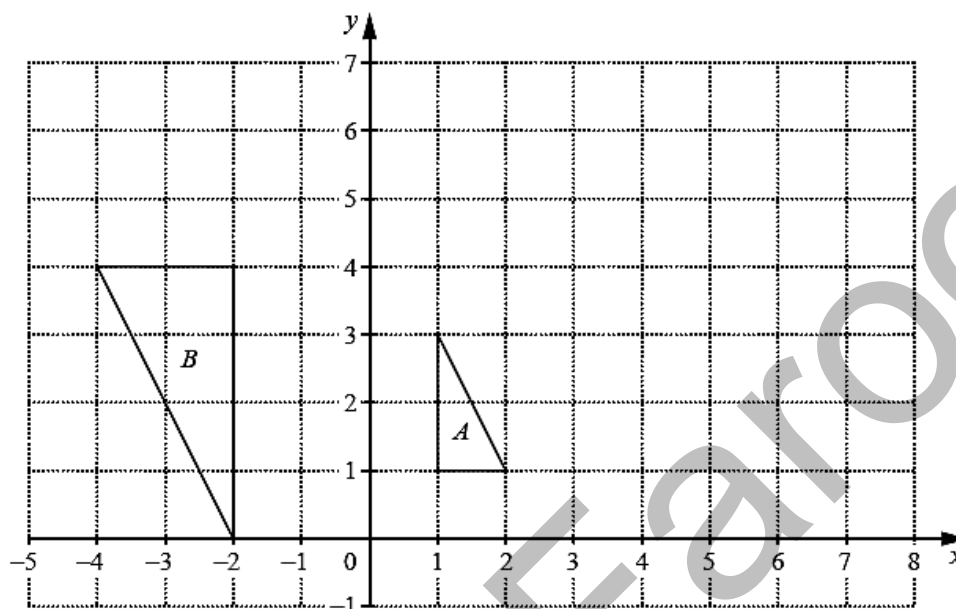**(b)** The rotation $90°$ clockwise, centre $(1, 1)$, maps triangle A onto triangle D.

On the diagram, draw and label triangle D. [2]

7

The diagram shows triangles *A* and *B*.

(a) Triangle *A* is mapped onto triangle *B* by an enlargement.

Find the scale factor, and the centre, of this enlargement.

Answer scale factor = ... centre = ... [2]

8 The diagram shows triangle A.
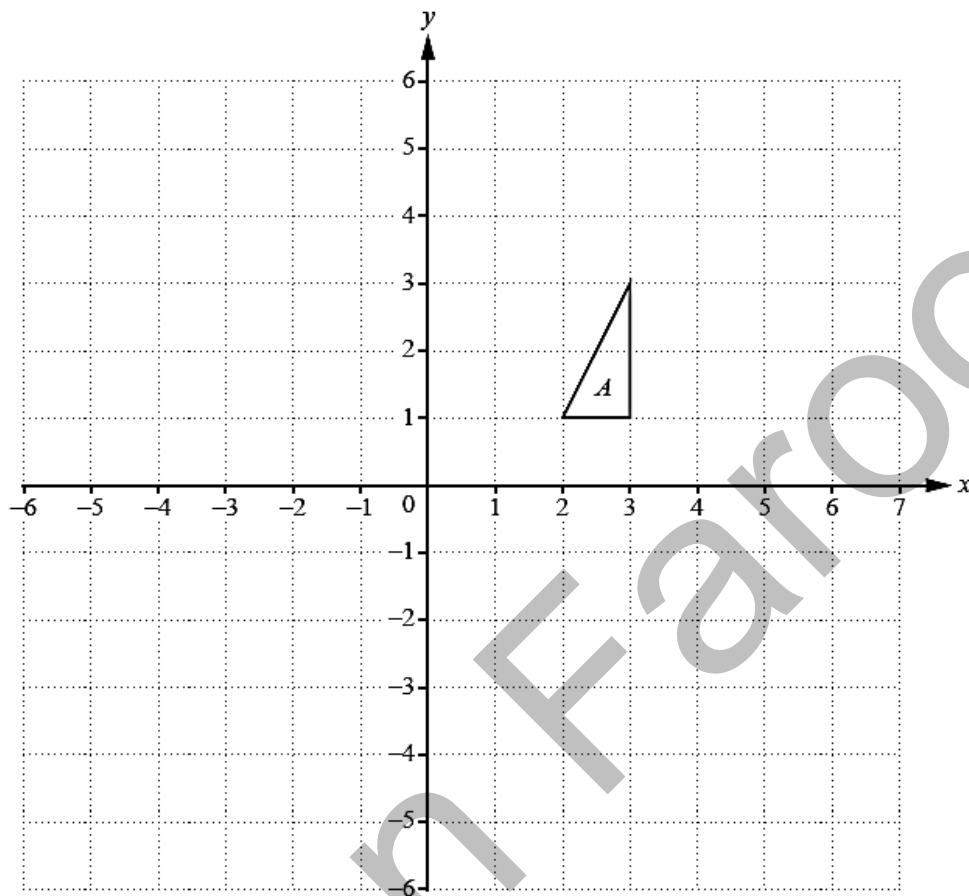
(a) Triangle B is the image of triangle A after reflection in the line $y = -1$.

Draw and label triangle B on the diagram. [1]

9
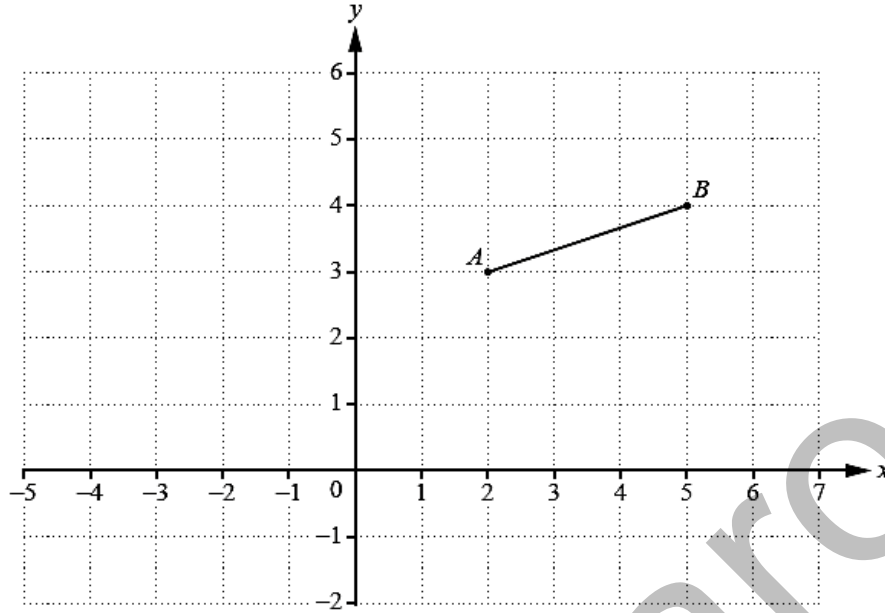
The diagram shows a line segment *AB* joining *A* (2, 3) and *B* (5, 4) .

(a) Find the coordinates of the midpoint of *AB*.   *Answer* (.................... , ...) [1]

(b) *AB* is mapped onto *CD* by the translation $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$.   *Answer* (.................... , ...) [1]

Find the coordinates of *C*.

(c) *AB* is mapped on to *FG* by a rotation of 90° clockwise with centre (1, 4).

Find the coordinates of *G*.   *Answer* (.................... , ...) [1]

(d) Find the equation of *AB*.   *Answer* .... [2]

Answers: **(a)** $\left(3\frac{1}{2}, 3\frac{1}{2}\right)$ **(b)** (–1, 4) **(c)** (1, 0) **(d)** $y = \frac{1}{3}x + \frac{7}{3}$
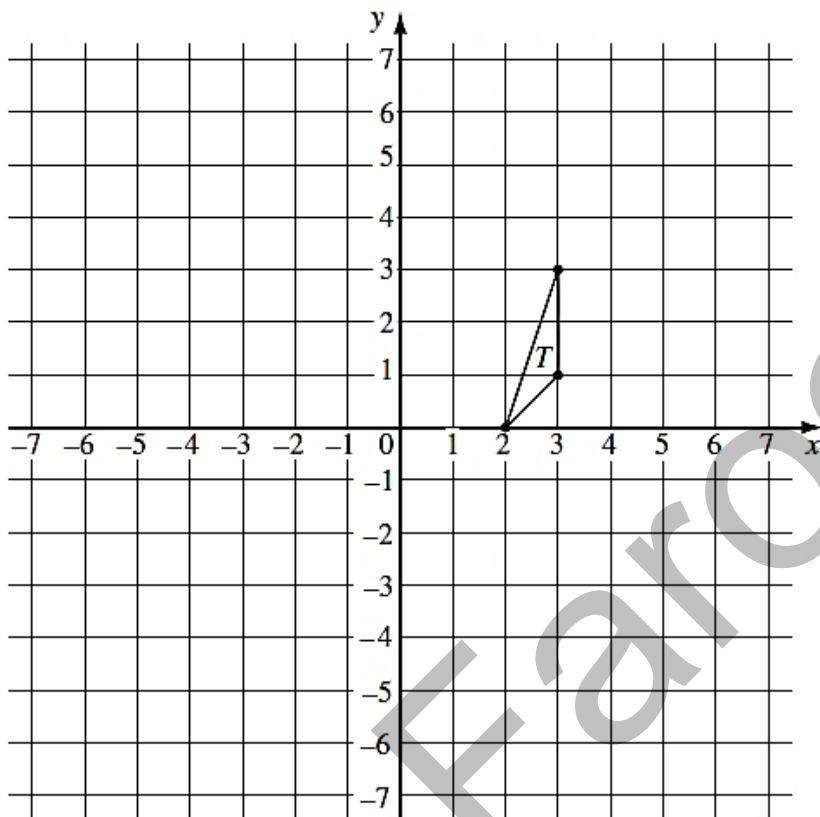
The diagram shows triangle T with vertices $(2, 0)$, $(3, 1)$ and $(3, 3)$.

(a) Triangle A is the image of triangle T under the enlargement with centre $(0, 0)$ and scale factor -2.

(i) Draw and label triangle A on the diagram. [1]

(ii) Describe fully the single transformation which maps A onto T.

Answer (a) (ii) ....

.... [1]

(b) Triangle B is the image of triangle T under the transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

Draw and label triangle B on the diagram. [2]

(c) Find the matrix representing the transformation which maps A onto B.

Answer (c) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

11   25   The diagram below shows the point P and triangles A, B, and C.
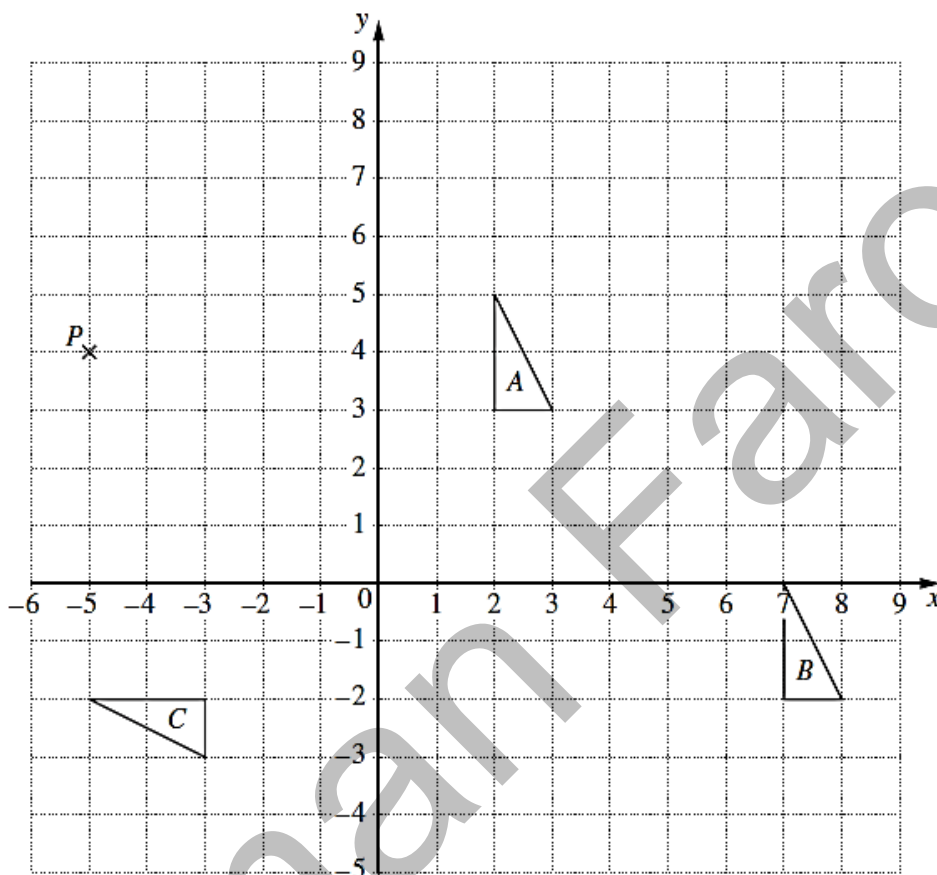
[4]

(a)   The translation T maps ΔA onto ΔB.
Given that $T(P) = Q$, write down the coordinates of Q.

(b)   Describe fully the single transformation which maps ΔA onto ΔC.

(d) ΔA is mapped onto ΔE by a rotation of $90°$ clockwise about the point (4, 2).
Draw and label ΔE on the diagram above.

Answer (a)   (........ , ...)   [1]

wer (b) ....[2]

12   The diagram below shows the point P and triangles A, B and C.

(a) The reflection, M, maps $\triangle A$ onto $\triangle B$.
Given that $M(P) = Q$, write down the coordinates of Q.

Answer (a)   (............. , ...)   [1]

(b) The rotation, R, maps $\triangle A$ onto $\triangle C$.

Find

(i)   the coordinates of the centre of this rotation,

(ii) the angle and direction of this rotation.

Answer (b)(i)   (............. , ...)   [1]

(ii) .... [1]

(c) The matrix $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ represents the transformation T.

Given that $T(A) = D$, draw and label $\triangle D$ on the diagram.

Answer (c)

[2]
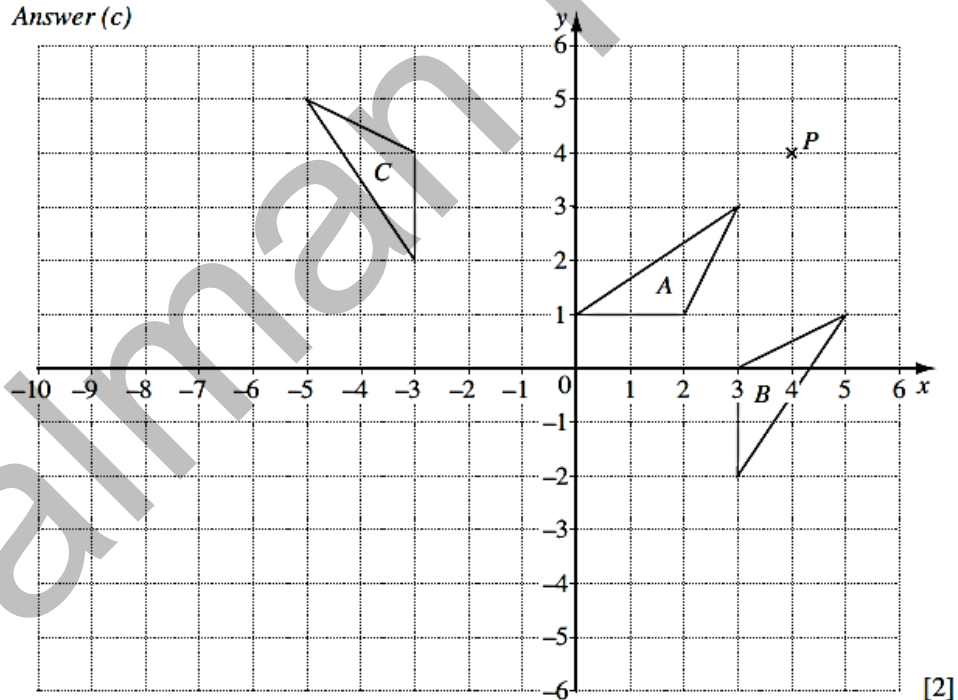
(d) Given that $FT(A) = A$, find the matrix representing the transformation F.

Answer (d) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

541

N05/1/Q22

13

[3]
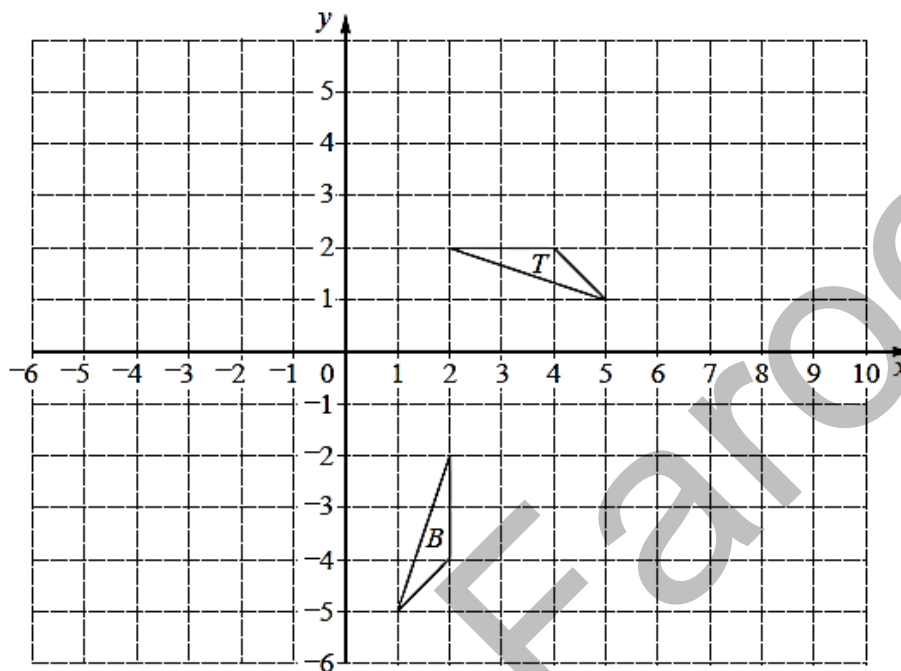
The diagram shows triangles T and B.

(a)   The enlargement, with centre (0, 0) and scale factor 2, maps ΔT onto ΔA.
Draw ΔA on the diagram above.

(b)   Describe fully the **single** transformation which maps ΔT onto ΔB.

Answer (b) ....[2]

(c)   A transformation is represented by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

This transformation maps ΔT onto ΔC.
Draw ΔC on the diagram above.

N06/1/Q24

14

The diagram below shows three triangles Q, R and S.

(a) Triangle T is the image of triangle Q under a translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Draw and label triangle T.

Answer (a)

[1]
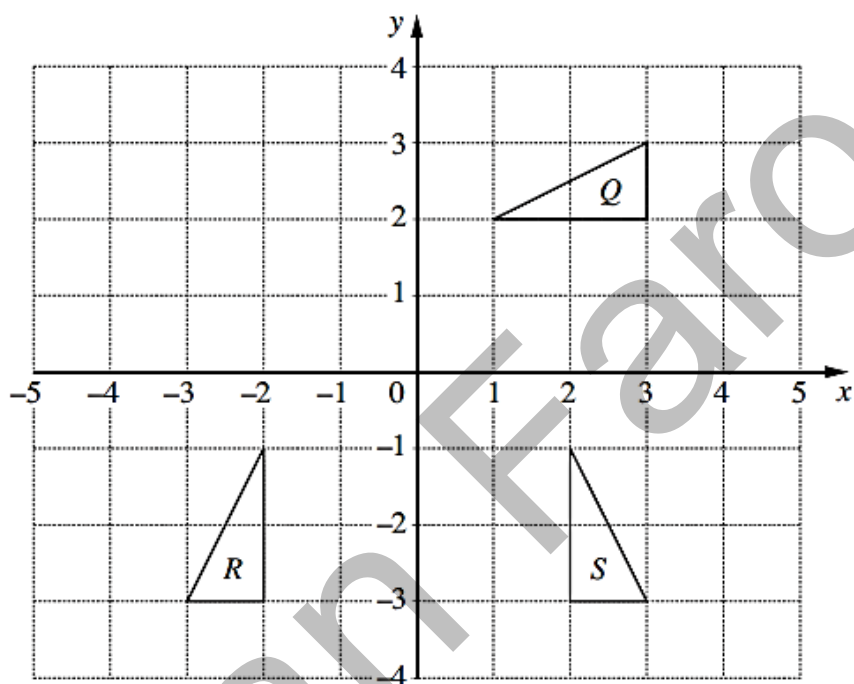
(b) Describe fully the **single** transformation that maps triangle Q onto triangle R.

Answer (b) ....

....[2]

(c) Find the matrix representing the transformation that maps triangle Q onto triangle S.

Answer (c) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

Answers: (a) Triangle with vertices at (−1,3), (1,3) and (1,4), (b) reflection in $y = -x$, **N08/1/Q14**
(c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

15   The diagram below shows triangle A and triangle B.

(a)   Describe **fully** the **single** transformation that maps triangle A onto triangle B.

Answer (a) ....

....[2]

(b)   Triangle A is mapped onto triangle C by a rotation, centre the origin, through 90° anticlockwise.

(i)   Draw, and label, triangle C on the diagram.

Answer (b)(i)

[1]
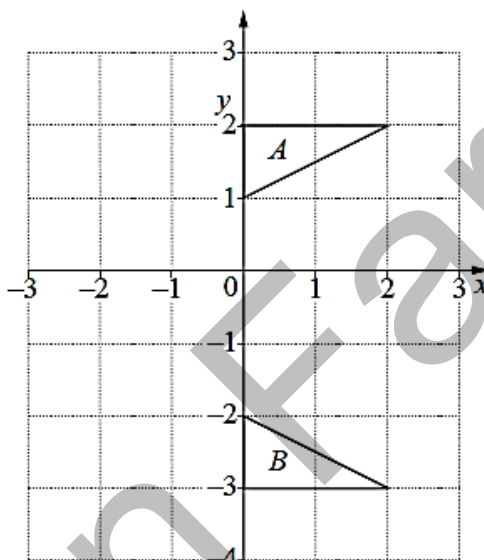
(ii) Write down the matrix that represents this transformation.   [1]

Answers: (a) Reflection in y = -½ (b)(i) Triangle with vertices (-1,0), (-2,0) and (-2,2) (ii) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ **N10/11/Q24**
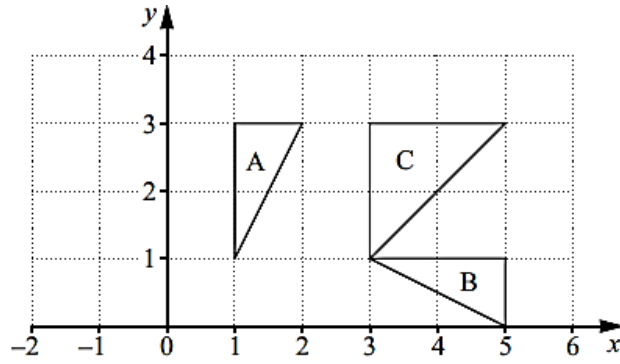
The diagram shows triangles A, B and C.

(a) Triangle A is mapped onto triangle B by an **anticlockwise** rotation.

    (i) Write down the angle of rotation.   *Answer* .... [1]

    (ii) Find the coordinates of the centre of rotation.   *Answer* (.......... , ...)   [1]

Answer. (a)(i) 270° (ii) (2, 0) (b)(i) 2 (ii) $x = -1$

17

The diagram shows triangles A and B and the point $P(0, 4)$.
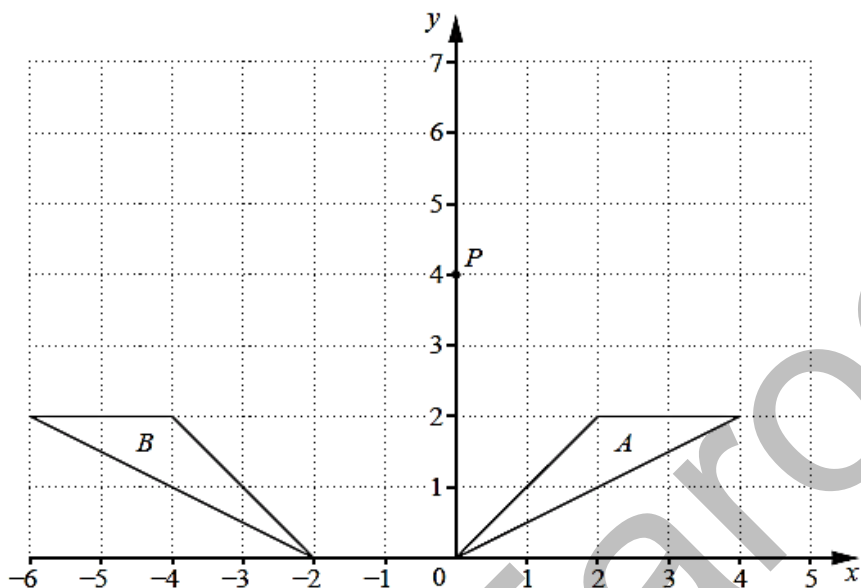
(a) Describe fully the **single** transformation that maps triangle A onto triangle B.

Answer..

....[2]

(b) Triangle A is mapped onto triangle C by an enlargement, centre P, scale factor $-\frac{1}{2}$.

On the diagram, draw triangle C. [2]

(c) Find the value of $\dfrac{\text{area of triangle } A}{\text{area of triangle } C}$.   Answer ....[1]

Answers: (a) reflection in $x = -1$ (b) Triangle with vertices (0,6), (-1,5), (-2,5) (c) 4   **N13/11/Q25**
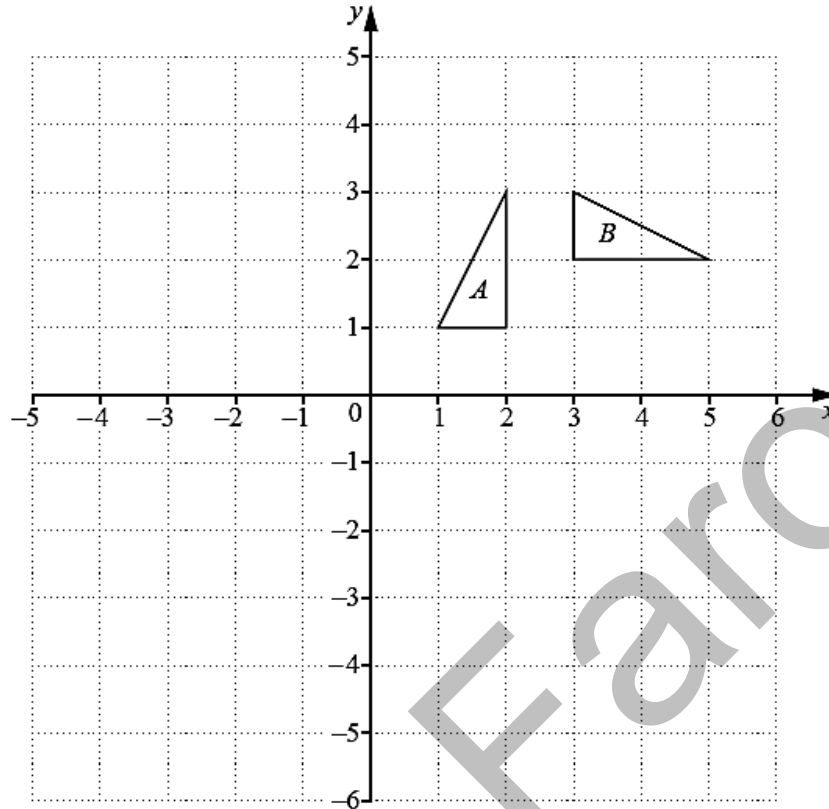
(a) Describe the **single** transformation that maps triangle A onto triangle B.

Answer ....

.... [2]

(b) Triangle A is mapped onto triangle C by an enlargement, centre $(0, 2)$ and scale factor -2.

Draw, and label, triangle C on the diagram. [2]

Answers: **(a)** Rotation of 90° clockwise, centre (3, 1) **(b)** Triangle with vertices (−2, 4), (−4, 0), (−4, 4).   **N16/11/Q15**

Transformations Paper 2

1
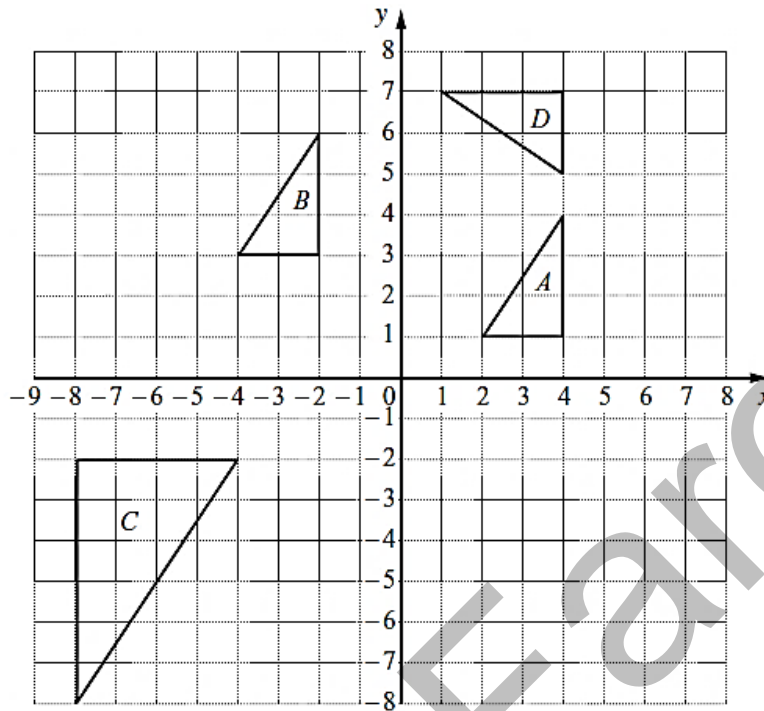
The diagram shows triangles A, B, C and D.

(a) Describe fully the single transformation which maps A onto B.   [2]

(b) Find the matrix that represents the single transformation which maps A onto C.   [2]

(c) A is mapped onto D by a clockwise rotation.

     Find

     (i)   the angle of this rotation,   [1]

     (ii)   the coordinates of the centre of this rotation.   [1]

J03/2/Q11

Answers: **(a)** Translation $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$; **(b)** $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$; **(c)(i)** 270°, **(ii)** (1, 4)

2 **(c)**
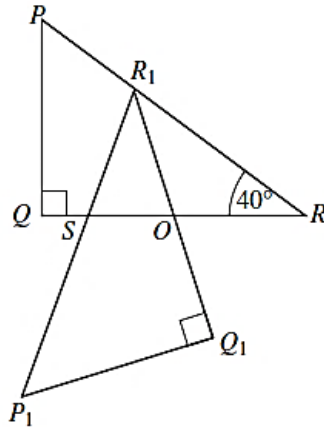
In the diagram, PQR is a triangle with $P\hat{Q}R = 90°$ and $Q\hat{R}P = 40°$.

The point O is the midpoint of QR.

Triangle $P_1Q_1R_1$ is the image of triangle PQR under an anticlockwise rotation about the point O.

The point R_1 lies on PR.

The line QR intersects the line P_1R_1 at the point S.

Find

(i) $R\hat{R}_1Q_1$, [1]

(ii) the angle of rotation , [1]

(iii) $O\hat{S}P_1$. [1]

(c)(i) 40°, (ii) 100°, (iii) 120°.   **J04/2/Q4c**

3
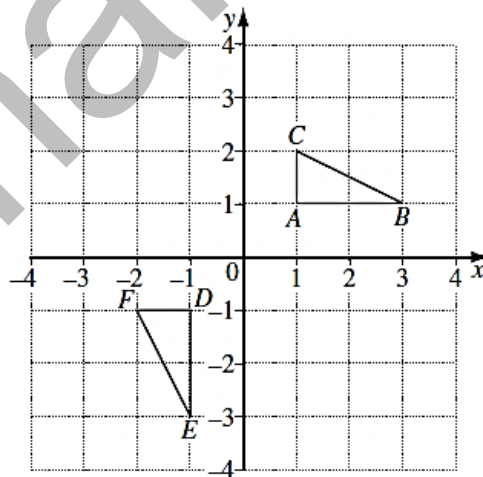
Triangle ABC has vertices A (1, 1), B (3, 1) and C (1, 2).
Triangle DEF has vertices D (–1, –1), E (–1, –3) and F (–2, –1).

The matrix **P** represents the **single** transformation, T, that maps triangle ABC onto triangle DEF.

(a) (i) Describe T **fully**. [2]

   (ii) Write down the matrix **P**. [1]

(b) Another transformation is represented by the matrix **Q**, where $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

   (i) This transformation maps *B* onto *Y*.

     Find the coordinates of *Y*. [1]

   (ii) This transformation maps *K* onto *C*.

     Find the coordinates of *K*. [2]

   (iii) Describe, **fully**, the **single** transformation which is represented by **Q**. [2]

   (iv) The matrix **R** is given by **Q = RP**.

     By considering the effects of transformations on triangle *ABC*, or otherwise, find **R**. [2]

(c) The point *H* lies on *DC* produced, where $\overrightarrow{DH} = \begin{pmatrix} 18 \\ h \end{pmatrix}$.

   Calculate

   (i) the ratio *DC* : *DH*, [1]

   (ii) the value of *h*. [1]

Answers: **(a)(i)** Reflection in the line $y = -x$, **(ii)** $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$; **(b)(i)** (−1, 3), **(ii)** (2, −1), **(iii)** Rotation of 90°   **J05/2/Q11**

anticlockwise about the origin, **(iv)** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; **(c)(i)** 1 : 9, **(ii)** 27.

---

4 (a) The diagrams show triangles *A*, *B*, *C* and *D*.

   (i) The single transformation P maps Δ*A* onto Δ*B*.
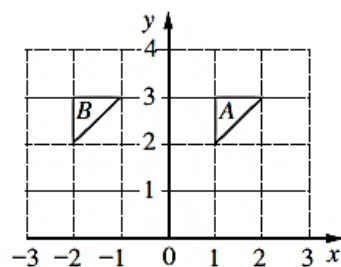
     Describe, **fully**, the transformation P. [2]

   (ii) The single transformation Q maps Δ*A* onto Δ*C*.
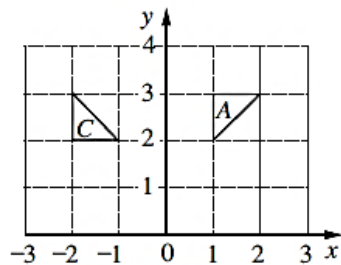
     Describe, **fully**, the transformation Q. [2]

(iii) The reflection R maps ΔA onto ΔD.

Find the matrix that represents the reflection R.

[2]
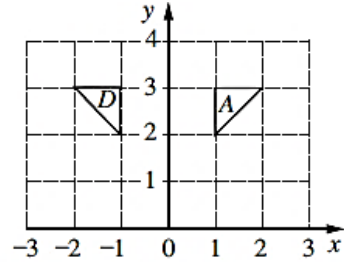
(b) The diagram shows the points E (1, 3), F (2, 3) and G (–1, 3). An enlargement, centre E, maps F onto G.
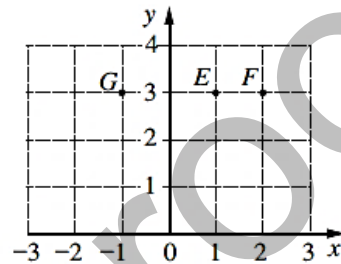
Write down

(i) the scale factor, [1]

(ii) the coordinates of the image of (0, 4). [1]

Answer: **(a)(i)** Translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$, **(ii)** Rotation, 90°AC, centre (0, 1), **(iii)** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$;

(b)(i) –2, **(ii)** (3, 1); **(c)(i)** 2, **(ii)** $\begin{pmatrix} 2 & -1\frac{1}{2} \\ 1 & -\frac{1}{2} \end{pmatrix}$; **(d)** $\begin{pmatrix} 11 \\ 5 \end{pmatrix}$.

5 **(a)** The diagram shows triangles A, B, C and D.
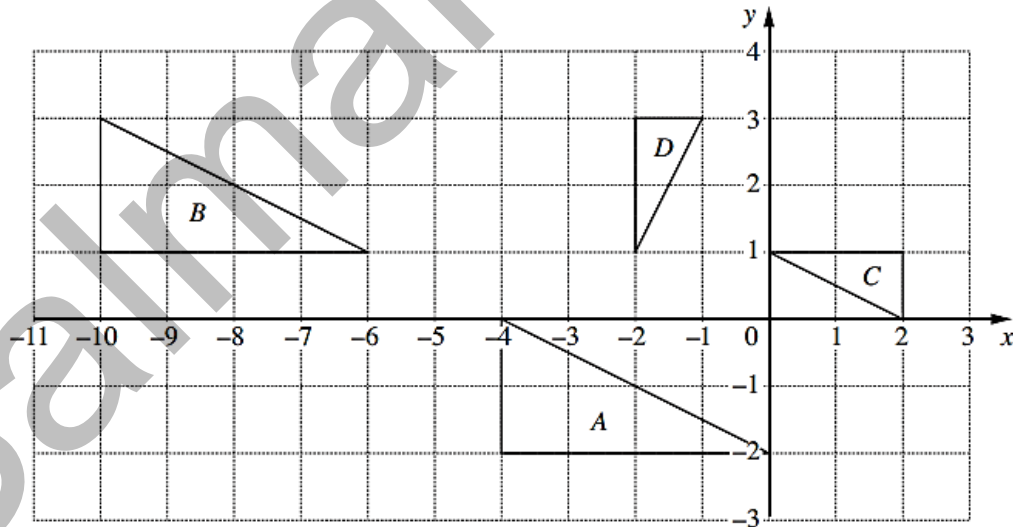
(i) Describe fully the **single** transformation that maps ΔA onto ΔB. [2]

(ii) Describe fully the **single** transformation that maps ΔB onto ΔC. [2]

(iii) Describe fully the **single** transformation that maps ΔC onto ΔD. [2]

(iv) Write down the matrix that represents the transformation which maps ΔC onto ΔA. [1]

Answers: **(a)(i)** Translation, $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$; **(ii)** Enlargement, SF $-\frac{1}{2}$, centre (−2, 1);

(iii) Rotation, 90° anticlockwise, centre (−1, 0); **(iv)** $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$.

6   **(b)**
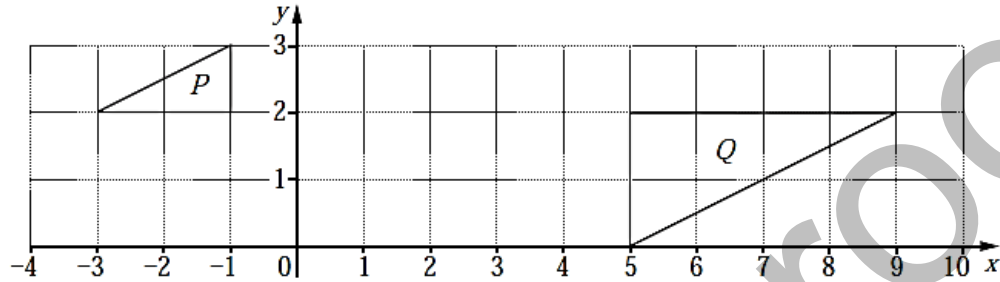
The diagram shows the triangles P and Q.

(i) The enlargement E maps triangle P onto triangle Q.

For this enlargement,

(a) write down the scale factor, [1]

(b) find the coordinates of the centre of enlargement. [2]

(b)(i)(a) −2, **(i)(b)** (1, 2), **(ii)** Shear, SF2, *x* axis invariant;
(iii)(a) 4 **(iii)(b)** (−13, 2).

7   **(a)** The diagram shows triangles A, B, C and D.
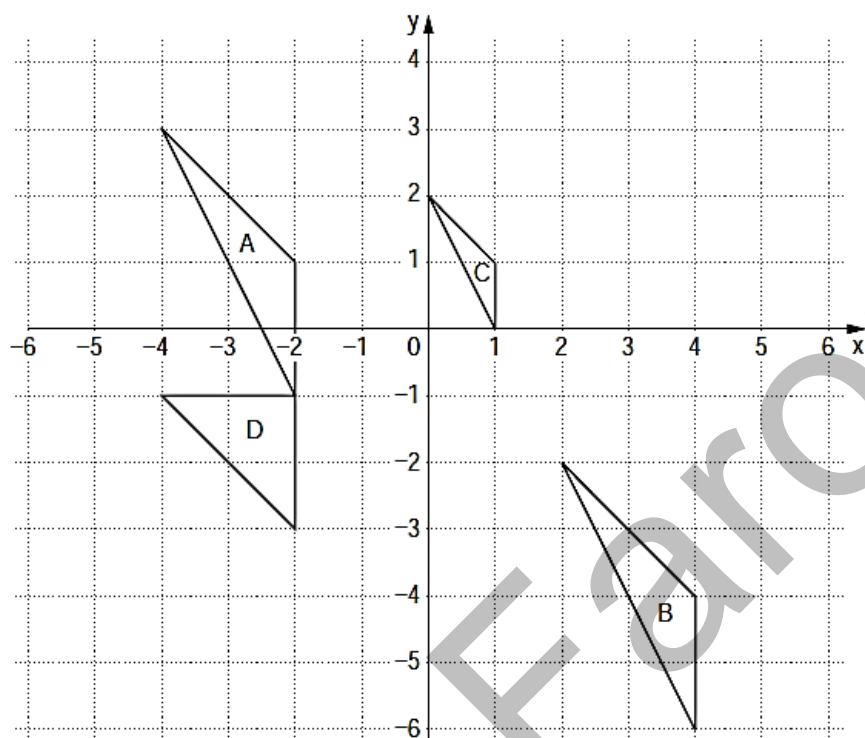
   (i)   Triangle A is mapped onto triangle B by the translation **T**.

   Write down the column vector that represents **T**.   [1]

   (ii)   Describe **fully** the **single** transformation that maps triangle A onto triangle C.   [3]

   (iii)   Triangle A is mapped onto triangle D by a **single** transformation.

   What is the name of this transformation?   [1]

   (iv)   Triangle D has one line of symmetry.

   Find the equation of this line of symmetry.   [2]

(b) K is the point (p, q).

   (i)   The transformation **U** is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.
   This transformation maps the point K onto the point L.

   Find, in terms of p and q, the coordinates of L.   [2]

   (ii)   The transformation **V** is a rotation $90°$ clockwise about the point $(0, 0)$.
   This transformation maps the point K onto the point M.

   Find, in terms of p and q, the coordinates of M.   [2]

   (iii)   The point L is mapped onto the point M by the single transformation **W**.

   Find the matrix that represents the transformation **W**.   [1]

---

8   **(a)**   The diagram shows triangles A, B and C.
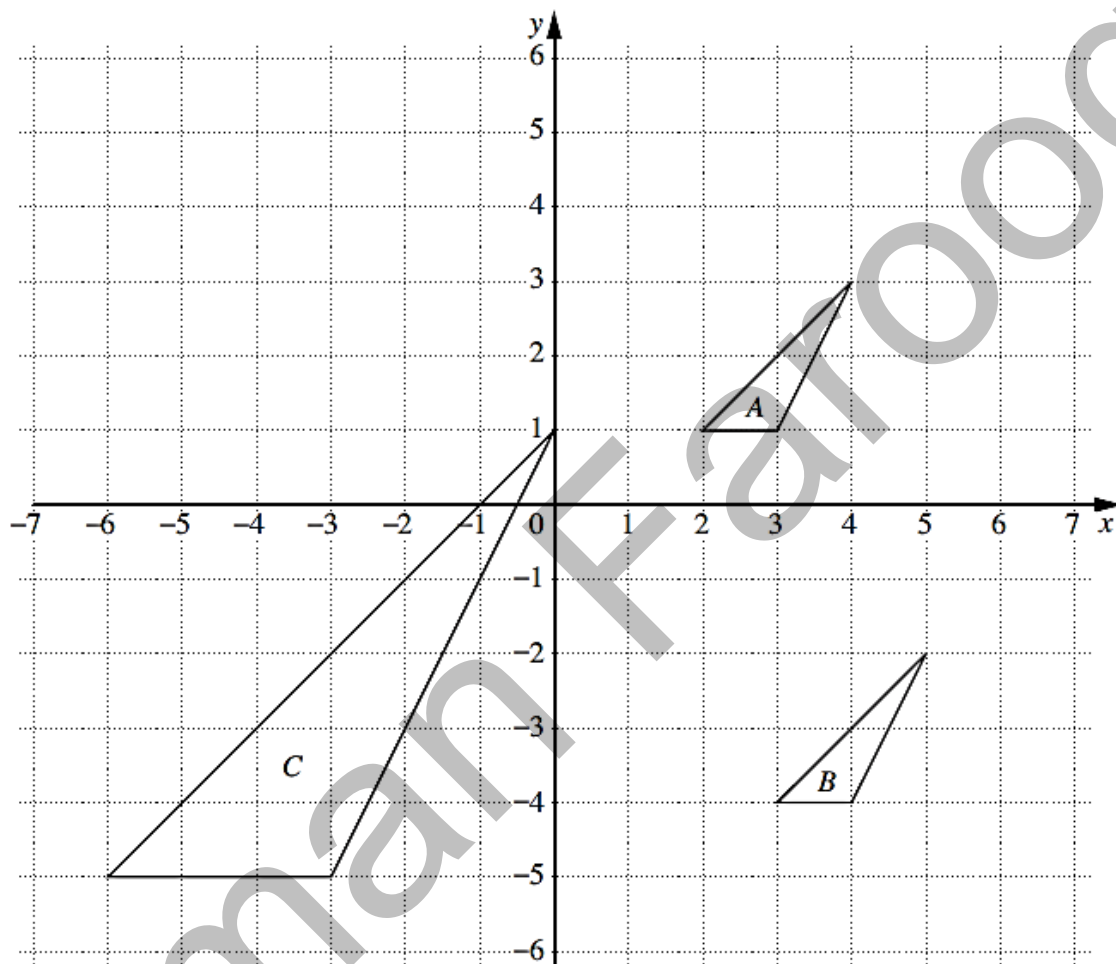
(i)   Describe fully the **single** transformation that maps

(a)   triangle A onto triangle B,   .[2]

(b)   triangle A onto triangle C.   .[2]

(ii) One vertex of triangle A is $(2, 1)$.

Find the coordinates of this point when it is

(a) reflected in the line $y = -x$,

Answer   (............ , ...)   [1]

(b) rotated through 90° anticlockwise about $(1, -1)$.

Answer   (............ , ...)   [1]

9   **(b)** The diagram shows triangles A, B and C.
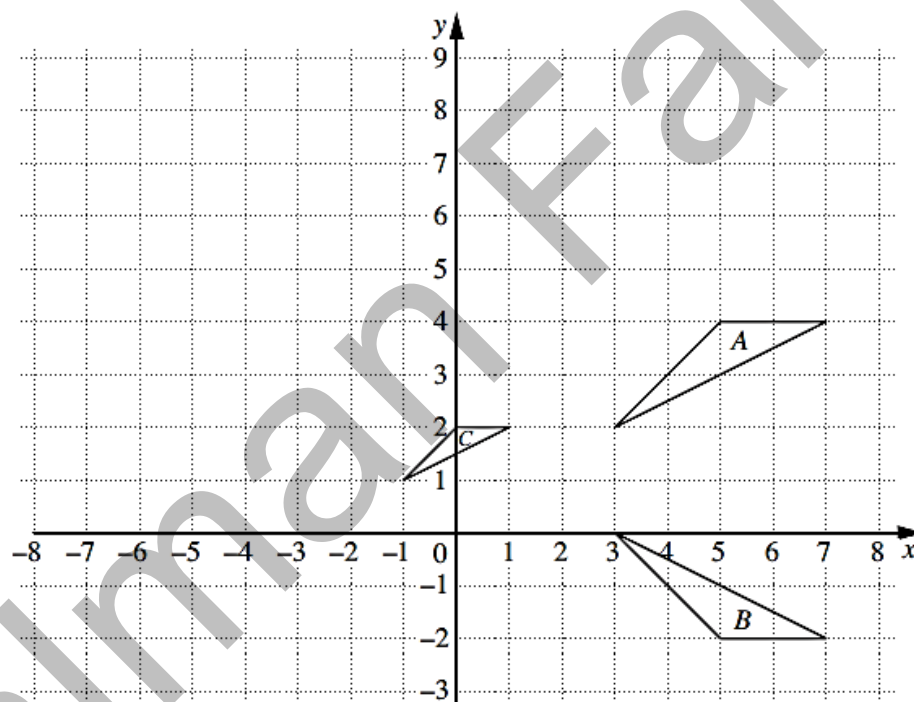
(i) Describe fully the **single** transformation that maps triangle A onto triangle B.

Answer ....

....[2]

(ii) Describe fully the **single** transformation that maps triangle A onto triangle C.

Answer ....

....[2]

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(iii) Another transformation is represented by the matrix **P**, where $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.
This transformation maps triangle A onto triangle D.

Find the vertices of triangle D.

Answer (............ , ...) (............ , ...) (............ , ...) [2]

(iv) Describe fully the **single** transformation represented by the matrix **P**.

Answer ....

.... [2]

(b) (i) Reflection in the line y = 1   **(ii)** Enlargement, scale factor ½, centre (–5, 0)   **J11/22/Q8b**
(iii) (–2, 3), (–4, 5), (–4, 7)   **(iv)** Rotation, 90° anticlockwise about centre (0, 0)

10

(b) The diagram shows triangles A and B.
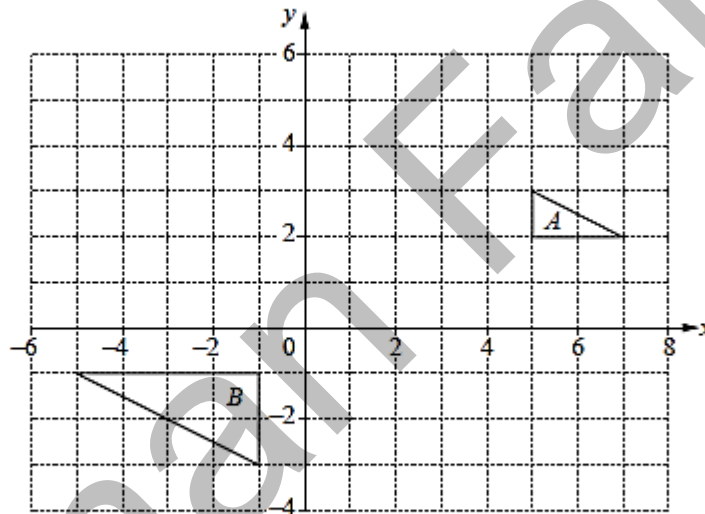
(i) Describe fully the single transformation that maps triangle A onto triangle B.

Answer .... [3]

(b) Stretch, Factor 2, x-axis invariant   **J14/21/Q10b**

11 Answer the whole of this question on a sheet of graph paper.

Triangle A has vertices $(6, -2)$, $(8, -2)$ and $(6, -5)$.

(a) Using a scale of 1 cm to represent 1 unit on each axis, draw axes for values of x and y in the ranges $-6 \leqslant x \leqslant 12$ and $-6 \leqslant y \leqslant 6$.

Draw and label triangle A. [1]

(b) The translation T is represented by the column vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

The translation T maps triangle A onto triangle B, so that $T(A) = B$.

Draw and label triangle *B*. [1]

(c) The transformation R is a rotation through 90° clockwise, centre (3, 4).
The transformation R maps triangle *A* onto triangle *C*, so that R(*A*) = *C*.

Draw and label triangle *C*. [2]

(d) Given that TR(*A*) = *D*, draw and label triangle *D*. [2]

12
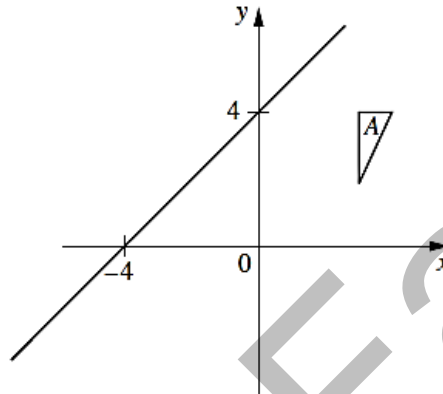
The diagram shows triangle *A* and the straight line $y = x + 4$.
Triangle *A* has vertices (3, 2), (3, 4) and (4, 4).

(a) Using a scale of 1 cm to represent 1 unit on each axis, draw, on a sheet of graph paper, axes for
values of *x* and *y* in the ranges $-6 \leqslant x \leqslant 6$ and $-6 \leqslant y \leqslant 10$.

Draw and label triangle *A*.
Draw the straight line $y = x + 4$. [1]

(b) The transformation M is a reflection in the line $y = x + 4$.
The transformation M maps triangle *A* onto triangle *B*, so that M(*A*) = *B*.

Draw and label triangle *B*. [2]

(c) Triangle *C* has vertices (−1, 2), (1, 2) and (1, 1).
The rotation R maps triangle *A* onto triangle *C*, so that R(*A*) = *C*.

Find

(i) the angle and direction of this rotation,

(ii) the coordinates of the centre of this rotation. [2]

(d) Given that MR(*A*) = *D*, draw and label triangle *D*. [2]

13 **(a)** P is the point $(2, 9)$ and Q is the point $(4, 6)$.

Find

  (i) the length of PQ, [2]

  (ii) the equation of the line PQ. [2]

(b)
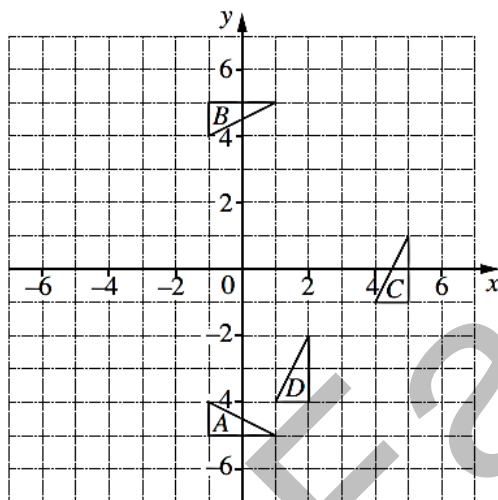
The diagram shows triangles A, B, C and D.

  (i) Find the matrix representing the transformation that maps triangle A onto triangle B. [1]

  (ii) Describe fully the **single** transformation that maps triangle B onto triangle C. [2]

  (iii) Triangle C is mapped onto triangle D by the translation T.

  (a) Write down the column vector that represents T. [1]

  (b) The transformation R that maps triangle A onto triangle C is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

  Show that the transformation R followed by T maps (h, k) onto $(-k-3, h-3)$. [1]

  (c) Find the value of h and the value of k for which the transformation R followed by T maps (h, k) onto itself. [2]

  (d) The **single** transformation that is equivalent to R followed by T is a rotation.

  Write down the coordinates of the centre of this rotation. [1]

Answers: (a)(i) 3.61, (ii) $3x + 2y = 24$, (b)(i) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, (ii) Reflection in the line $y = x$, (iii)(a) $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$, **N07/2/Q11**

(b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} h \\ k \end{pmatrix} + \begin{pmatrix} -3 \\ -3 \end{pmatrix}$ seen, (c) $h = 0, k = -3$, (d) $(0, -3)$.

14 **(b)**
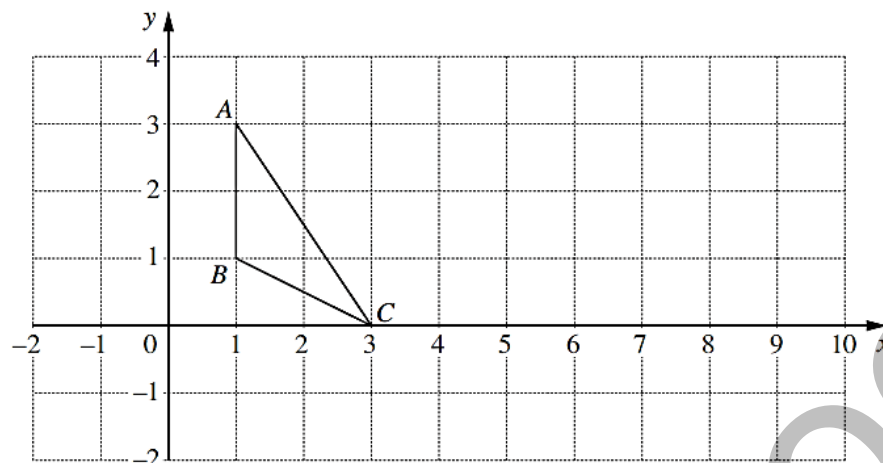
The diagram shows triangle *ABC*.

(i) Find the area of triangle *ABC*. [1]

(ii) An enlargement, scale factor 4, maps triangle *ABC* onto triangle *LMN*.
The point *A* maps onto the point *L*(10 , 3).

(a) Find the coordinates of the centre of enlargement. [1]

(b) Write down the area of triangle *LMN*. [1]

Answers: (a)(i)(a) 37, (b) $\begin{pmatrix} 16 \\ -21 \end{pmatrix}$, (ii) $\begin{pmatrix} 14 \\ -28 \end{pmatrix}$, (iii) (−6,51) (b)(i) 2, (ii)(a) (−2,3),

(b) 32, (iii)(a) (3,1), (b) 2.

15 **(a)** *PQRS* and *P′Q′R′S′* are congruent quadrilaterals.
R is the same point as *S′*.
S is the same point as *R′*.

A single transformation maps *P* onto *P′*,
Q onto *Q′*, *R* onto *R′* and *S* onto *S′*.
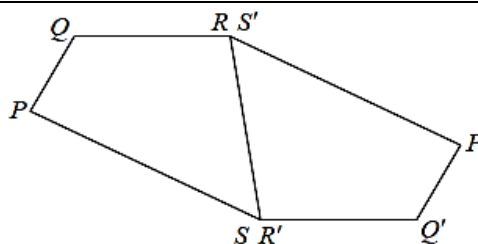
(i) Describe fully this transformation. [3]

(ii) Write down two facts connecting *PQ* and *Q′P′*. [1]

(b)
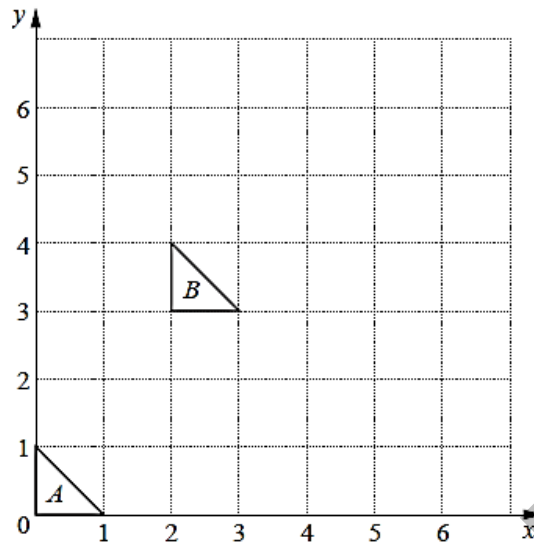
The diagram shows triangle A and triangle B.

(i) A translation, T, maps triangle A onto triangle B.

State the column vector representing this translation. [1]

16 **(a)** Some transformations of the plane are given in the following table.

M_x	Reflection in the x-axis
M_y	Reflection in the y-axis
M_d	Reflection in the line $y = -x$
R_{90}	Rotation of $90°$, anti-clockwise, centre the origin
R_{180}	Rotation of $180°$, centre the origin
R_{270}	Rotation of $270°$, anti-clockwise, centre the origin.

You may use the grid on the next page to help answer the following questions.

(i) The point A has coordinates $(2, 3)$.

    (a) Find the coordinates of $M_y(A)$.

                    Answer   (............. , ...)   [1]

    (b) Find the coordinates of $M_d M_y(A)$.

                    Answer   (............. , ...)   [1]

    (c) The inverse of R_{90} maps B onto A.

        Find the coordinates of B.   *Answer*   (............. , ...)   [2]

(ii) **(a)** Write down the matrix which represents M_x.

                    Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

    (b) Which single transformation given in the table is equivalent to $R_{180}M_x$?

                    Answer   .... [1]
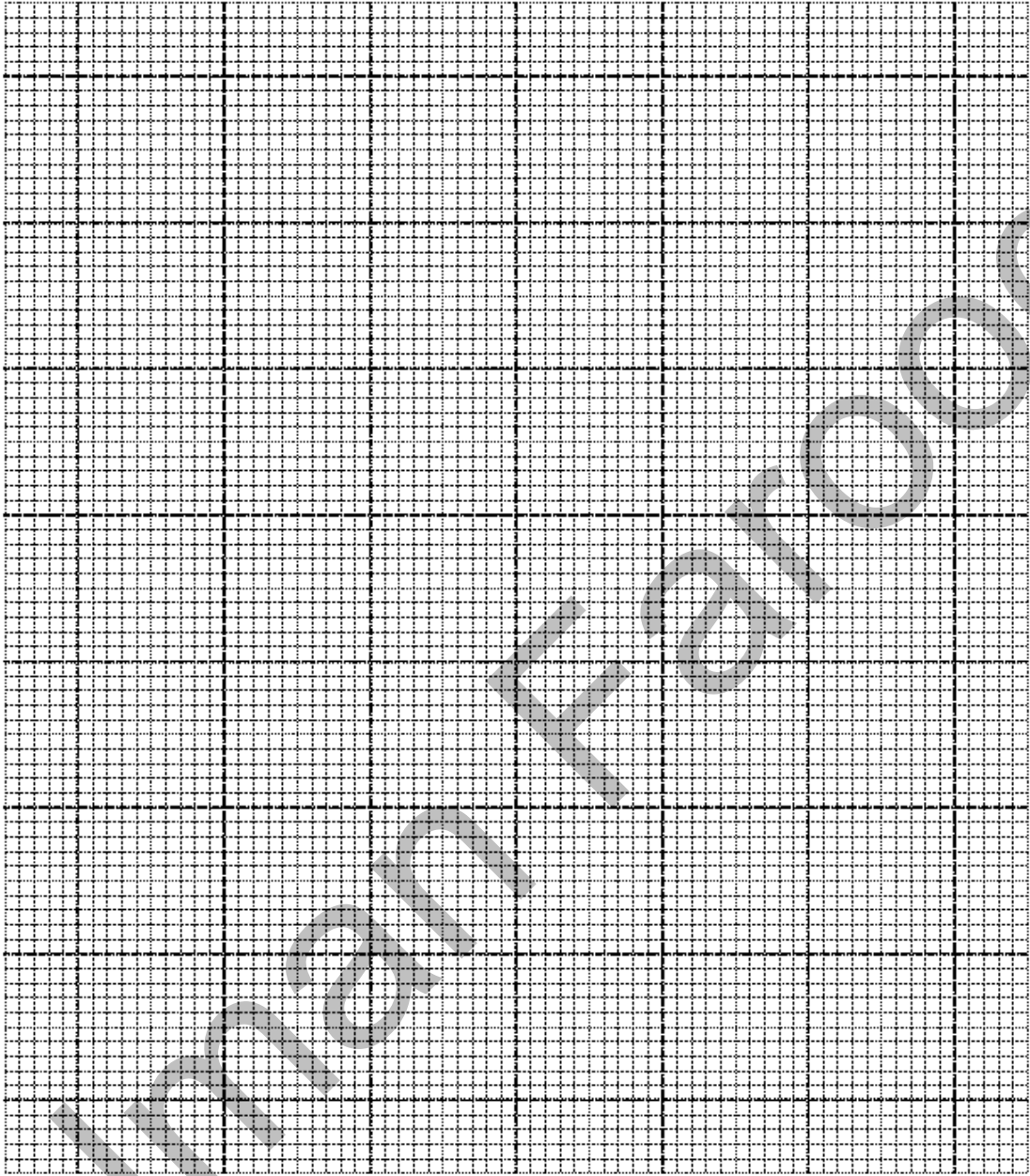
(b) The points P and Q have coordinates $(4, 0)$ and $(9, 0)$ respectively.
The points P' and Q' have coordinates $(4, 4)$ and $(7, 8)$ respectively.

(i) Write down the length of PQ. *Answer* $PQ = $ .... units [1]

(ii) Calculate the length of $P'Q'$. *Answer* $P'Q' = $ .... units [2]

(iii) PQ is mapped onto $P'Q'$ by a single rotation.
By using the grid below,

(a) find, by drawing, the coordinates of the centre of this rotation,

Answer (..........., ...) [2]

(b) measure the clockwise angle of rotation. *Answer* .... [1]
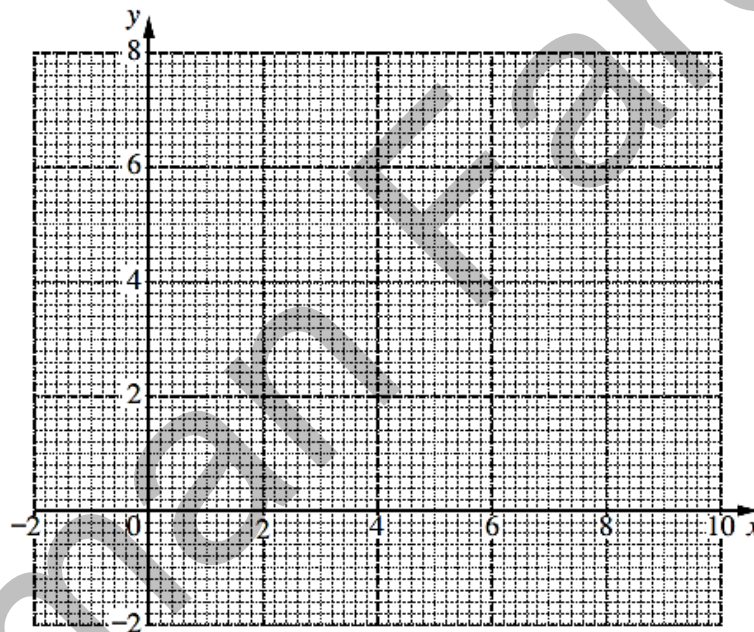
17

(a) $\begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ 11 \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$.

Find x and y.

Answer $x =$ ....

$y =$ .... [2]

(b) (i) The transformation A is represented by the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Find, in terms of a, b, c and d as appropriate,

(a) the image of (1, 0) under the transformation A,

Answer   (.........., ...) [1]

(b) the image of (0, 1) under the transformation A.

Answer   (.........., ...) [1]

(ii) The transformation B maps (1, 0) onto (1, 3) and (0, 1) onto (−3, −2).

Write down the matrix that represents transformation B.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(iii) Describe fully the transformation given by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

....[2]

Answers: (a) $x = 5$ $y = 4$ (b)(i)(a) (a, c) (b) (b, d) (ii) $\begin{pmatrix} 1 & -3 \\ 3 & -2 \end{pmatrix}$ (iii) Reflection in the x - axis   **N11/22/Q5**

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18 **(b)**
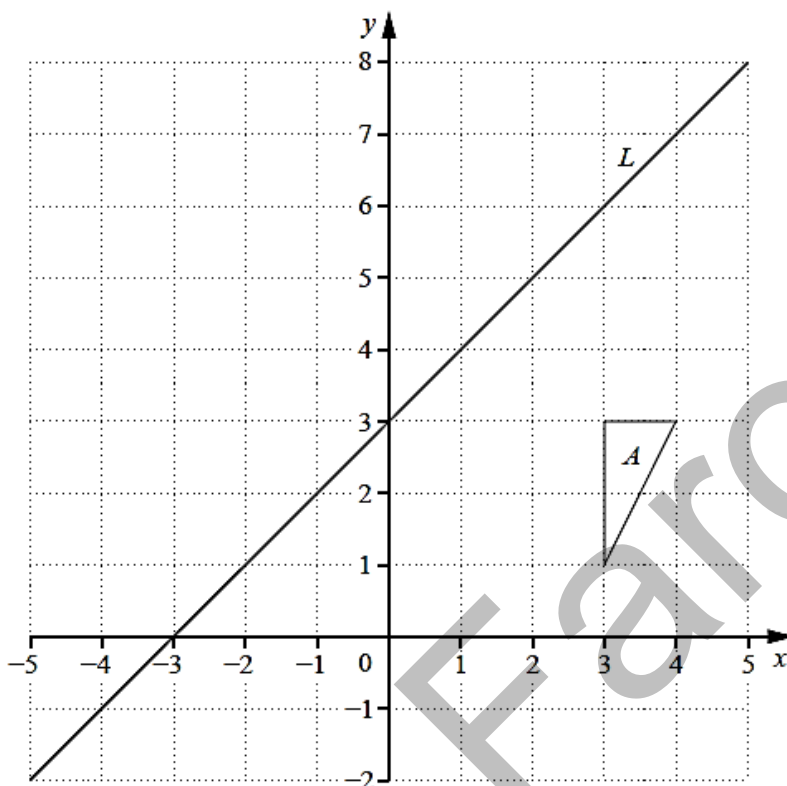
The grid shows triangle *A* and line *L*.

(i) Triangle *A* is mapped onto triangle *B* by a reflection in line *L*.

Draw and label triangle *B*.   [2]

(ii) Triangle *A* is mapped onto triangle *C* by a clockwise rotation of 90°, centre (0, 3).

Draw and label triangle *C*.   [2]

(iii) Triangle *C* is mapped onto triangle *D* by a reflection in line *L*.

Describe the single transformation that maps triangle *B* onto triangle *D*.

Answer ....

.... [3]

19

The diagram shows triangle A and octagon B.

(a) Find the gradient of the line of symmetry of triangle A. *Answer* .... [1]

(b) Triangle A is mapped onto triangle C by a reflection in the line $y = x$.

Draw and label triangle C. [2]

(c) Write down the equation of the line of symmetry of octagon B that is parallel to the y-axis.

Answer .... [1]

(d) State the order of rotational symmetry of octagon B. *Answer* .... [1]

(e) Octagon B is mapped onto octagon D by an enlargement, scale factor 2 and centre $(-3, -3)$.

Draw and label octagon D. [2]

(f) A mat is made from six identical octagons, each **similar** to octagon B, and two squares, as shown in the sketch below.
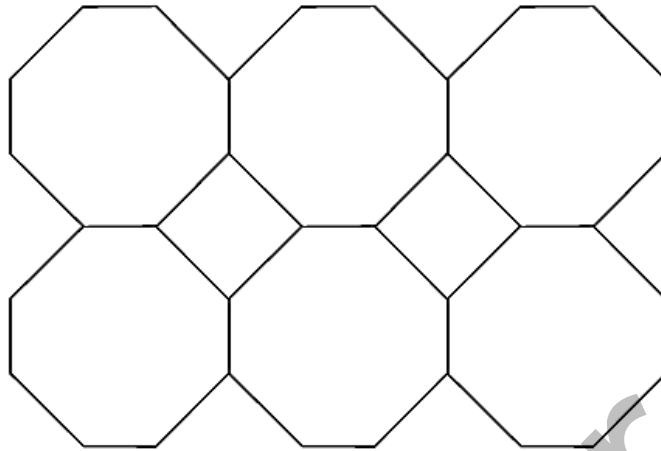
The lengths of the short sides of the octagons are each 15 cm.

(i) Calculate the area of one of these octagons.

Answer .... cm² [2]

(ii) Find the length of a diagonal of one of the squares.

Answer .... cm [1]

(iii) Calculate the total area of the mat.

Answer .... cm² [2]

Answers: **(a)** −1; **(b)** correct triangle; **(c)** $x = -2.5$; **(d)** 4; **(e)** correct octagon; **(f)(i)** 1575; **(ii)** 30; **(iii)** 10 350.

N14/21/Q9

20 **(c)**
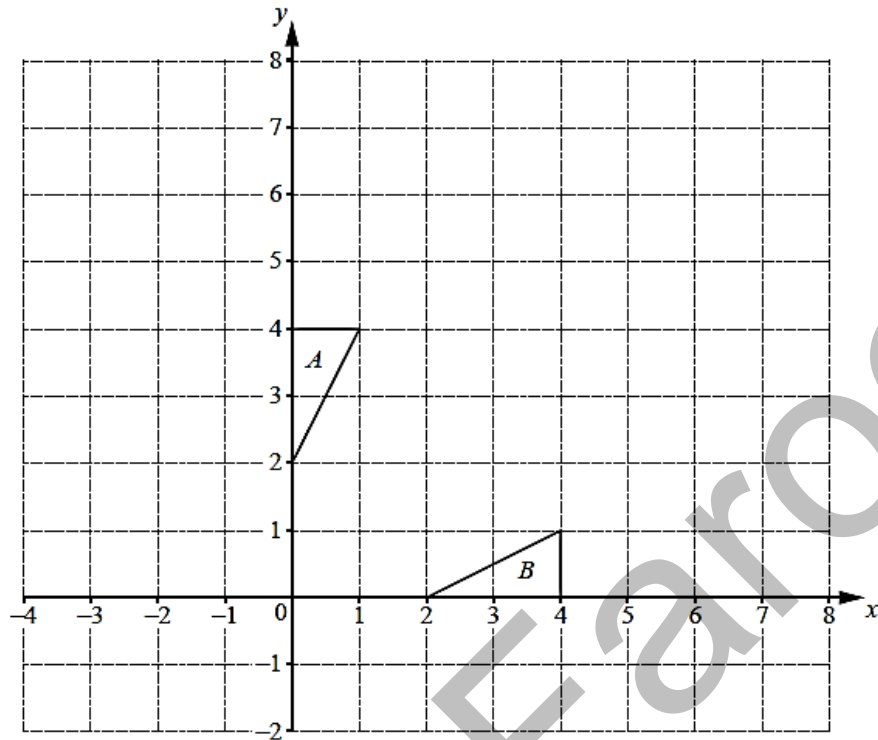
The diagram shows triangle A and triangle B.

(i) Triangle A is mapped onto triangle C by the translation P with vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Draw and label triangle C. [2]

(ii) Triangle A is mapped onto triangle B by a reflection Q.

Write down the equation of the line of this reflection.

Answer .... [1]

(iii) Triangle C is mapped onto triangle D by reflection Q.

Describe fully the **single** transformation that maps triangle B onto triangle D.

Answer .... [2]

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(iv) Transformation R is a reflection in the line $y = 0$.

RQ(A) = E.

(a) Find the coordinates of the vertices of triangle *E*.

Answer .... [1]

(b) Describe fully the **single** transformation that maps triangle *A* onto triangle *E*.

Answer ....

.... [2]

(c) Find the matrix which represents the transformation that maps triangle *A* onto triangle *E*.

Answer [1]

N15/21/Q7

21 **(b)**
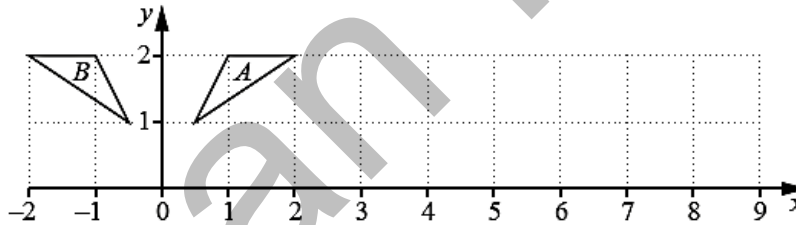
Triangle *A* has vertices $(\frac{1}{2}, 1)$, (1, 2) and (2, 2).
Triangle *B* has vertices $(-\frac{1}{2}, 1)$, (−1, 2) and (−2, 2).

(i) Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

Answer ....

.... [2]

N16/21/Q11

22 **(a)** Triangle *A* is shown on the grid.
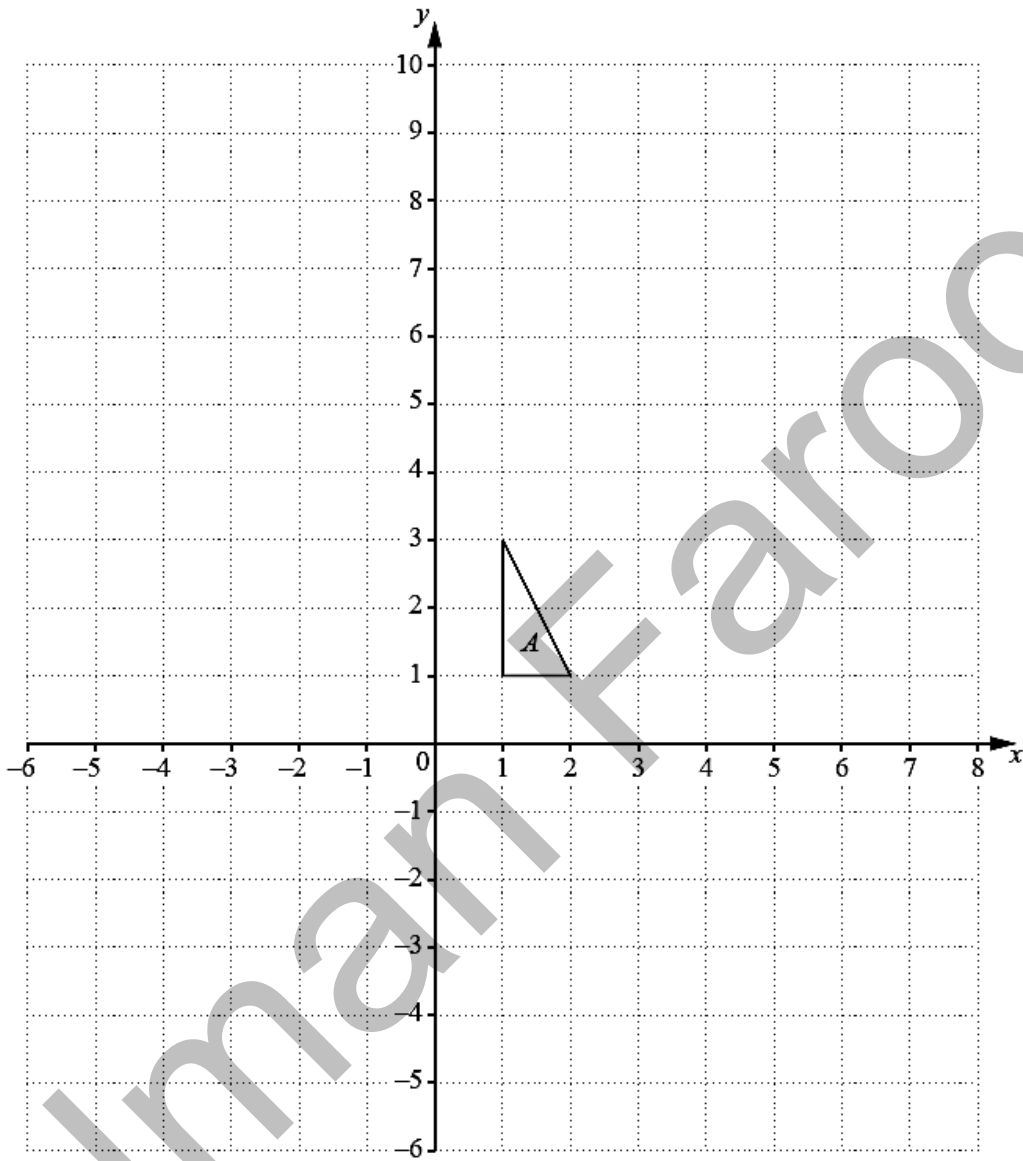
(i) Triangle *A* is mapped onto triangle *B* by a rotation of 180° about point (2, −1).

Draw and label triangle *B* on the grid. [2]

(ii) Triangle *A* is mapped onto triangle *C* by the transformation represented by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

Draw and label triangle *C* on the grid. [2]

(iii) Write down the matrix that represents the transformation that maps triangle *C* onto triangle *A*.

Answer .... [1]

(iv) Describe fully the **single** transformation that maps triangle *C* onto triangle *B*.

....

.... [3]

Volume 5

Everyday Graphs Paper 1

1

Five items bought at a shop are shown on the receipt.

The part showing the cost of the apples is missing.

```
Apples
Roll              1.35
Mineral water     1.20
Cheese            1.64
Tomatoes          1.20
        Total  $ 5.90
```

(a) How much did the apples cost?   *Answer (a)* $.. [1]

(b) The total cost of $5.90 when converted to euros is 6.80 euros.

   (i) Using these totals, draw a graph on the axes in the answer space which will enable you to convert dollars ($) to euros.

   (ii) Use your graph to estimate the cost of the mineral water in euros.

Answer (b)(i)   [1]
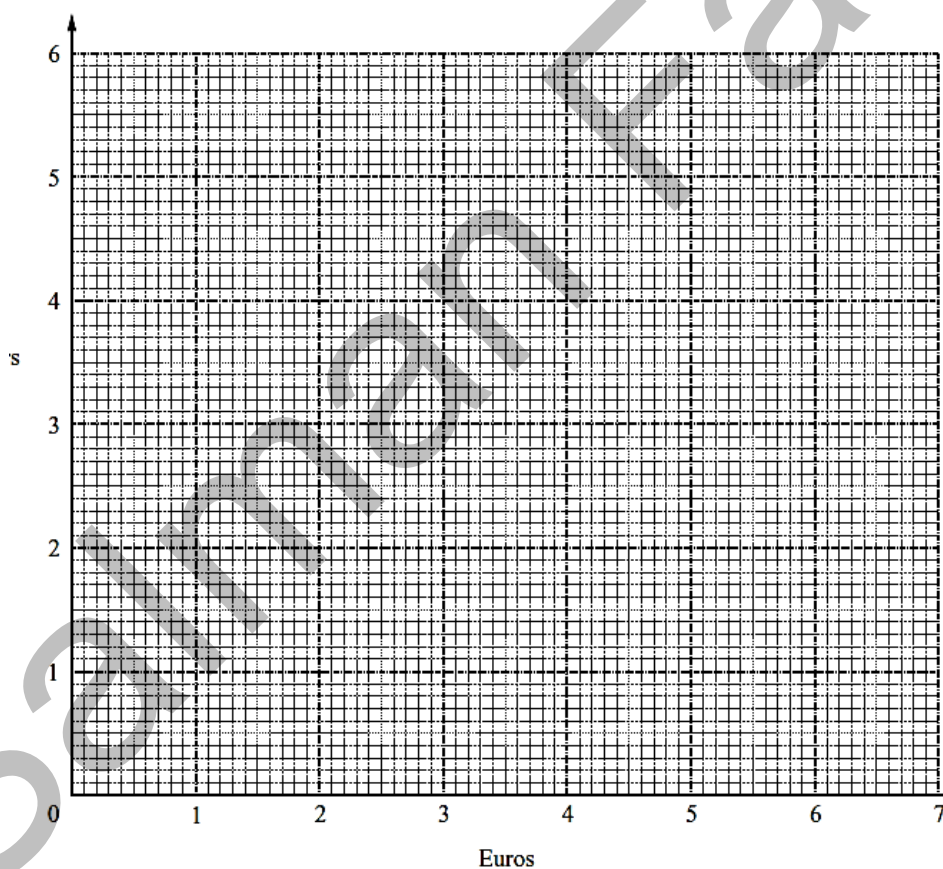
Euros

Answer (b)(ii) .... euros [1]

Answers: **(a)** $0.51; **(b)(i)** Straight line joining (0 , 0) to (6.80 , 5.90), **(ii)** 1.35 to 1.40 euros.   J03/1/Q12

2

String is used to measure distances on two globes, G_1 and G_2.

(a) The length of string required to go around the equator on globe G_1 is 157.5 cm.
The actual length of the equator is 40 000 km.
On the axes below, draw a graph which will enable you to convert lengths of string on the globe G_1 to actual distances.

swer (a)

Actual distance in kilometres

Length of string in centimetres [1]
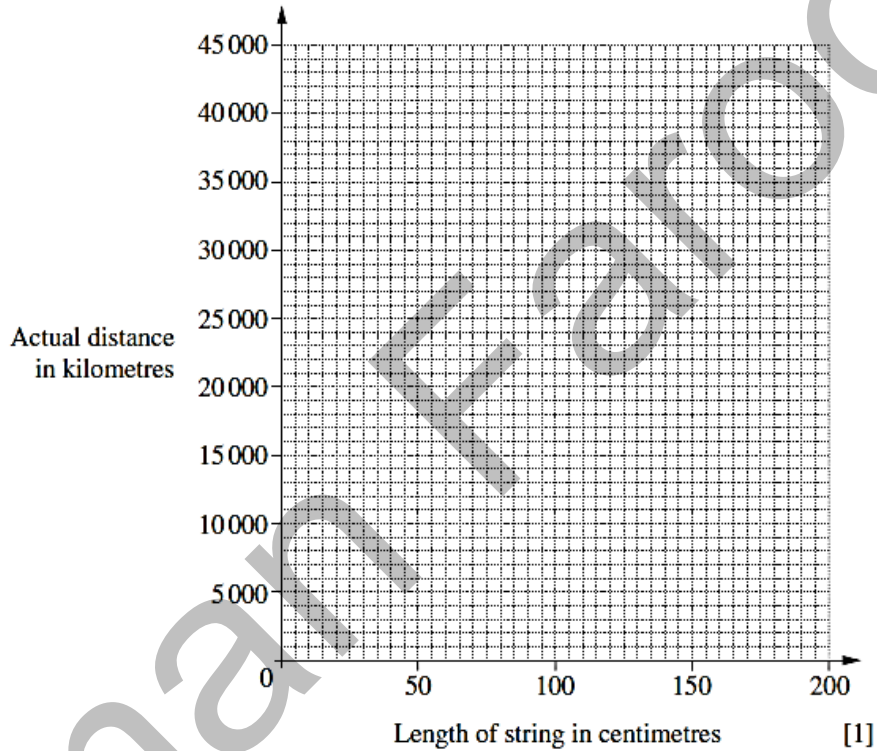
(b) The flight path between two places, *A* and *B*, on globe G_1 requires 35 cm of string.

(i) Use your graph to estimate the actual distance between *A* and *B*.

Answer (b) (i) .... km [1]

(ii) On the other globe G_2, the same flight path between *A* and *B* requires 17.5 cm of string.

Write down the value of $\dfrac{\text{volume of globe } G_2}{\text{volume of globe } G_1}$.

Answer (b)(ii) .... [1]

3 Every month a salesman's pay is made up of a fixed amount plus a bonus.
The bonus is a percentage of his monthly sales.

(a) In 2006 the bonus paid was $m\%$ of his monthly sales.
The graph shows how the salesman's monthly pay varied with his monthly sales.
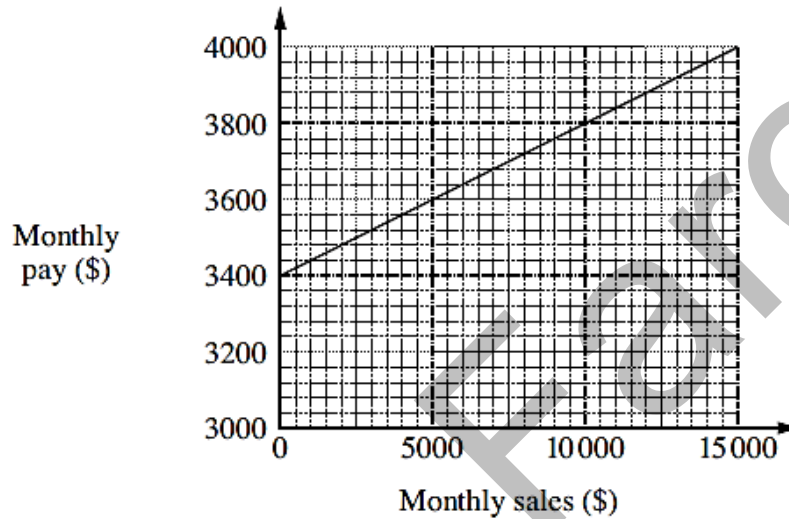
Use the graph to find

(i) the fixed amount,

Answer *(a)*(i) $ ....[1]

(ii) the value of m.

(ii) $m =$ ....[2]

(b) In 2007 the fixed amount was $3500 per month and the bonus was 5% of his monthly sales.
In July his sales were $12 000.
Calculate the salesman's pay for July.   *Answer* *(b)* $ ....[2]

Answers: **(a)(i)** $3400,   **(ii)** 4,   **(b)** $4100.

Kinematics Paper 1

1
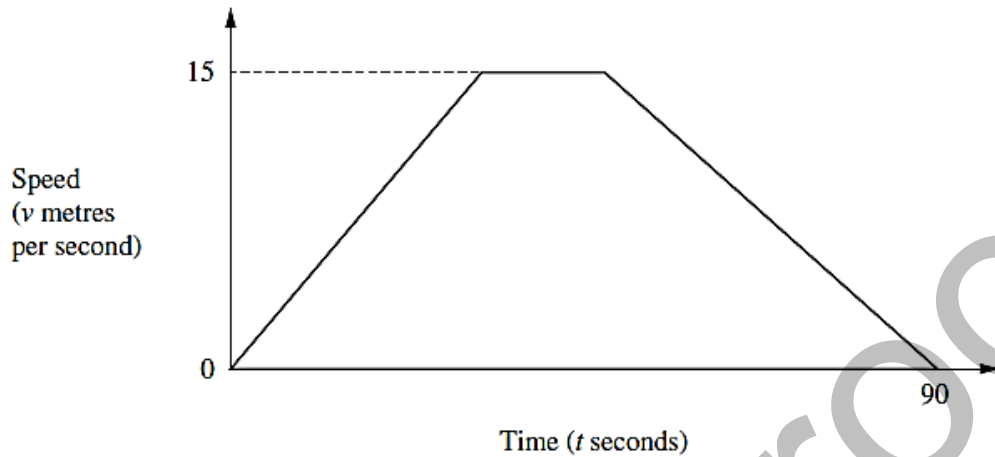
The diagram shows the speed – time graph of a bus over a period of 90 seconds.
The bus reaches a maximum speed of 15 metres per second.

(a) Express 15 metres per second in kilometres per hour. *Answer* (a) .... km/h [1]

(b) Given that the acceleration was $0.5 \, m/s^2$, calculate the time taken, in seconds, to reach its maximum speed. (b) .... s [1]

(c) The total distance travelled during the 90 seconds was 750 metres.
Calculate the length of time that the bus was travelling at its maximum speed.

(c) .... s [2]

2
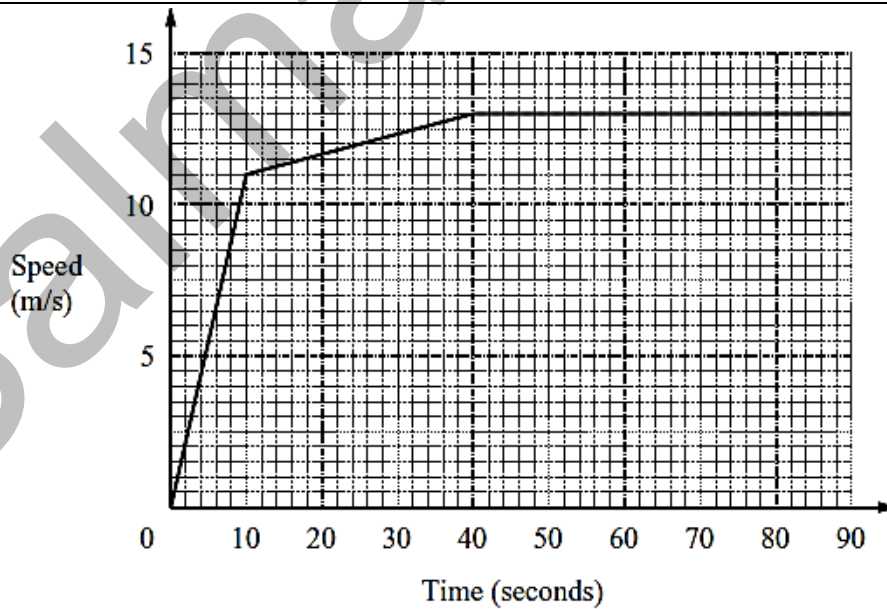
The speed–time graph shows the performance of a cyclist during the first 90 seconds of a race.

(a) Calculate the acceleration of the cyclist during the first 10 seconds. [1]

(b) Calculate the distance, in metres, travelled by the cyclist in the first 90 seconds. [3]

(c) Calculate the time taken for the cyclist to travel 1 kilometre. [2]

Answers: (a) 1.1 m/s²; (b) 1065 m; (c)85 s.   J03/1/Q24

3   A car accelerates uniformly from rest for 30 seconds.
     Its speed after 30 seconds is 18 m/s.
     The speed remains constant for the next 10 seconds.

(a) Draw the speed-time graph for the first 40 seconds of the journey.

Answer (a)

Speed
(metres per second)

Time (seconds)
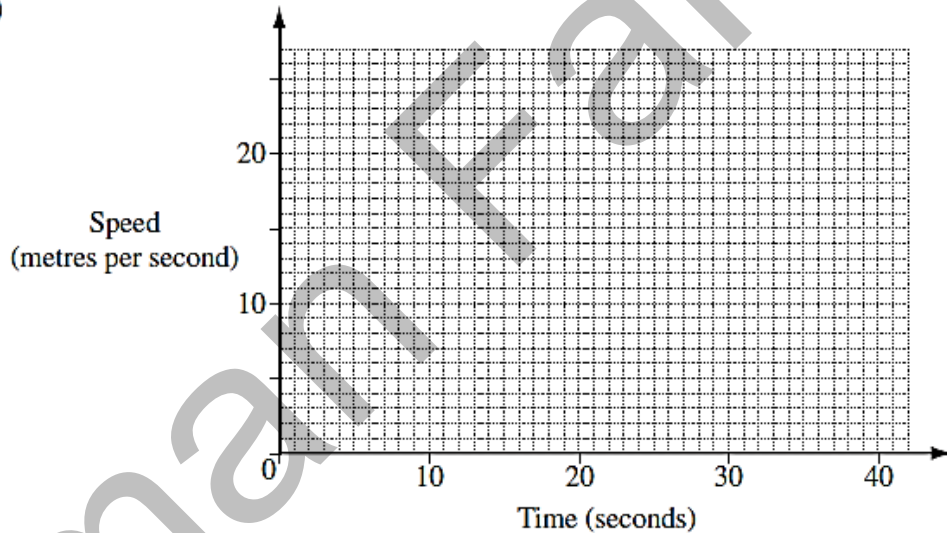
[1]

(b) Calculate

(i)   the car's acceleration during the first 30 seconds,

(ii) its average speed for the first 40 seconds.

Answer (b)(i) ....m/s² [1]

(ii) ....m/s [2]

Answers: (a) Straight lines from (0, 0) to (30, 18) to (40, 18); (b) 0.6 m/s²; (c) 11.25 m/s.   J05/1/Q23

577

4   A cyclist took 30 seconds to ride from *A* to *B*.
The diagram is the speed-time graph of his ride.
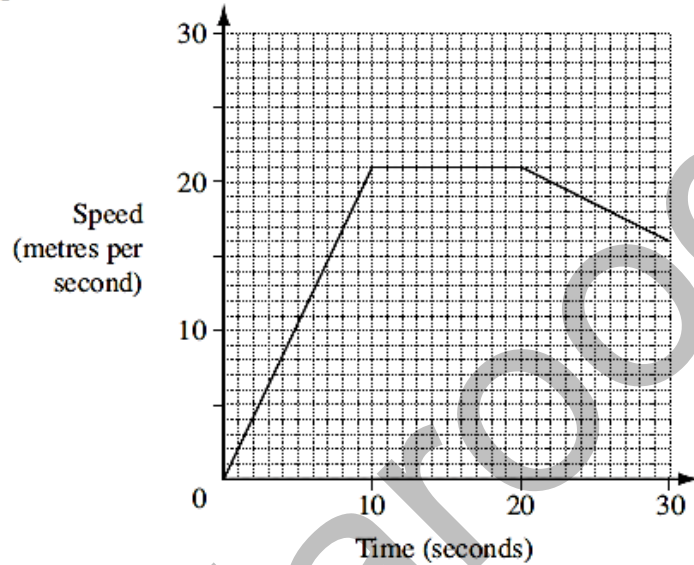
Calculate

(a)   the distance from *A* to *B*,   *Answer* (a) .... m [2]

(b)   his retardation during the final 10 seconds.   (b) .... m/s² [1]

Answer: (a) 500 m   (b)   0.5 m/s²   **J06/1/Q15**

5

(a)   A car decelerates uniformly from 20 m/s to 5 m/s in 25 seconds.
Calculate the retardation.

(b)   Express 20 metres per second in kilometres per hour.

Answer (a) .... m/s² [1]

(b) .... km/h [1]

J07/1/Q4

Answer: (a) 0.6 m/s²   (b) 72 km/h

6

The diagram is the speed-time graph of the first 20 seconds of a motorcyclist's journey.
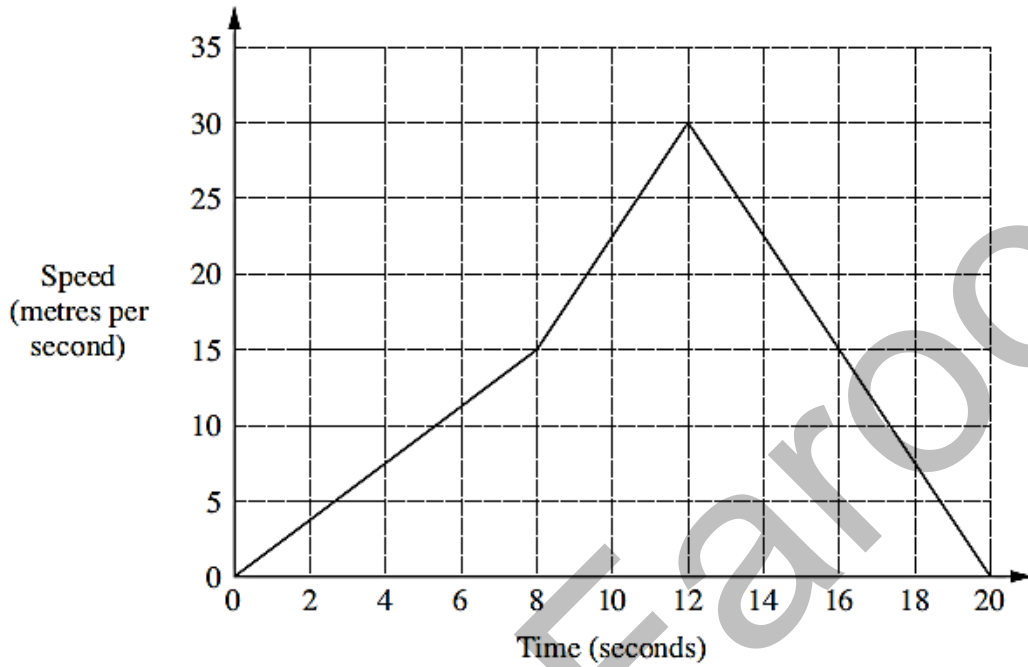
(a) Calculate the motorcyclist's retardation during the final 8 seconds.

(b) Calculate the distance travelled in the 20 seconds.

Answer (a) .... m/s^2 [1]

(b) .... m [2]

Answers: **(a)** 3.75 m/s² **(b)** 270 m

7

22 A walker leaves his house at 10 00 and walks towards a shopping centre at a constant speed of 5 km/h.
A cyclist leaves the same house 10 minutes later.
He travels along the same road at a constant speed of 20 km/h until he reaches the shopping centre which is 6 km from the house.
The cyclist stops at the shopping centre for 14 minutes.
He then returns to the house along the same road at a constant speed of 20 km/h.

(a) The distance-time graph for the walker is drawn below.

On the same axes, draw the distance-time graph for the cyclist.
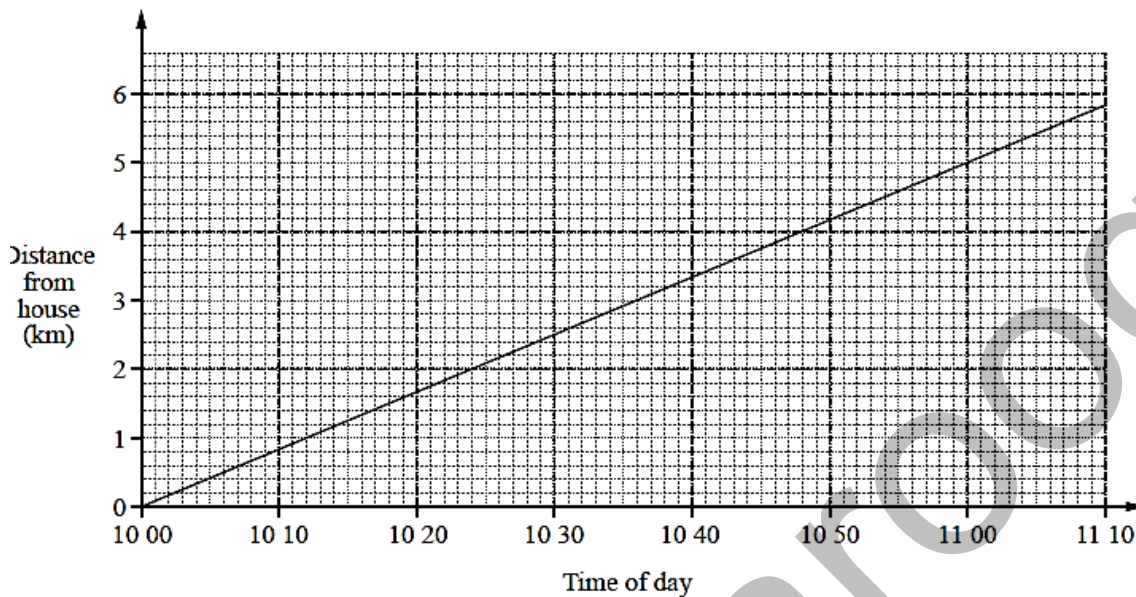
[3]

(b) Using the graphs, find

(i) the time when the cyclist, on his return journey, meets the walker,

Answer (b)(i) .... [1]

(ii) the distance from the house when this meeting takes place.

Answer (b)(ii) ....km [1]

Answer: (a) correct distance-time graph   (b)(i) 10 48   (b)(ii) 4   **J10/11/Q22**

8   The diagram is the speed-time graph for the first 20 seconds of a cyclist's journey.

580
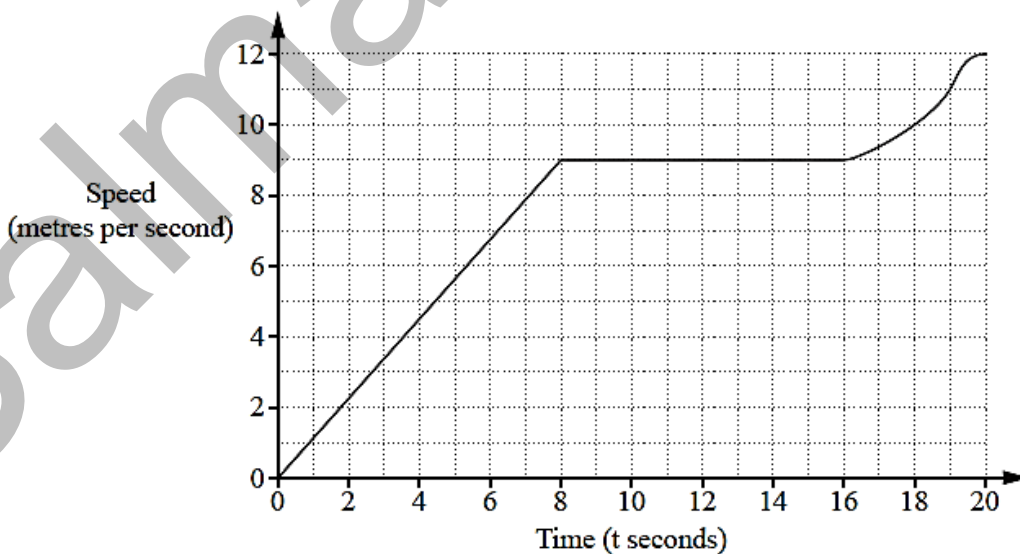
(a) Calculate the distance travelled in the first 16 seconds.

Answer *(a)* .... m [1]

(b) By drawing a tangent, find the acceleration of the cyclist when $t = 18$.

Answer *(b)* .... m/s^2 [2]

(c) On the grid in the answer space, sketch the distance-time graph for the first 16 seconds of the cyclist's journey.

Answer (c)
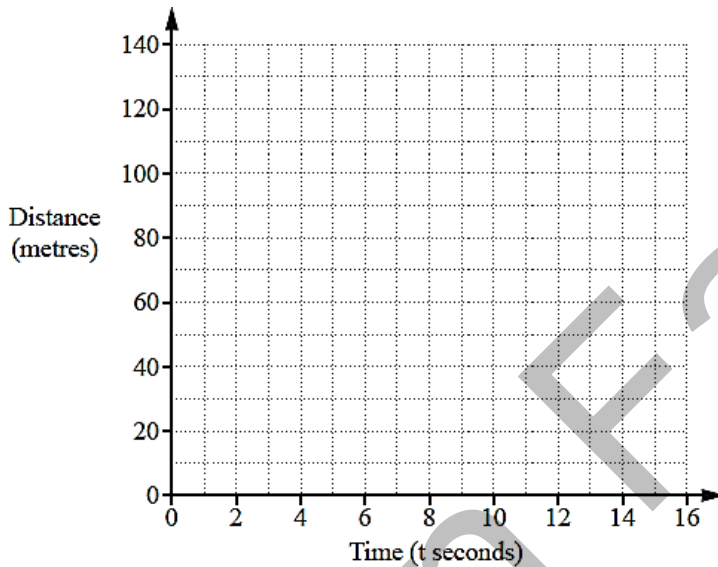
[2]

Answer: **(a)** 108   **(b)** 0.7 ±0.2 with tangent at $t = 18$ **(c)** Correct distance/time graph   **J10/12/Q25**

9 The diagram shows the speed-time graph of a car travelling between two road junctions.
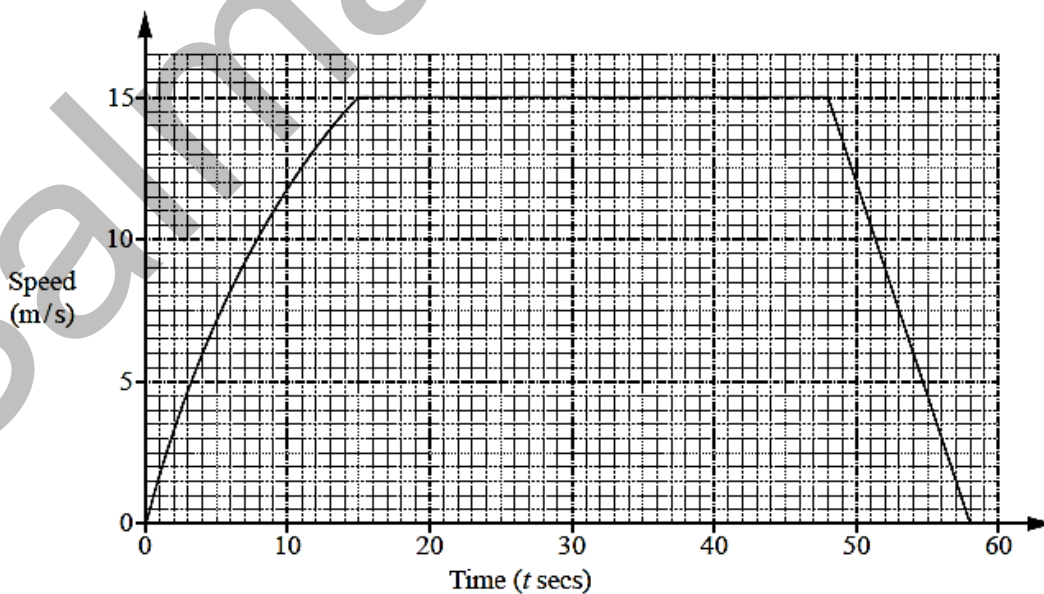
(a) Calculate the retardation of the car between $t = 48$ and $t = 58$.

Answer ....m/s^2 [1]

(b) By drawing a tangent, estimate the acceleration of the car when $t = 8$.

Answer ....m/s^2 [2]

(c) Calculate the distance travelled by the car between $t = 15$ and $t = 58$.

Answer ....m [2]

Answer: **(a)** 1.5 **(b)** 0.7 to 1 **(c)** 570

10
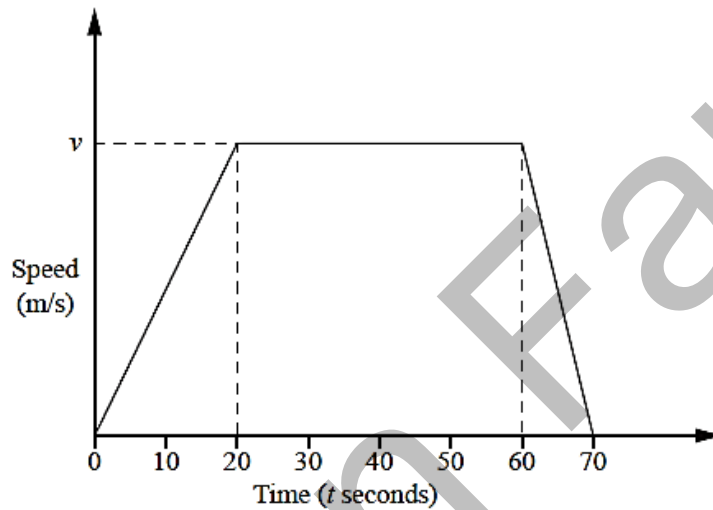
The diagram shows the speed-time graph for 70 seconds of a car's journey.
After 20 seconds the car reaches a speed of v m/s.
During the 70 seconds the car travels 1375 m.

(a) Calculate v.

Answer $v =$ ....[2]

(b) Calculate the acceleration of the car during the first 20 seconds.

Answer ....m/s^2 [1]

Answer: **(a)** 25 **(b)** 1.25

11
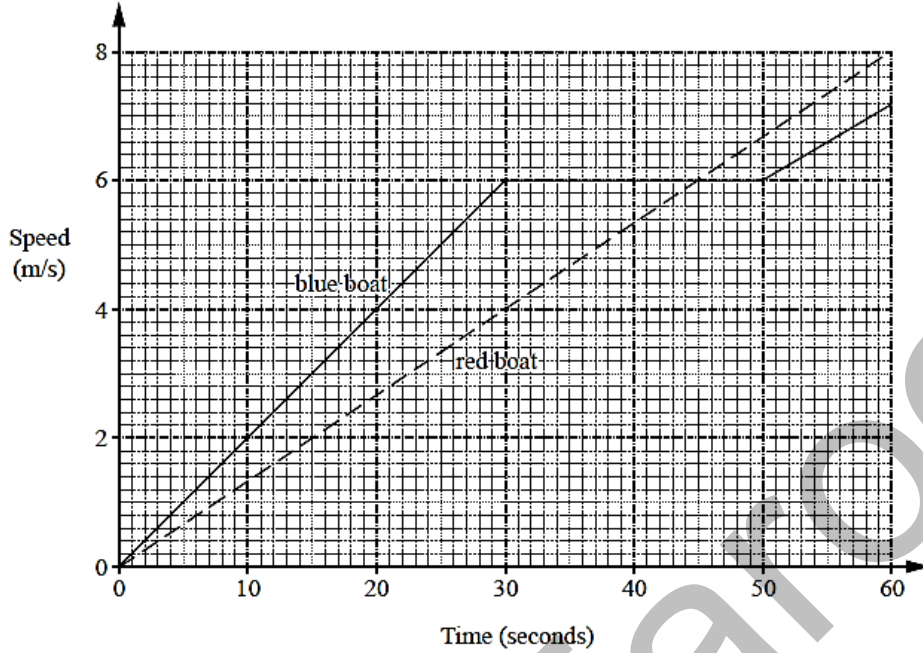
Two boats, one red and one blue, leave a harbour at the same time.
They travel in the same direction.
The speed-time graphs for the boats are shown, for the first minute of their journey.

(a) Find the acceleration of the blue boat in the last 10 seconds.

Answer ....m/s² [1]

(b) Find which boat is ahead after one minute and by what distance.

Answer .... is ahead by .... m [3]

Answers: (a) 0.12 (b) Blue boat, 36

J14/11/Q18

12   Kim and Lee run a 2000 metre cross-country course that starts at P and ends at Q.
Lee starts 1 minute after Kim.
Their distance-time graphs are shown in the diagram.
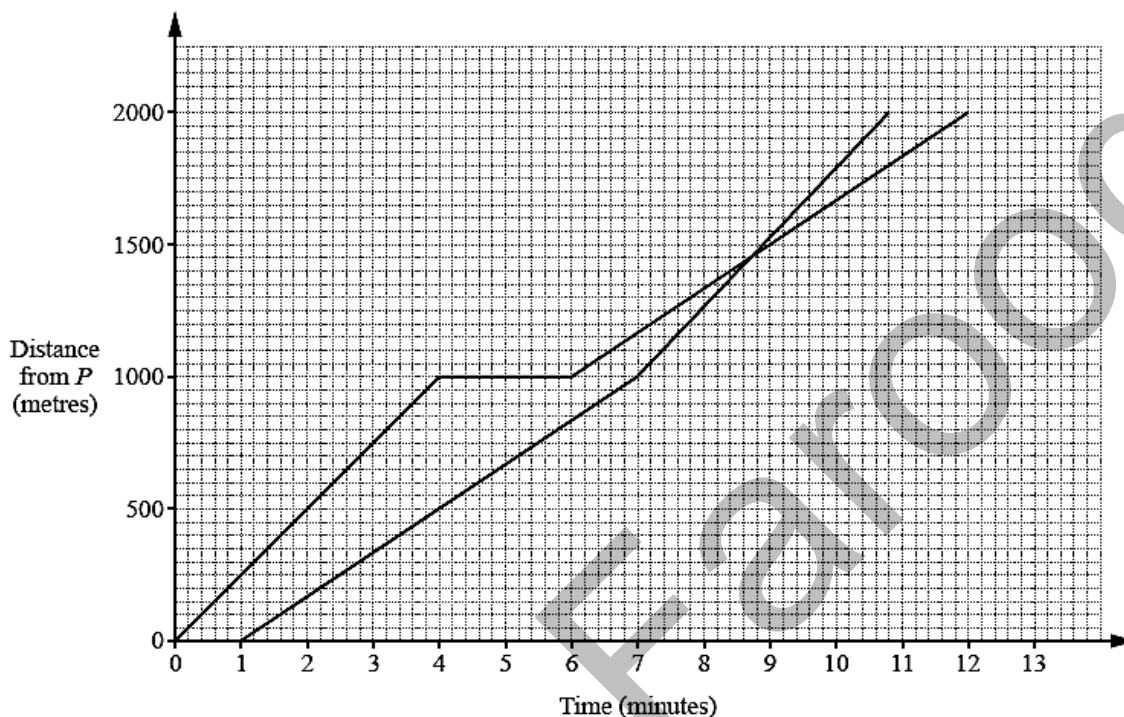
(a) Find the distance Lee has run when he overtakes Kim.

Answer .... m [1]

(b) Find how much longer Kim takes to complete the course than Lee.

Answer .... minutes [1]

(c) Melvin starts 3 minutes after Kim.
He runs the course in the **opposite** direction to that taken by Kim and Lee.
He runs at a constant speed and takes 10 minutes to reach P.

(i) On the diagram, draw the distance-time graph for Melvin. [1]

(ii) Express Melvin's speed in km/h.   *Answer* .... km/h [1]

Answers: **(a)** 1450 **(b)** 2.2 **(c)(ii)** 12

13 The diagram shows the speed-time graph of a train which slows down from 20m/s to a stop in T seconds.
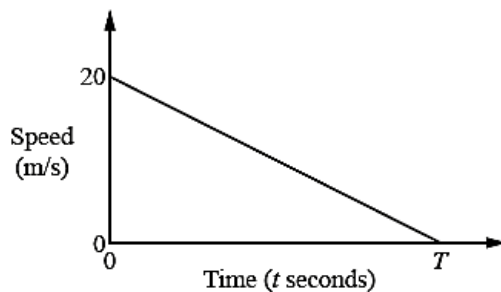
(a) (i) Find an expression, in terms of T, for the retardation of the train.

Answer .... m/s² [1]

(ii) Find the speed of the train when $t = \frac{3}{4}T$.

Answer .... m/s [1]

(b) The distance travelled by the train between $t = 0$ and $t = T$ is 150m.

(i) Find T.

Answer T = ... [1]

(ii) On the diagram, sketch the distance–time graph of the train.
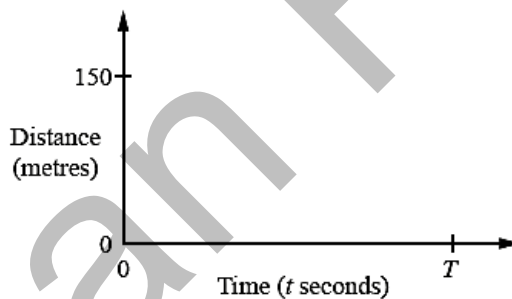
[1]

Answers: (a)(i) $\frac{20}{T}$ (a)(ii) 5 (b)(i) 15 (b)(ii) Curve, concave down, from (0,0) to (T,150)

J16/11/Q24

585

14
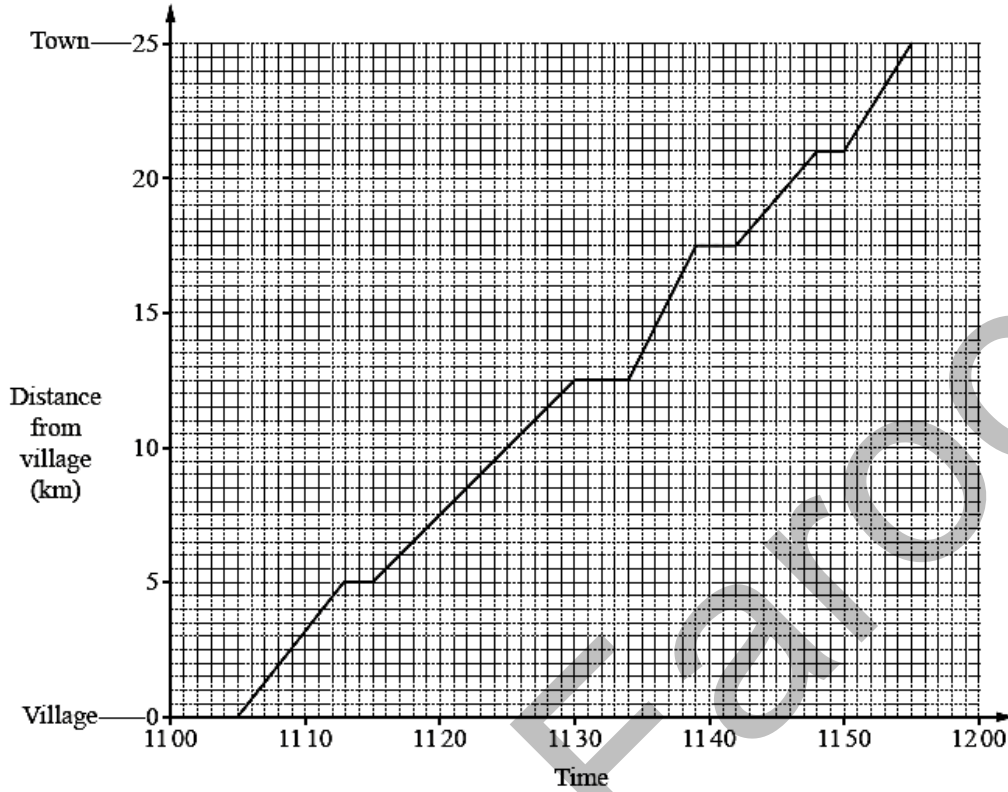
The distance-time graph shows the journey of a red bus travelling from a village to a town.

(a) Find the total length of time for which the bus is stopped during the journey.

Answer ....minutes [1]

(b) Find the average speed of the bus over the **whole** journey from the village to the town.

Answer ....km/h [1]

(c) A yellow bus leaves the **town** at 11 25 and travels non-stop along the same road to the village at a constant speed of 50 km/h.

(i) On the graph draw the distance-time graph for the yellow bus. [1]

(ii) At what time does the yellow bus meet the red bus? *Answer* ....[1]

Answers: **(a) 11 (b) 30 (c)(i)** line joining (1125, 25) to (1155, 0) **(ii)** 1136 – 1137   **J17/11/Q20**

586

15   A car starts from rest and accelerates at a constant rate to a speed of 20 m/s in 10 seconds.

(a) Find the acceleration.

It then travels at a constant speed of 20 m/s for the next 10 seconds.

(b) Find the total distance travelled in the 20 seconds.

(c) On the axes below, draw
- (i) the speed-time graph for the first 20 seconds of the car's journey,
- (ii) the distance-time graph for the same 20 seconds.

Answer (a) .... m/s² [1]

(b) .... m [1]

Answer (c) (i)

Speed
(metres per
second)

30
20
10
0

Time (seconds)

10   20

[1]

(ii)

Distance
(metres)

400
300
200
100
0

Time (seconds)

10   20

[2]

587
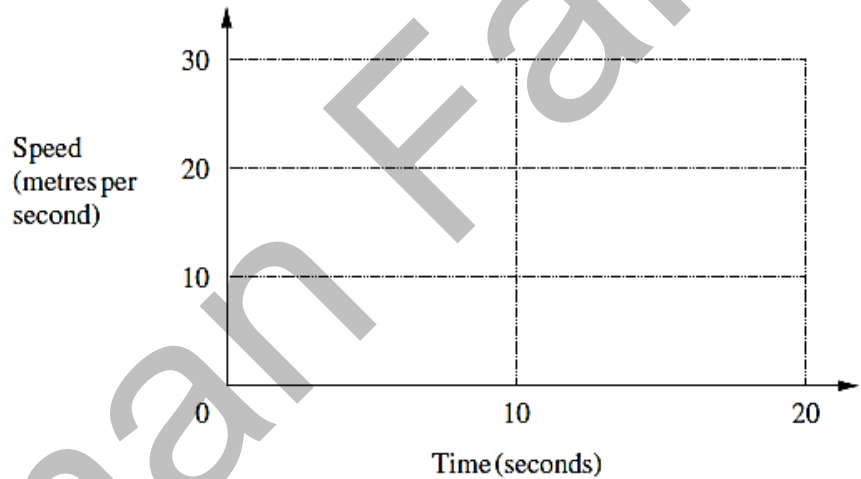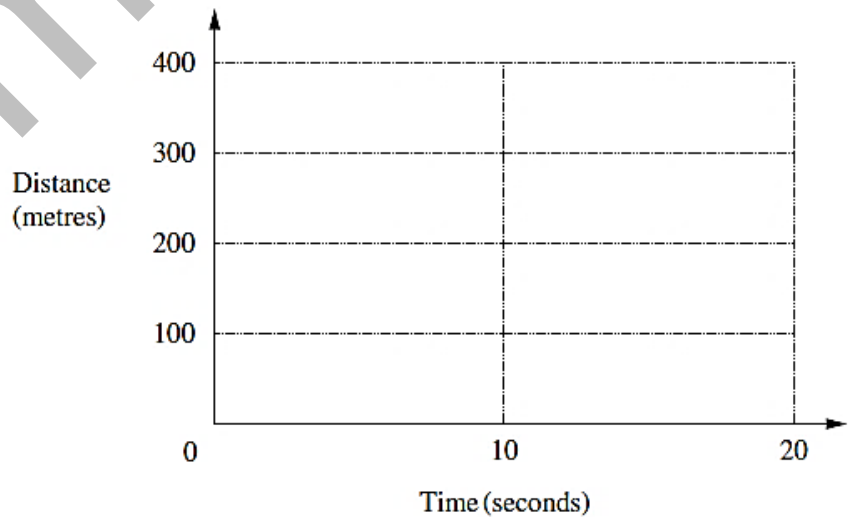
Answers: **(a)** 2m/s²; **(b)** 300m; **(c)(i)** a straight line from (0, 0) to (10, 20) and a horizontal straight line from (10, 20) to (20, 20), **(ii)** a curve from (0,0) to (10, 100) and a straight line from (10, 100) to (20, 300).

16   The diagram shows the distance – time graphs of the journeys of Ali and Bala from home to school.
They leave home together and follow the same route.
Ali runs to school and Bala cycles.
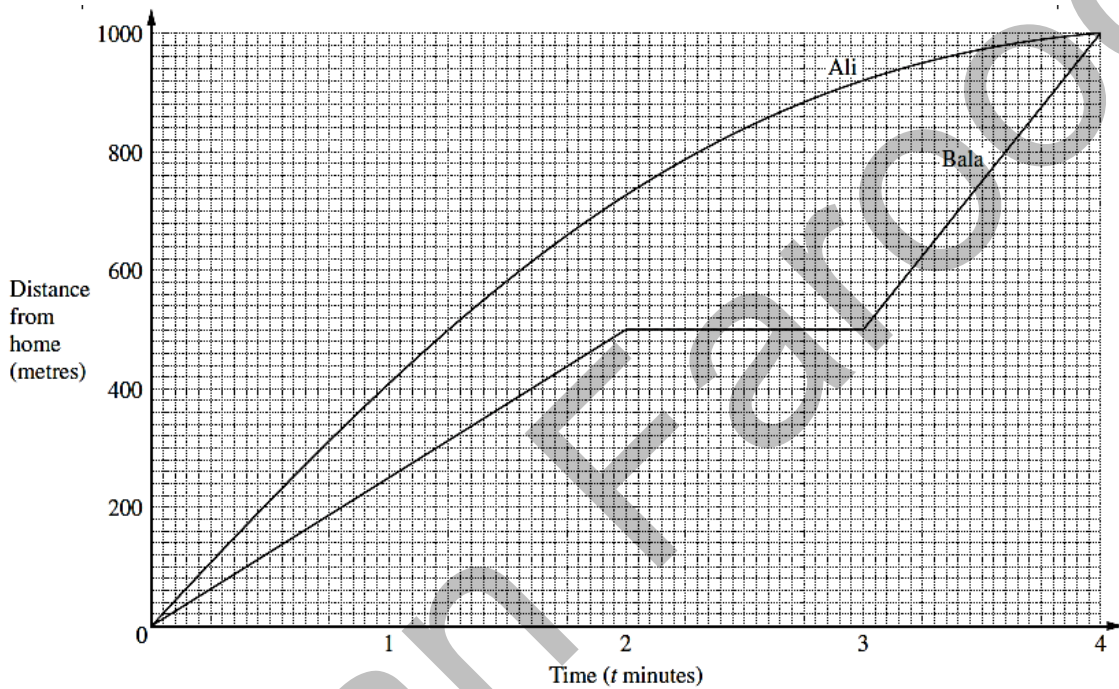
(a)   How long does it take Ali to run the first 700 m?   *Answer (a)* ....minutes [1]

(b)   Estimate the distance between Ali and Bala when $t = 3$.   *(b)* ....m [1]

(c)   Find Bala's speed when $t = 2.6$.   *(c)* ....m/minute [1]

(d)   Find Bala's speed when $t = 3.5$.   *(d)* ....m/minute [1]

Answers: **(a)** 1.9 minutes; **(b)** 420 m; **(c)** 0 m/minute; **(d)** 500 m/minute.

17
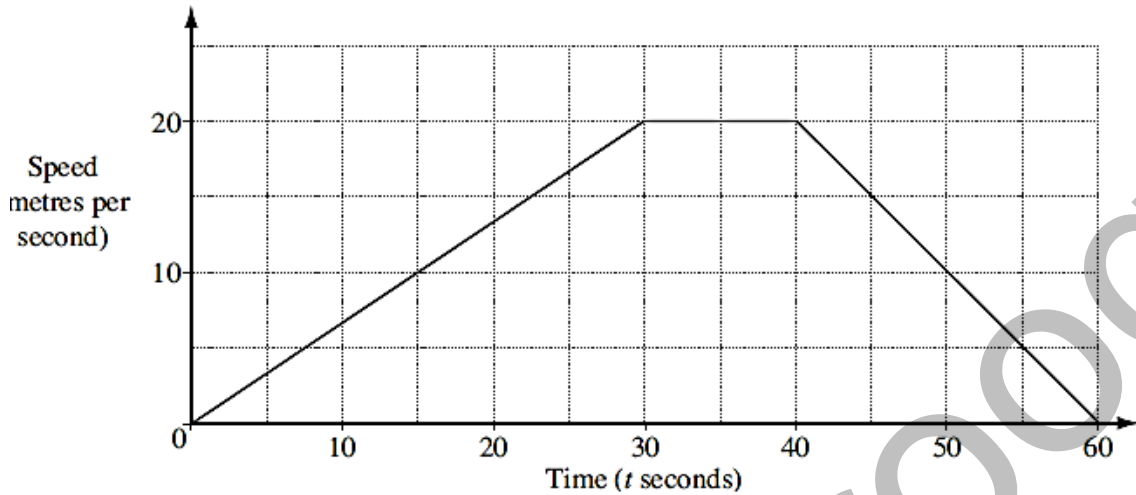
The diagram shows the speed–time graph of a car's journey.

(a) Find the speed when $t = 20$.   *Answer (a)* ....m/s [1]

(b) Find the acceleration when $t = 20$.   *(b)* ....m/s^2 [1]

(c) Find the distance travelled in   *(c)(i)* ....m [1]

    (i)   the first 40 seconds,   *(ii)* ....m [1]

    (ii) the first 60 seconds.

(d) Part of the distance–time graph for the same journey is shown in the answer space.
    Complete this graph.

(d)

[2]
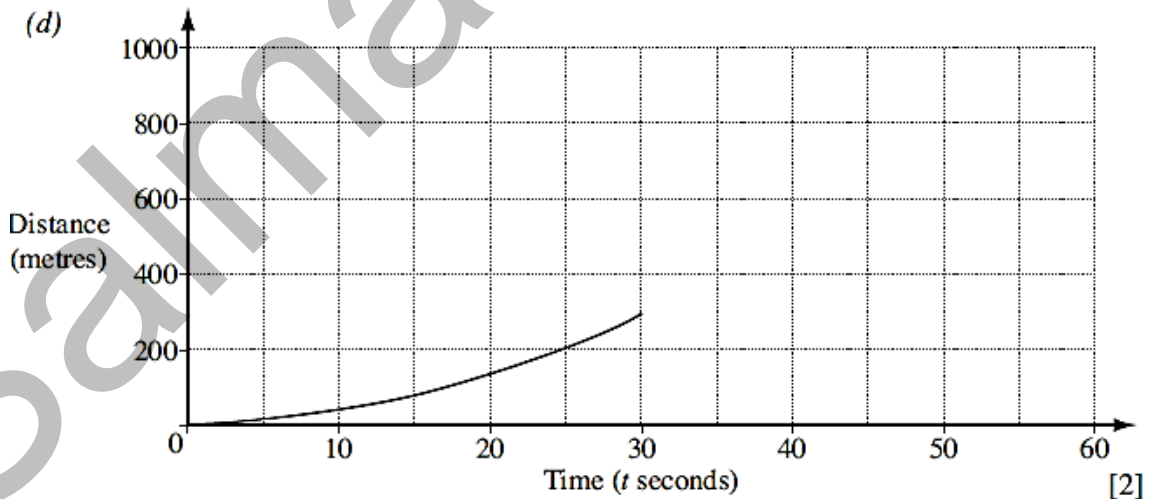
Answers: **(a)** 13 – 14 m/s; **(b)** $\frac{2}{3}$ m/s^2; **(c)(i)** 500 m, **(ii)** 700; **(d)** straight line from (30, 300) to (40, 500), curve with decreasing gradient from (40, 500) to (60, 700).

N05/1/Q20

18

Speed (metres per second)

Time (*t* seconds)
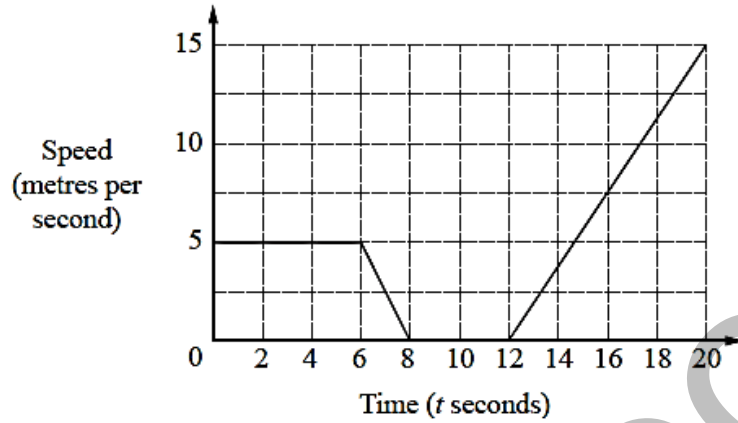
The diagram is the speed-time graph for the first 20 seconds of a journey.

(a) Find

(i) the acceleration when $t = 16$,

(ii) the distance travelled in the first 20 seconds.

Answer (a) **(i)** ....m/s^2 [1]

(ii) ....m [1]

(b) On the grid in the answer space, sketch the distance-time graph for the same journey.

Answer (b)

Distance travelled (metres)

Time (*t* seconds)
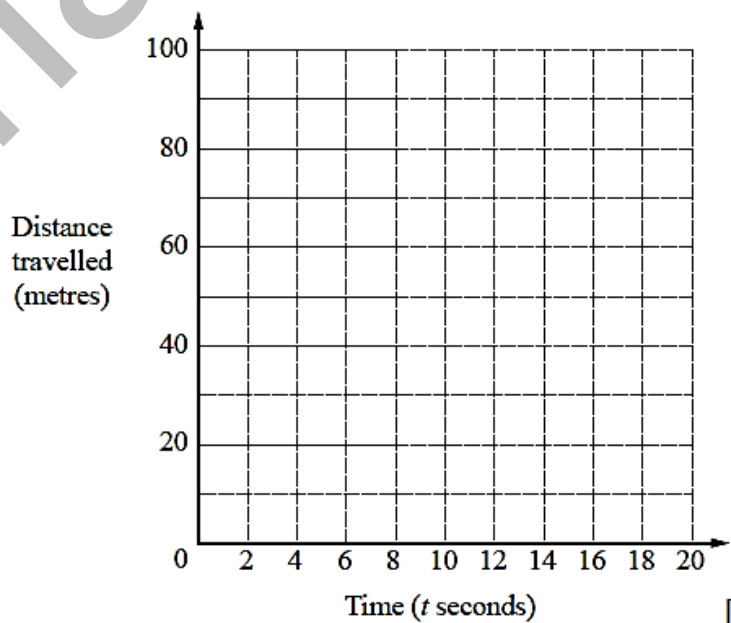
[3]

Answer: (a)(i) $\frac{15}{8}$ m/s², (ii) 95 m.

19   **22**
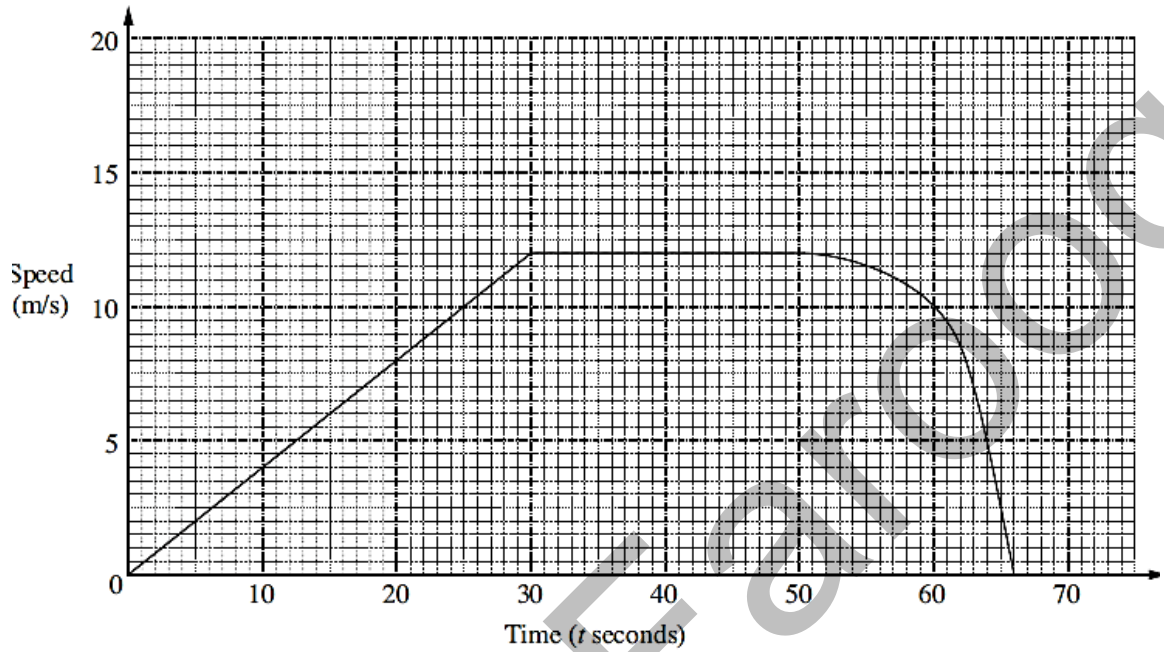
Time (*t* seconds)

The diagram is the speed-time graph of a cyclist's journey.

(a) Calculate the time taken to travel the first 300 metres.

(b) By drawing a tangent, find the retardation of the cyclist when $t = 55$.

Answer (a) .... s [2]

(*b*) .... m/s² [2]

Answer: (a) 40 seconds   (b) 0.12 to 0.24 (+ or −)

20
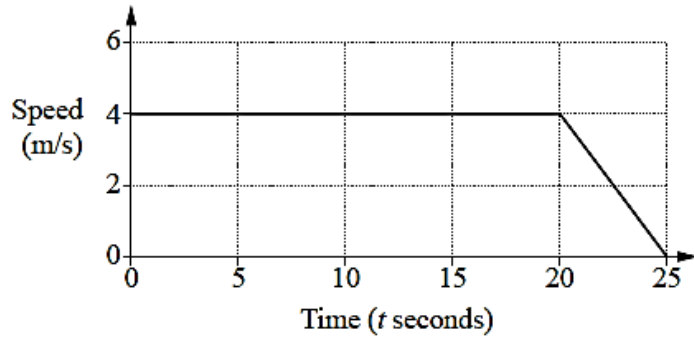
The diagram is the speed-time graph of the last 25 seconds of a car's journey.
From $t = 0$ to $t = 20$ the car moves with a constant speed of 4 m/s.
From $t = 20$ to $t = 25$ the car moves with a constant retardation.

(a) Calculate the retardation when $t = 22.5$.

Answer (a) .... m/s^2 [1]

(b) Show that the distance travelled during the 25 seconds is 90 m.

Answer (b) ....

....[1]

(c) On the grid below, draw the distance-time graph for the 25 seconds.

Answer (c)

[2]

N10/11/Q25

21
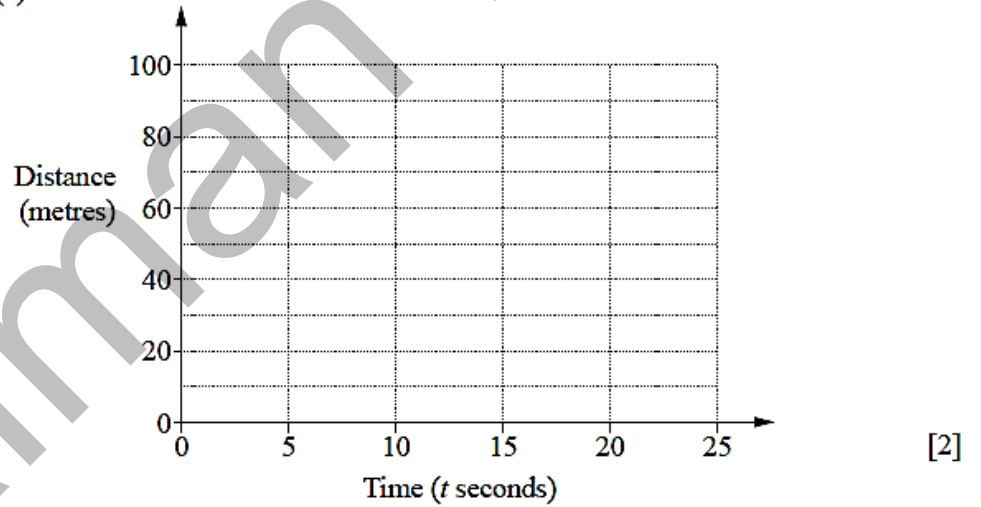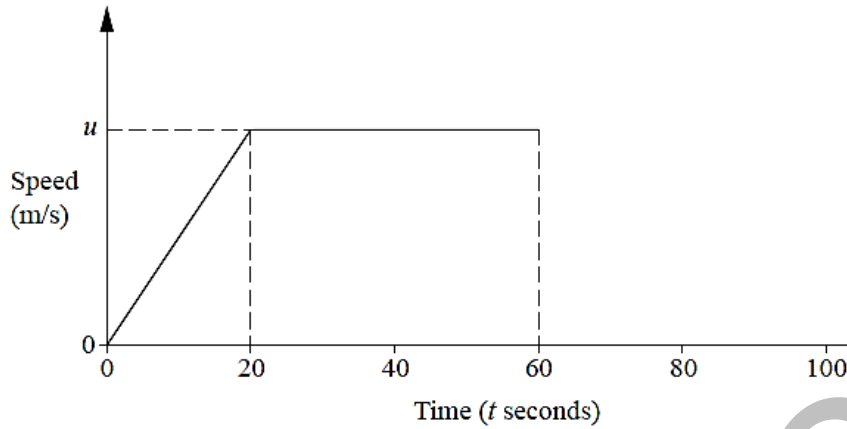
The diagram is the speed-time graph of part of the journey of a car.
From $t = 0$ to $t = 20$ the car moves with a constant acceleration.
From $t = 20$ to $t = 60$ the car moves with a constant speed of u metres per second.

(a) When $t = 20$ the car has travelled D metres from the start.

Calculate the value of t when the car has travelled $2D$ metres **from the start**.

Answer *(a) t =* .... [2]

(b) At $t = 60$, the car slows down with a constant deceleration.
This deceleration is half of the acceleration between $t = 0$ and $t = 20$.

During this period of deceleration, calculate the value of t when the car has a speed
of $\frac{u}{4}$ metres per second.   *Answer*   *(b) t =* .... [2]

Answer: **(a)** 30 **(b)** 90   **N10/12/Q25**

22
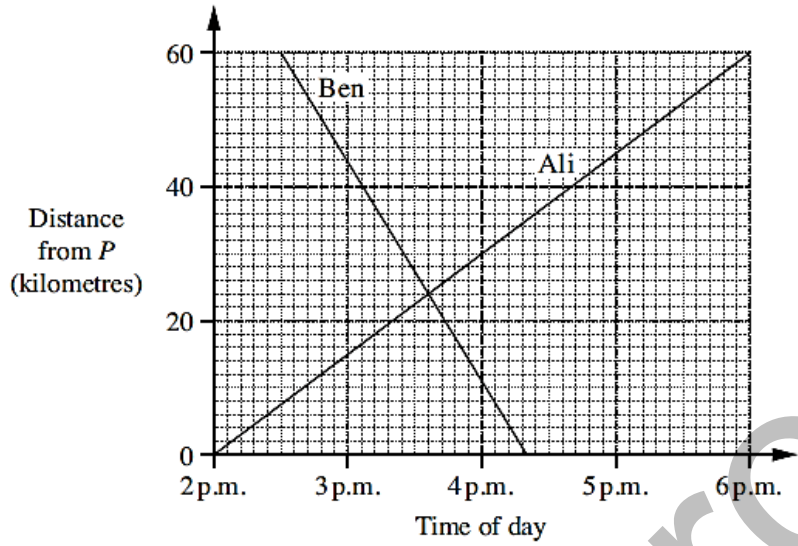
Ali and Ben each made a journey between two towns, P and Q, that are 60 km apart.
These two journeys are shown on the travel graph.

(a) Calculate Ali's speed.

Answer .... km/h [1]

(b) Find the number of minutes after 3 p.m. that Ali and Ben passed each other.

Answer .... [1]

(c) Find how far Ben had travelled when he met Ali.

Answer .... km [1]

(d) Chris left P at 3 p.m. and travelled to Q at a speed of 30 km/h.

On the diagram, draw the graph that represents Chris's journey. [1]

Answers: (a) 15 (b) between 33 and 39 inclusive (c) 36 (d) straight line from (3, 0) to (5, 60)   **N11/11/Q23**

23
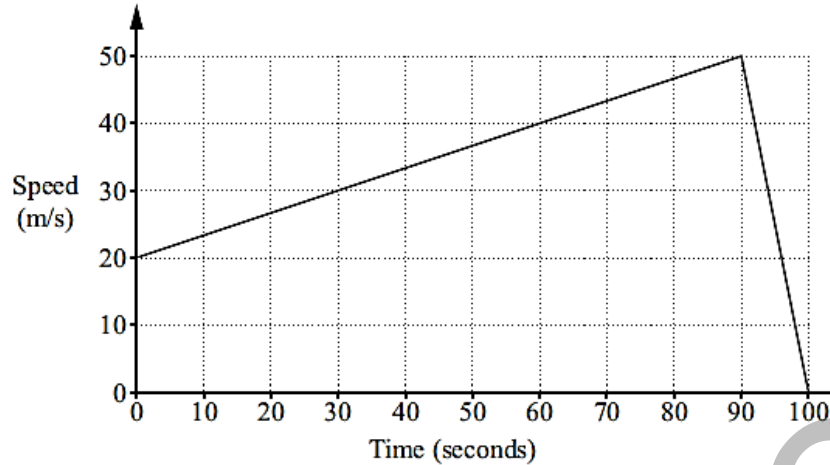
The diagram is the speed-time graph of the last 100 seconds of a train's journey.

(a) Calculate the train's retardation during the last 10 seconds of the journey.

Answer .... m/s² [1]

(b) Calculate the distance travelled in the 100 seconds. Answer .... m [2]

Answer: (a) 5 (b) 3 400

24

The diagram is the speed-time graph of part of a train's journey.

The train slows down uniformly from a speed of 50 m/s to a speed of 10 m/s in a time of 20 seconds.

During the next 30 seconds, it accelerates uniformly to a speed of u metres/second.
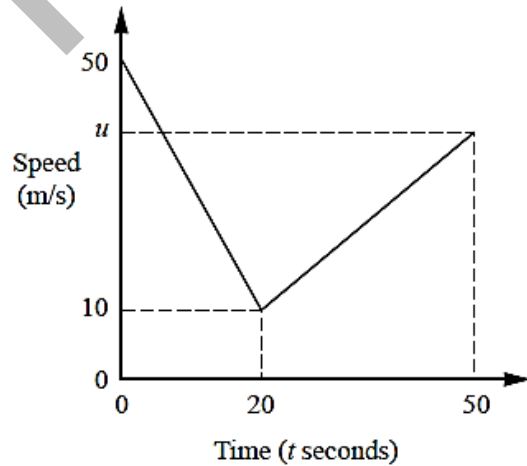
(a) Calculate the retardation from $t = 0$ to $t = 20$.

Answer .... m/s² [1]

(b) Calculate the speed of the train when $t = 15$.

Answer .... m/s [1]

(c) Calculate the distance travelled by the train from $t = 0$ to $t = 20$.

Answer .... m [1]

(d) The size of the acceleration is half the size of the retardation. Find the value of u.

Answer $u = $ .... [1]

595

25

The diagram is the speed-time graph of part of a train's journey.

The train slows down uniformly from a speed of u m/s to a speed of 6 m/s in 10 seconds.

During the next 20 seconds it travels at a constant speed of 6 m/s.

It then slows down uniformly to a stop after a further 30 seconds.
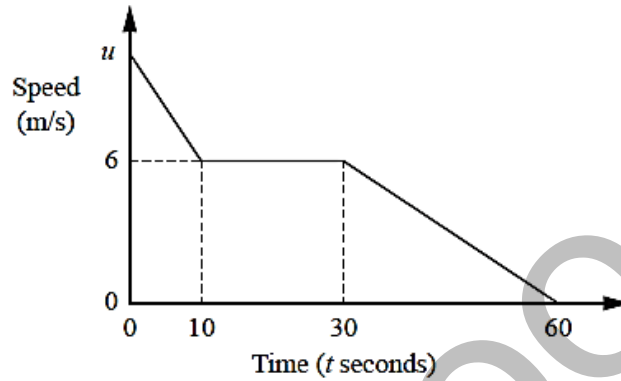
(a) Calculate the retardation from $t = 30$ to $t = 60$.

Answer ....m/s² [1]

(b) Calculate the speed of the train when $t = 40$.

Answer .... m/s [1]

(c) The distance travelled by the train from $t = 0$ to $t = 10$ is 85 m.

Find u.

Answer $u =$ .... [2]

26
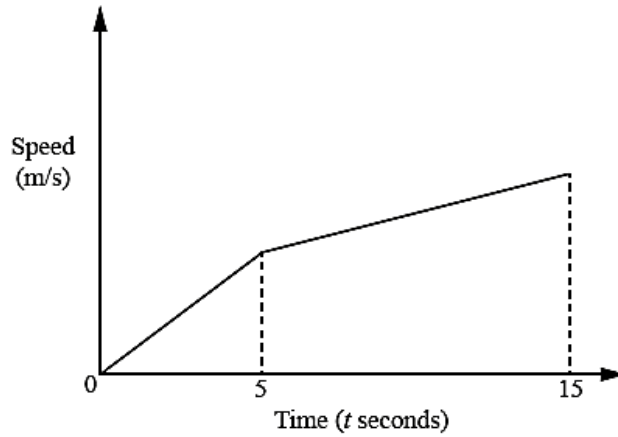
The diagram shows the first 15 seconds of a car's journey.
The car starts from rest.
The acceleration of the car from $t = 0$ to $t = 5$ is $4\,\text{m/s}^2$.
The acceleration of the car from $t = 5$ to $t = 15$ is $2\,\text{m/s}^2$.

(a) Find the speed of the car when

  (i) $t = 5$,

  Answer .... m/s [1]

  (ii) $t = 15$.

  Answer .... m/s [1]

(b) Find the distance travelled by the car between $t = 5$ and $t = 15$.

  Answer .... m [2]

Answers: **(a)(i)** 20 **(ii)** 40 **(b)** 300

N15/11/Q20

27 The diagram is the speed-time graph of part of a car's journey.
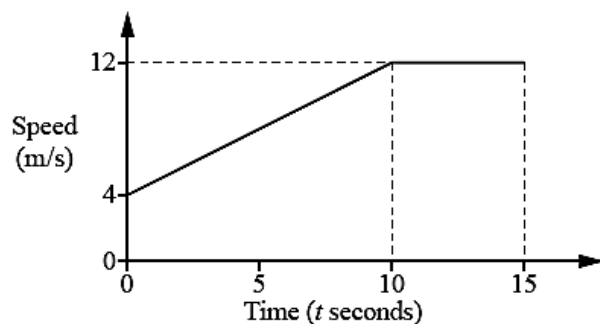
(a) Find the speed when $t = 8$.

Answer .... m/s [2]

b) Find the distance travelled by the car from $t = 0$ to $t = 10$.

Answer .... m [2]

(c) On the diagram below sketch the distance-time graph for $t = 0$ to $t = 15$.
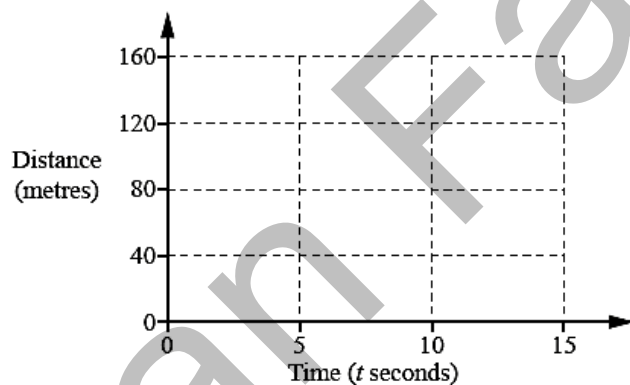
[2]

Answers: (a) 10.4 (b) 80 (c) Curve, concave upwards, from (0, 0) to (10, their (b)) and straight line from (10, their (b)) to (15, 60 + their (b))   **N16/11/Q22**

28 The diagram is the speed-time graph of a train which travels between two stations.

Speed (m/s) vs Time (t seconds)
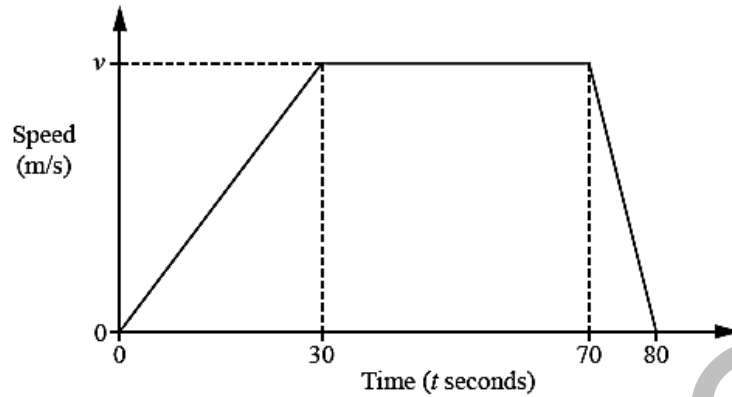
(a) Find an expression, in terms of v, for the retardation of the train.

Answer .... m/s² [1]

(b) The distance between the two stations is 1.2 km.

Find v.

Answer $v =$ .... [3]

Answers: (a) $\dfrac{v}{10}$ (b) 20

N17/11/Q22

Kinematics Paper 2

1 **(a)**
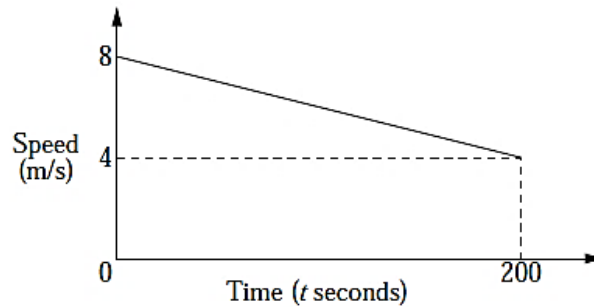
Ali was on a training run.
The diagram is the speed-time graph of part of his run.
At $t = 0$, his speed was 8 m/s.
His speed decreased at a constant rate until it was 4 m/s at $t = 200$.

(i) Calculate

(a) his retardation during the 200 s, [1]

(b) the distance he ran during the 200 s, [2]

(c) his speed at $t = 150$. [1]

(ii) Ben ran at a constant speed in the same direction as Ali.
At $t = 0$, Ali and Ben were level.
They ran the same distance in the next 150 seconds.

Calculate Ben's speed. [2]

Answers: (a)(i)(a) 0.02 m/s², (i)(b) 1200 m, (i)(c) 5 m/s, (ii) 6.5 m/s; **J09/2/Q5**

600

2 **(a)**
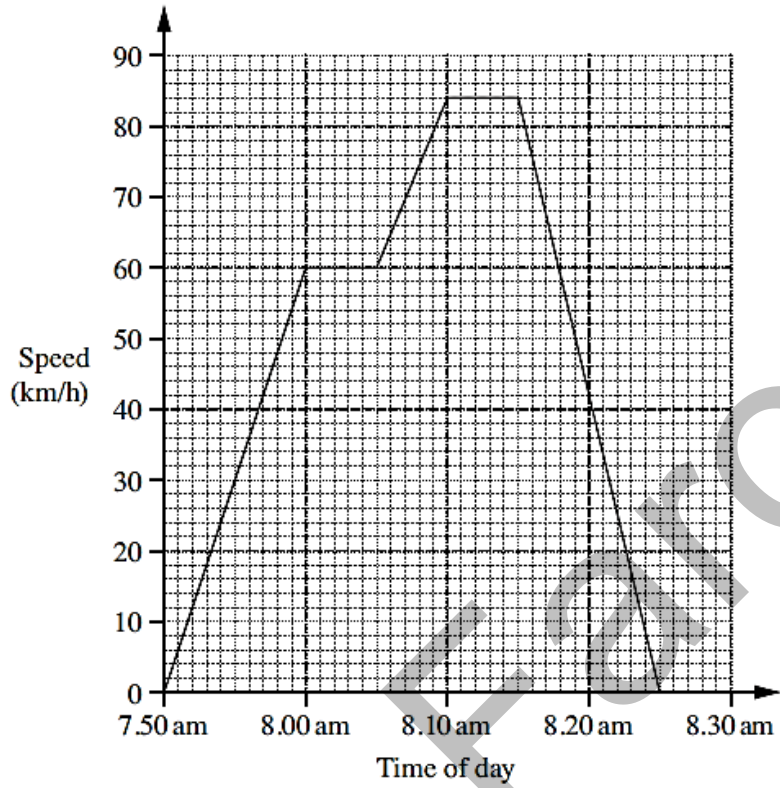
The speed-time graph represents Brian's car journey to work on Monday.

(i) How long does his journey take?

Answer .... minutes [1]

(ii) During the first 10 minutes he travels with a constant acceleration.

Find this acceleration in kilometres per hour per hour.

Answer .... km/h^2 [1]

(iii) How far does Brian travel at his maximum speed?

Answer .... km [1]

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(b) On Tuesday Brian leaves home at 8.00 am and travels 30 km to work.
On the first part of his journey he travels for 15 minutes at a speed of 40 km/h.
On the second part of his journey he travels 12 km in 8 minutes at a constant speed.
On the third part of his journey he travels at a constant speed.
He arrives at work at 8.47 am.

(i) Find the distance he travels on the first part of his journey.

Answer .... km [1]

(ii) On the axes below, draw a distance-time graph to represent his journey to work on Tuesday.

[2]
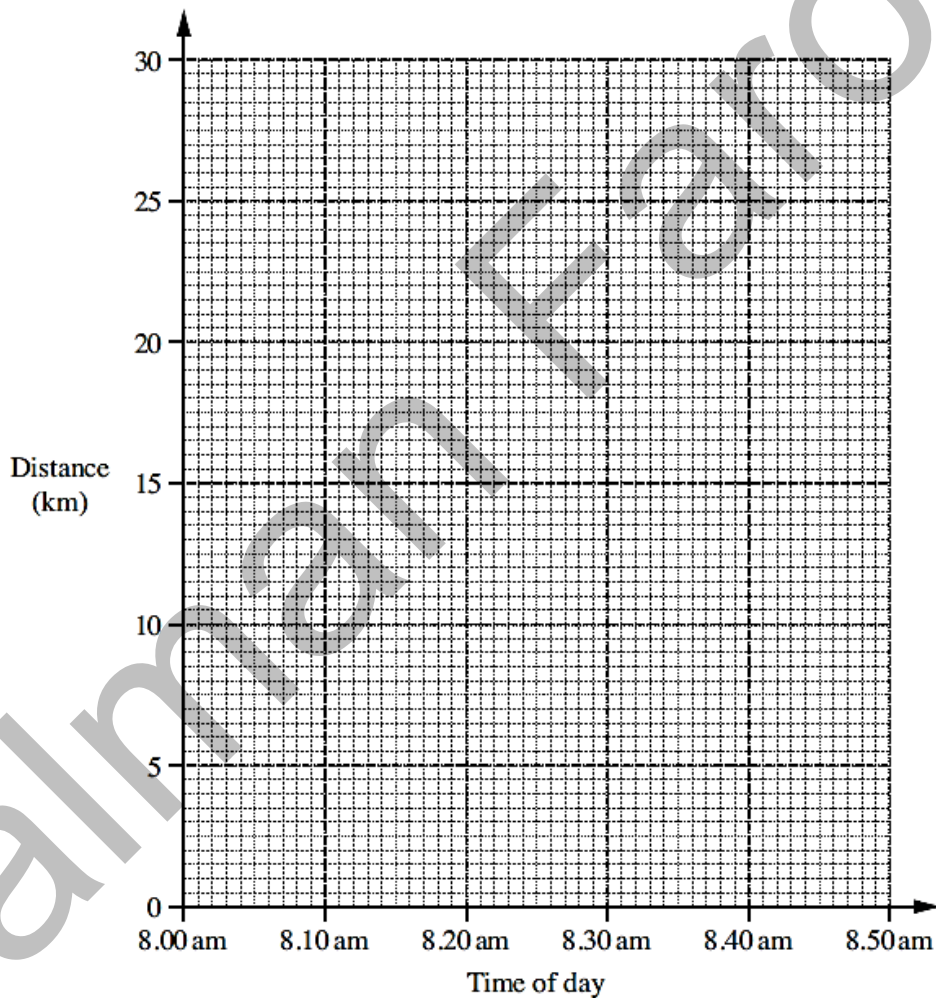
(iii) Calculate the speed, in kilometres per hour, for the third part of his journey.

Answer .... km/h [2]

Answers: (a) (i) 35 (ii) 360 (iii) 7 (b)(i) 10 (iii) 20

J11/21/Q11

3

The distance-time graph shows Ravi's cycle journey.
He sets out from home and cycles to a park.
After a short stop at the park, he then continues his journey to a shopping centre.
He stops for lunch at the shopping centre before cycling home.

(a) At what time does Ravi arrive at the park?   *Answer*   ....[1]

(b) How many minutes does Ravi spend at the shopping centre?

    Answer   .... minutes [1]

(c) How far is the park from the shopping centre?   *Answer*   ....km [1]

(d) At what speed does Ravi cycle home?   *Answer*   ....km/h [1]
    Give your answer in kilometres per hour.

(e) Between which two places did Ravi cycle slowest?   *Answer*   .... and ....[1]

(f) Salim, Ravi's brother, sets out from home at 11 15.
    He cycles directly to the shopping centre at a constant speed of 15 km/h.

    Who arrives at the shopping centre first?
    How many minutes later does his brother arrive?

    Answer .... arrives first and his brother arrives .... minutes later. [2]

Answers: (a) 11 30 (b) 39 (c) 8 (d) 24 (e) park and shopping centre (f) Salim, 9   **J11/22/Q5**

4 **(c)**

The travel graph represents Sahid's journey to and from a football match.
He travels from home straight to the football stadium and watches the match.
On his way home he stops at a café.

(i) How many minutes does Sahid spend at the football stadium?

Answer .... minutes [1]

(ii) What is his speed, in kilometres per hour, on his journey to the football stadium?

Answer ....km/h [1]

(iii) What is the distance between the football stadium and the café?

Answer ....km [1]

(iv) Between which two places does he travel the fastest?

Answer .... and ....[1]

(c)(i) 138   (ii) 44   (iii) 28   (iv) Stadium, café   **J12/21/Q12c**

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5
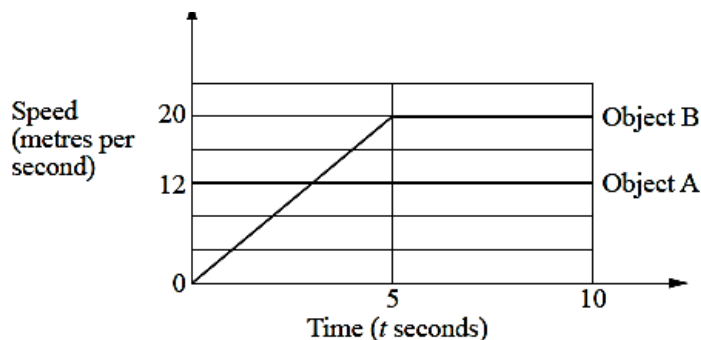
The diagram shows the speed-time graphs of two objects, A and B, for the first 10 seconds of their motion.

Object A travelled at a constant speed of 12 m/s throughout the 10 seconds.

Object B started from rest, and accelerated at a constant rate, attaining a speed of 20 m/s after 5 seconds. It then travelled at a constant speed of 20 m/s.

(a) Calculate

    (i)   the distance travelled by object B during the first 5 seconds of its motion,   [1]

    (ii)   the average speed of object B for the first 10 seconds of its motion,   [2]

    (iii)   the value of t when both objects were travelling at the same speed,   [2]

    (iv)   the value of t when both objects had travelled the same distance.   [2]

(b) The diagram below shows the distance-time graph for object B.
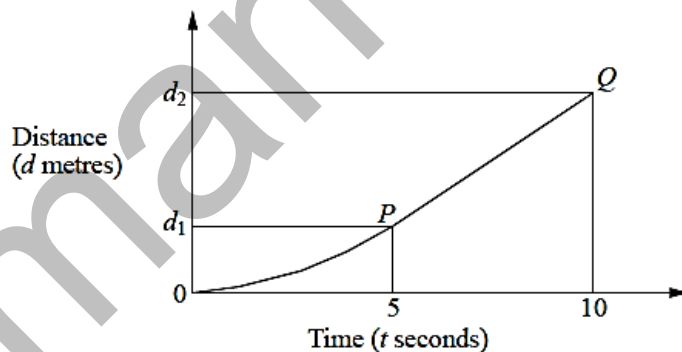
In the diagram, OP is a curve and PQ is a straight line.

    (i)   State the values of d_1 and d_2.   [1]

    (ii)   What does the gradient of the straight line PQ represent?   [1]

    (iii)   Write down the gradient of the tangent to the curve at $t = 2.5$.   [1]

(c) After 10 seconds, both objects slowed down at the same constant rate.
Object A came to rest after a further 9 seconds.
After how many seconds **from the start of its motion** did object B come to rest?   [2]

Answers: (a)(i) 50 m (ii) 15 ms^{-1} (iii) 3 seconds (iv) 6.25 seconds (b)(i) 50 m and 150 m   **N09/2/Q11**
(ii) the constant speed of object B, (iii) 10 ms^{-1}, (c) 25 seconds.

11 (a) One day, two brothers, Zac and Tom, leave their home at different times.
They meet at the library before going to the swimming pool.
The travel graph represents Zac's journey to the swimming pool.
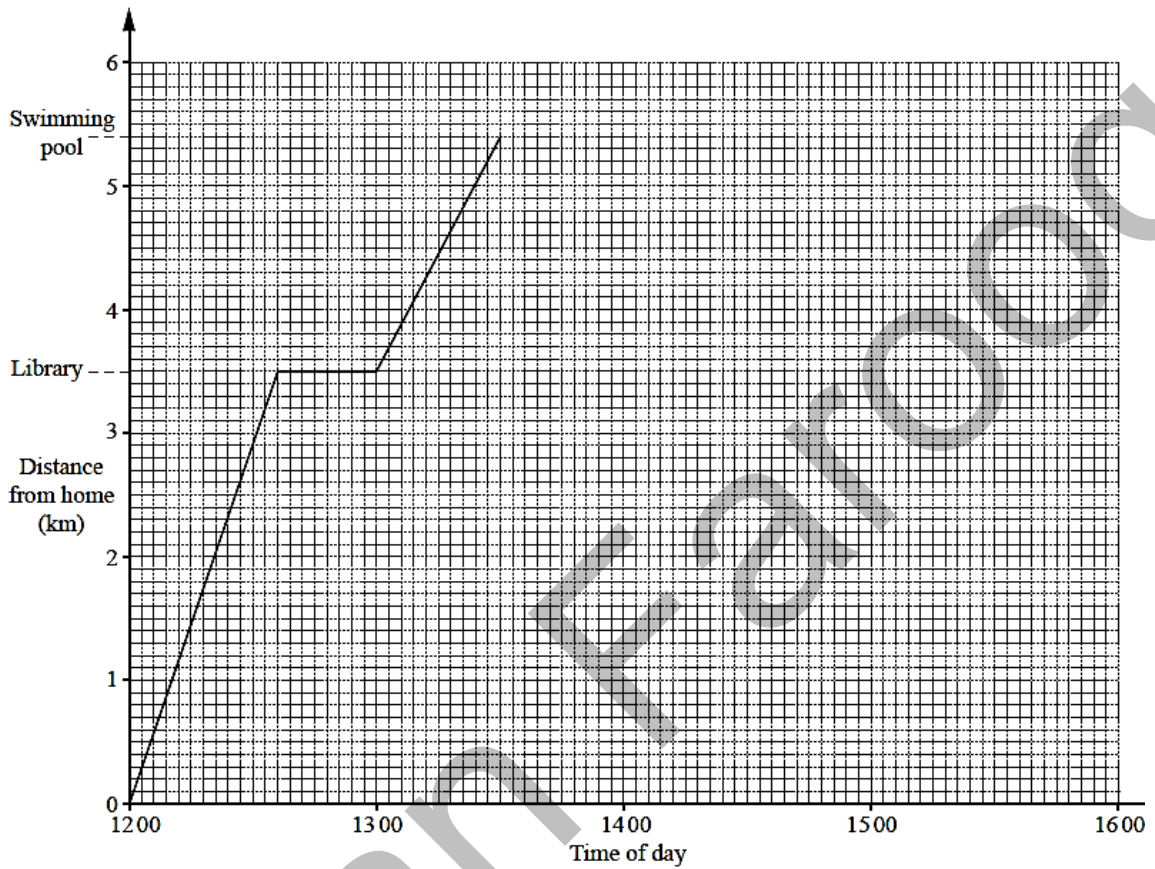
(i) How much time does Zac spend at the library? *Answer* .... minutes [1]

(ii) Tom leaves their home at 12 30 and cycles to the library at 14 km/h.
Calculate the time Tom arrives at the library. *Answer* .... [1]

23

(iii) How far is the swimming pool from the library? *Answer* .... km [1]

(iv) Zac stays at the swimming pool for an hour and a quarter.
He then walks home at a constant speed, arriving at 15 39 .

(a) Complete his travel graph. [2]

(b) Calculate Zac's speed, in kilometres per hour, as he walks home.

Answer .... km/h [1]

Answers: **(a)(i)** 23 to 25; **(ii)** 12 45; **(iii)** 1.9; **(iv)(a)** Straight lines to (14 45, 5.4) and from (14 45, 5.4) **N14/21/Q11**
to (15 39, 0); **(b)** 6; **(b)(i)** Correct sectors and labels; **(ii)** $\frac{5}{12}$; **(iii)** $\frac{41}{66}$.

Graphs of Functions Paper 1

1 **(a)** The diagram shows the graphs of

$y = 2^x$ and
$y = 2x + 1$.

   (i) State the gradient of the line
   $y = 2x + 1$.

   (ii) Find the value of x such that
   $x > 0$ and $2x + 1 = 2^x$.
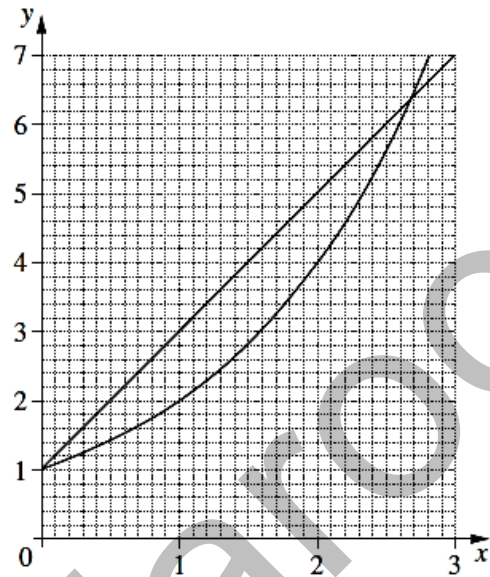
Answer (a)(i) .... [1]

(ii) $x =$ .... [1]

(b) The diagram shows the graph of

$y = ka^x$.

State the value of

   (i) k ,

   (ii) a .
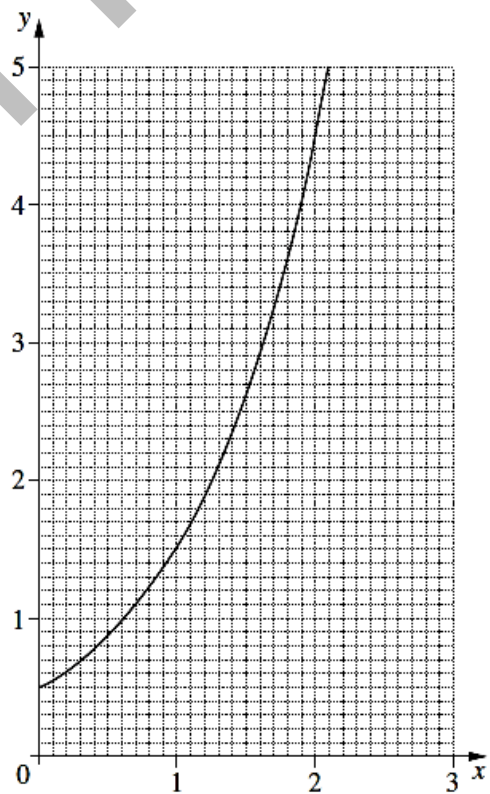
2 (a) The diagram in the answer space is a sketch of the graph of $y = \frac{3}{x}$ for $x > 0$.
Complete the sketch for $x < 0$.

(b) Sketch the graph of $y = x$ on the diagram in the answer space.

(c) The graphs of $y = \frac{3}{x}$ and $y = x$ meet at $x = k$.

Find the values of k.

Answer (a) (b)

[2]

(c) .... [2]
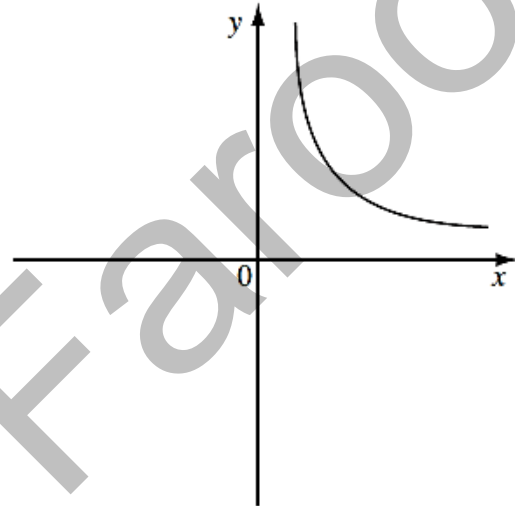
3 (a) Given that $2y = 3^x$, find x when $y = 40\frac{1}{2}$.

Answer (a) $x =$ ....[1]

(b) The points, $A(0, \frac{1}{2})$ and $B(2, 4\frac{1}{2})$, lie on the curve as shown in the diagram.
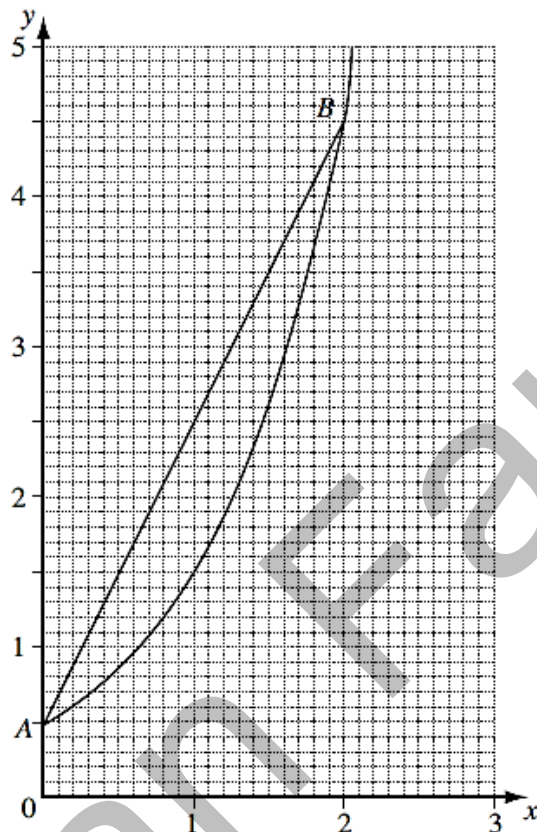
(i) Calculate the gradient of the straight line AB.

(ii) Using the diagram, estimate the value of x at which the gradient of the curve is equal to the gradient of the straight line AB.

Answer (b)(i) .... [1]

(ii) $x =$ .... [1]

Answer. (a) 4   (b)(i) 2   (ii) 1.1 to 1.3

4
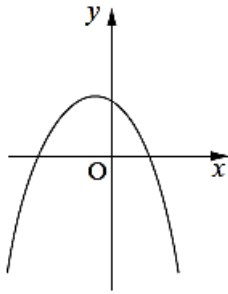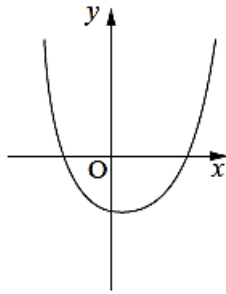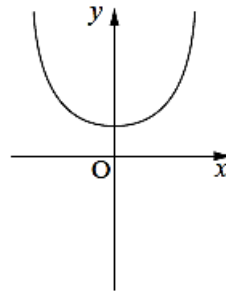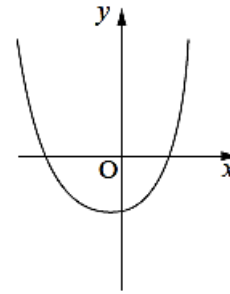
Figure 1   Figure 2   Figure 3   Figure 4

Which of the figures shown above could be the graph of

(a) $y = x^2 + 2$,   *Answer* (*a*) Figure ....[1]

(b) $y = (x - 2)(x + 1)$,   *Answer* (*b*) Figure ....[1]

(c) $y = 2 - x - x^2$?   *Answer* (*c*) Figure..........................[1]

Answer: (a) 3   (b) 2   (c) 1   **J09/1/Q13**

5 The variables x and y are connected by the equation $y = x^3 - 6x$.

(a) Complete the table of values.

x	-2	-1	0	1	2	3
y		5	0		-4	9

[2]

(b) On the axes below, draw the graph of $y = x^3 - 6x$ for $-2 \leqslant x \leqslant 3$.

[2]
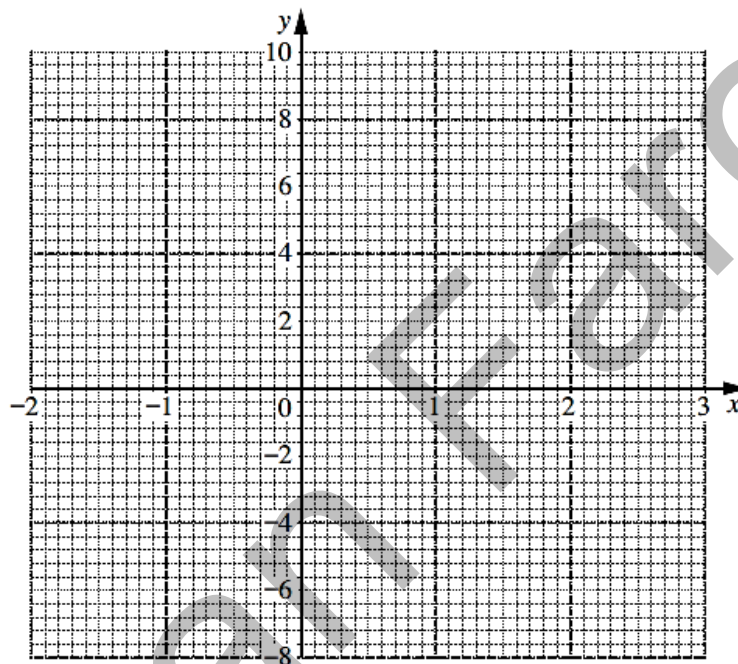
(c) Use your graph to find

(i) two solutions of the equation $x^3 - 6x = 0$,   *Answer* $x =$ .... or .... [2]

(ii) the least value of y when x is positive.   *Answer* $y =$ .... [1]

Answers: **(a)** 4, -5   **(c)** 0, 2.4 to 2.5   **(d)** follow through from graph   **J11/11/Q24**

6 The diagram shows the graph of $y = x^2 + x - 12$.

(a) The graph cuts the y-axis at K $(0, k)$.

Write down the value of k.

(b) The graph cuts the x-axis at L $(l, 0)$ and M $(m, 0)$.

Find the value of

(i) l,

(ii) m.
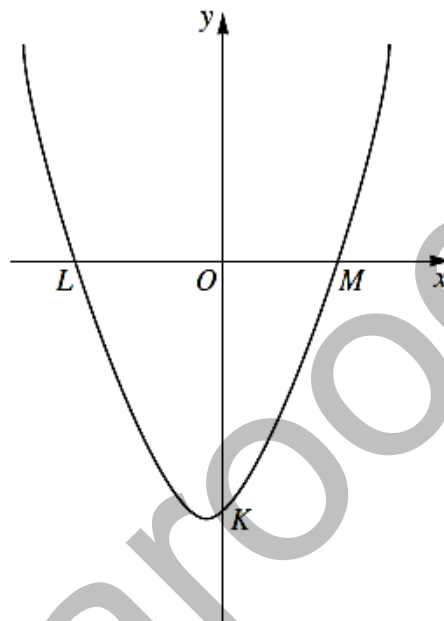
Answer (a) $k = $ .... [1]

(b) (i) $l = $ .... [1]

(ii) $m = $ .... [1]

Answers: **(a)** –12; **(b)(i)** –4, **(ii)** 3.

7
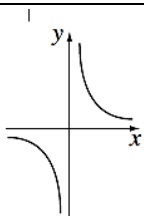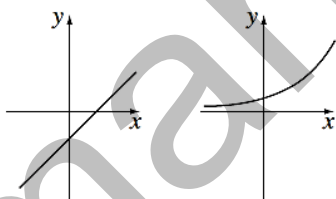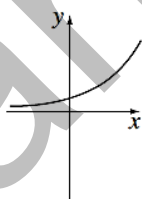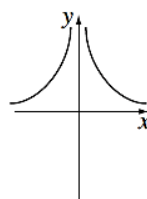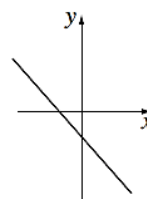
Figure 1   Figure 2   Figure 3   Figure 4   Figure 5   Figure 6
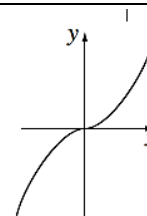
Which of the graphs shown above could be the graph of

(a) $y = x^3$,

(b) $y = \dfrac{1}{x^2}$,

(c) $y = x - 1$?

Answer (a) Figure................................. [1]

(b) Figure................................. [1]

(c) Figure................................. [1]

Answers: **(a)** Figure 6; **(b)** Figure 4; **(c)** Figure 2.

8 The curve $y = x^3 - 5$ is shown on the axes below.
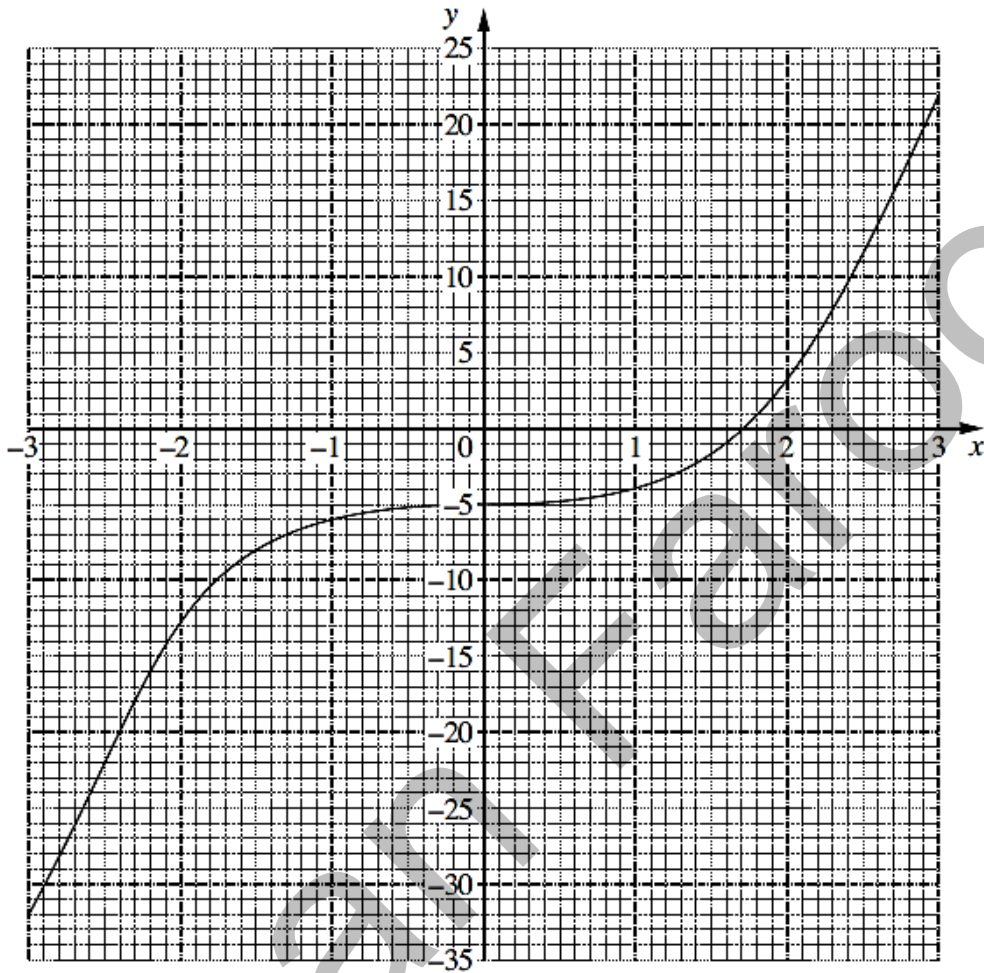
[1]

(a) Use the graph to find an approximate value of $\sqrt[3]{5}$.

(b) (i) On the axes above, draw the graph of $y = 15 - 5x$.

(ii) Write down the coordinates of the point where the graphs cross.

(iii) The x coordinate of the point where the graphs cross is a solution of the equation $x^3 = a + bx$.
Find the value of a and the value of b.

Answer (a) ....[1]

$(b)(ii)$ $(............,............)$ [1]

(iii) $a =$ .... $b =$ ....[1]

Answers: **(a)** approx. 1.7, **(b) (i)** st. line through (0, 15) and (3, 0), **(ii)** approx. (2.1, 4.5), **N08/1/Q17**
(iii) $a = 20$ and $b = -5$.

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9
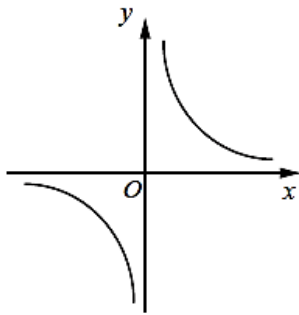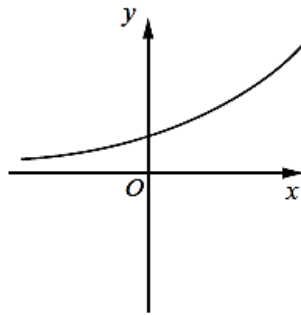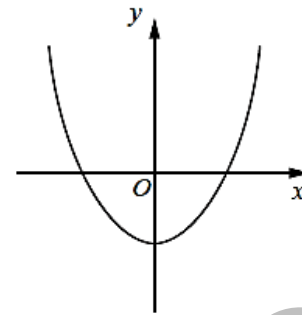
Figure A   Figure B   Figure C
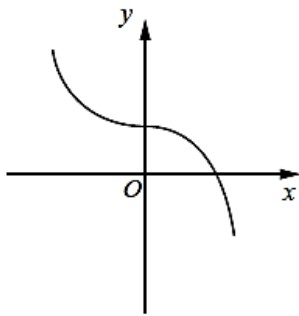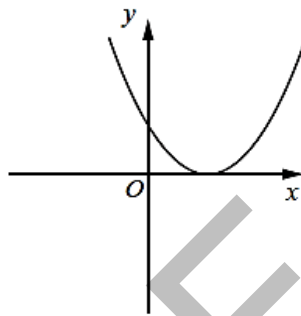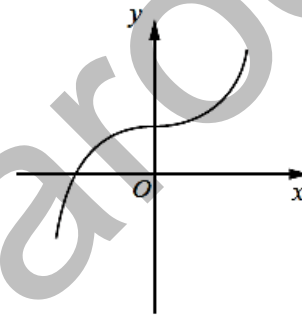
Figure D   Figure E   Figure F

State which figure could be the graph of

(a) $y = x^3 + 1$,

Answer   Figure .... [1]

(b) $y = x^2 - 3$,

Answer   Figure .... [1]

(c) $y = 3^x$,

Answer   Figure .... [1]

(d) $y = (x - 3)^2$,

Answer   Figure .... [1]

Answers: (a) F; (b) C; (c) B; (d) E.   **N14/11/Q20**

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Graphs of Functions Paper 2

1 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of x and the corresponding values of y, where

$$y = 30 - 18x + x^3.$$

x	-4	-3	-2	-1	0	1	2	3	4
y	38	57	58	47	30	13	2	3	22

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-4 \leqslant x \leqslant 4$.
Using a scale of 2 cm to represent 10 units, draw a vertical y-axis for $0 \leqslant y \leqslant 60$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find

   (i) the largest value of $30 - 18x + x^3$ in the interval $-4 \leqslant x \leqslant 4$, [1]

   (ii) the smallest value of x for which $30 - 18x + x^3 = 50$. [1]

(c) By drawing a tangent, find the gradient of the curve when $x = 1$. [2]

(d) A is the point $(0, 27)$ and B is the point $(3, 3)$.

   (i) Draw, on the axes used in part (a), the line which passes through A and B. [1]

   (ii) Find the equation of AB. [2]

   (iii) The x coordinates of the points where the line AB intersects the curve are the solutions of the equation $x^3 + ax + b = 0$.
Find the value of a and the value of b. [2]

2 Answer the whole of this question on a sheet of graph paper.

Temperatures were recorded over a nine hour period.
The table below shows the temperature, $y\,°C$, at various times.

Time (x hours)	0	1	2	3	4	5	6	7	8	9
Temperature ($y\,°C$)	2	-1	-2	-1.4	0	2	3.5	3.4	2.4	0.6

(a) Using a scale of 1 cm to represent 1 hour, draw a horizontal x-axis for $0 \leqslant x \leqslant 9$.
Using a scale of 2 cm to represent $1\,°C$, draw a vertical y-axis for $-2 \leqslant y \leqslant 4$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find an estimate for

   (i) the temperature when $x = 5.5$, [1]

   (ii) the difference between the highest and lowest temperatures, [1]

   (iii) how long, in hours and minutes, the temperature was above $2\,°C$. [2]

(c) (i) By drawing a tangent, find the gradient of the curve at the point where $x = 8$. [2]

(ii) State **briefly** what this gradient represents. [1]

(d) The curve from $x = 0$ to $x = 2$ has the equation $y = x^2 + Bx + C$.

Find the value of C and the value of B. [2]

Answers: (b)(i) $2.8 < t < 3.3$, (ii) $5.5 < t < 5.8$, (iii) 3h 12min to 3h 18min; (c)(i) $-1.4 \leqslant g \leqslant -1.1$,
(ii) Rate of cooling (in °C per hour); (d) $B = -4$, $C = 2$.

3 **Answer the whole of this question on a sheet of graph paper.**

The table gives the x and y coordinates of some points which lie on a curve.

x	1	1.5	2	2.5	3	4	5	6
y	140	110	100	98	100	110	124	140

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 6$.

Using a scale of 2 cm to represent 10 units, draw a vertical y-axis for $90 \leqslant y \leqslant 150$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to find

(i) the value of y when $x = 4.5$, [1]

(ii) the values of x for which $y = 128$. [1]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

(d) The line $y = k$ is a tangent to the curve.

Find the value of k. [1]

(e) The values of x and y are related by the equation

$$y = \frac{A}{x} + Bx.$$

(i) Use the fact that the point $(2, 100)$ lies on the curve to show that

$$200 = A + 4B.$$ [1]

(ii) Obtain a second equation connecting A and B.

Hence calculate the value of A and the value of B. [3]

Answers: (b)(i) 116 to 117, (ii) 1.1 to 1.2 *and* 5.2 to 5.3; (c) -40 to -22; (d) 98; (e) $A = 120$, $B = 20$.

4 Answer the whole of this question on a sheet of graph paper.

During one day, at a point P in a small harbour, the height of the surface of the sea above the seabed was noted.
The results are shown in the table.

Time (t hours) after 8 a.m.	0	1	2	3	4	5	6	7	8	9
Height (y metres) above the sea-bed	3.8	3.3	2.5	1.8	1.2	1.0	1.2	1.8	2.5	3.3

(a) Using a scale of 1 cm to represent 1 hour, draw a horizontal t-axis for $0 \leqslant t \leqslant 9$.

Using a scale of 2 cm to represent 1 metre, draw a vertical y-axis for $0 \leqslant y \leqslant 4$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) (i) By drawing a tangent, find the gradient of the curve at the point where $t = 4$. [2]

    (ii) Explain the meaning of this gradient. [1]

(c) On the same day, a straight pole was driven vertically into the seabed at the point P.
Work started at 8 a.m.
The pole was driven in at a constant rate.
The height, y metres, of the top of the pole above the seabed, t hours after 8 a.m., is given by the equation

$$y = 4 - \tfrac{1}{2}t.$$

    (i) Write down the length of the pole. [1]

    (ii) On the same axes as the curve, draw the graph of $y = 4 - \tfrac{1}{2}t$. [2]

    (iii) How many **centimetres** was the top of the pole above the surface of the sea at noon? [2]

    (iv) Find the value of t when the top of the pole was level with the surface of the sea. [1]

Answers: **(b)(i)** −0.5 to −0.3, **(ii)** rate at which the water level is changing (or equivalent); **(c)(i)** 4 m, **(ii)** 75 to 85 cm, **(iv)** 5.7 to 5.9. **J05/2/Q8**

5 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of x and the corresponding values of y, correct to one decimal place, where

$$y = \frac{x^2}{8} + \frac{18}{x} - 5.$$

x	1	1.5	2	2.5	3	4	5	6	7	8
y	13.1	7.3	4.5	3.0	2.1	1.5	1.7	p	3.7	5.3

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 8$.

Using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \leqslant y \leqslant 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find

(i) the value of x when $y = 8$, [1]

(ii) the least value of $\dfrac{x^2}{8} + \dfrac{18}{x}$ for values of x in the range $0 \leqslant x \leqslant 8$. [1]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 2.5$. [2]

(e) On the axes used in part (b), draw the graph of $y = 12 - x$. [2]

(f) The x coordinates of the points where the two graphs intersect are solutions of the equation

$$x^3 + Ax^2 + Bx + 144 = 0.$$

Find the value of A and the value of B. [2]

Answers: (a) 2.5; (c)(i) 1.4 to 1.5, (ii) 6.4 to 6.5; (d) –2.0 to –2.5; (e) A = 8, B = –136. **J06/2/Q8**

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6 Answer the whole of this question on a sheet of graph paper.

Adam stood on a slope, 15 m from the bottom.
He rolled a heavy ball directly up the slope.
After t seconds the ball was y metres from the bottom of the slope.
The table below gives some values of t and the corresponding values of y.

t	0	1	2	2.5	3	3.5	4	4.5	5	5.5
y	15	22	25	25	24	22	19	15	10	4

(a) Using a scale of 2 cm to represent 1 unit, draw a horizontal t-axis for $0 \leqslant t \leqslant 6$.

Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $0 \leqslant y \leqslant 30$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(b) Extend the curve to find the value of t when the ball reached the bottom of the slope. [1]

(c) (i) By drawing a tangent, find the gradient of the curve when $t = 3.5$. [2]

(ii) State briefly what this gradient represents. [1]

(d) Immediately after he rolled the ball, Adam ran down the slope at a constant speed of 1.5 m/s.

(i) Write down the distance of Adam from the bottom of the slope when

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(a) $t = 0$,

(b) $t = 4$. [2]

(ii) On the same axes, draw the graph that represents the distance of Adam from the bottom of the slope for $0 \leqslant t \leqslant 6$. [2]

(iii) Hence find the distance of Adam from the bottom of the slope when the ball passed him. [1]

Answer: **(b)** 5.7 to 5.9; **(c)(i)** −4 to −6, **(ii)** speed; **(d)(i)(a)** 15, **(b)** 9, **(ii)** line from (0, 15) to (6, 6), **(iii)** 7 to 7.4. **J07/2/Q8**

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7 **Answer the WHOLE of this question on a sheet of graph paper.**

The table below shows some values of x and the corresponding values of y, correct to one decimal place, for

$$y = \tfrac{4}{5} \times 2^x.$$

x	−2	−1	0	1	2	2.5	3	3.5	4
y	p	0.4	0.8	1.6	3.2	4.5	6.4	9.1	12.8

(a) Calculate p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \leqslant x \leqslant 4$.

Using a scale of 2 cm to represent 2 units, draw a vertical y-axis for $0 \leqslant y \leqslant 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) As x decreases, what value does y approach? [1]

(d) By drawing a tangent, find the gradient of the curve at the point $(3, 6.4)$. [2]

(e) (i) On the axes used in part (b), draw the graph of $y = 8 - 2x$. [2]

(ii) Write down the coordinates of the point where the line intersects the curve. [1]

(iii) The x coordinate of this point of intersection satisfies the equation

$$2^x = Ax + B.$$

Find the value of A and the value of B. [2]

Answers: **(a)** 0.2; **(c)** 0; **(d)** any value between 4 and 5; **(e)(ii)** (2.2, 3.6) **J08/2/Q8**
(iii) $A = -2\tfrac{1}{2}$ $B = 10$.

8   Answer THE WHOLE of this question on a sheet of graph paper.
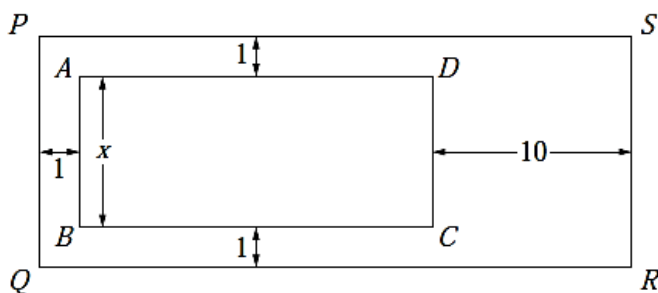
The diagram represents a rectangular pond, $ABCD$, surrounded by a paved region.
The paved region has widths 1 m and 10 m as shown.
The pond and paved region form a rectangle $PQRS$.
The area of the pond is $168\,\text{m}^2$.

(a) Taking the length of AB to be x metres, write down expressions, in terms of x, for

   (i) PQ,

  (ii) BC,

 (iii) QR.   [2]

(b) Hence show that the area, y square metres, of the paved region, is given by

$$y = 22 + 11x + \frac{336}{x}.$$ [2]

(c) The table below shows some values of x and the corresponding values of y.

x	3	3.5	4	5	6	7	8	9
y	167	156.5	150	144.2	144	147	152	p

   Calculate p.   [1]

(d) Using a scale of 2 cm to represent 1 metre, draw a horizontal x-axis for $3 \leqslant x \leqslant 9$.

   Using a scale of 2 cm to represent 5 square metres, draw a vertical y-axis for $140 \leqslant y \leqslant 170$.

   On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(e) By drawing a tangent, find the gradient of the curve at $(4, 150)$.   [2]

(f) Use your graph to find

   (i) the smallest area of the paved region,   [1]

  (ii) the length of PQ when the area of the paved region is smallest.   [1]

Answers: **(a)** $PQ = x + 2$, $BC = \dfrac{168}{x}$, $QR = \dfrac{168}{x} + 11$;   **(c)** 158;   **(e)** −6 to −12;   **J09/2/Q8**

(f) (i) $143 \leqslant \text{area} < 144$,   **(ii)** $7.4 \leqslant PQ \leqslant 7.6$.

9

Answer the WHOLE of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y for

$$y = \frac{12}{x} - x.$$

x	1	2	3	4	5	6
y	11	4	1	−1	p	−4

(a) Calculate p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 6$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-4 \leqslant y \leqslant 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $\frac{12}{x} - x = 2$ in the range $1 \leqslant x \leqslant 6$. [1]

(d) The equation $\frac{12}{x} = 2x$ can be solved using the intersection of your curve and a straight line.

    (i) State the equation of this straight line. [1]

    (ii) By drawing this straight line, solve the equation $\frac{12}{x} = 2x$. [2]

(e) The points A and B are $(1, 11)$ and $(4, -1)$ respectively.

Find the gradient of the line AB. [1]

(f) The line l is parallel to AB and is a tangent to the curve $y = \frac{12}{x} - x$.

    (i) Draw the line l. [1]

    (ii) Find the coordinates of the point where l crosses the y-axis. [1]

    (iii) Hence find the equation of the line l. [1]

Answers: (a) $p = -2.6$　(c) 2.55 to 2.65　(d)(i) $y = x$　(ii) 2.40 to 2.50　(e) −4　**J10/21/Q8**
(f)(ii) (0, 12)　(iii) $y = -4x + 12$

10   Answer the WHOLE of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y for

$$y = \frac{2^x}{4}.$$

x	-1	0	1	2	3	4	5
y	m	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	n

(a) Calculate the values of m and n.   [2]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-1 \leqslant x \leqslant 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $0 \leqslant y \leqslant 8$.

On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c) Use your graph to solve the equations

   (i) $\frac{2^x}{4} = 3$,   [1]

   (ii) $2^x = 6$.   [1]

(d) The equation $y = \frac{2^x}{4}$ can be written in the form $y = 2^t$.

   (i) Find an expression for t in terms of x.   [1]

   (ii) Hence, find the equation of the line that can be drawn on your graph to evaluate
        y when $t = -\frac{3}{4}$.   [1]

Answers: (a) $m = \frac{1}{8}$, $n = 8$;   (c) (i) 3.6,   (ii) 2.6;   (d)(i) $t = x - 2$,   (ii) $x = 1.25$.   **J10/22/Q6**

11   **9**   The table below shows some of the values of x and the corresponding values of y for

$$y = (2x - 3)(x + 2).$$

x	-3	-2	-1	0	1	2	3
y	9	0			-3	4	15

(a) Complete the table.   [1]

(b) On the axes below, plot the points from the table and join them with a smooth curve.

[2]
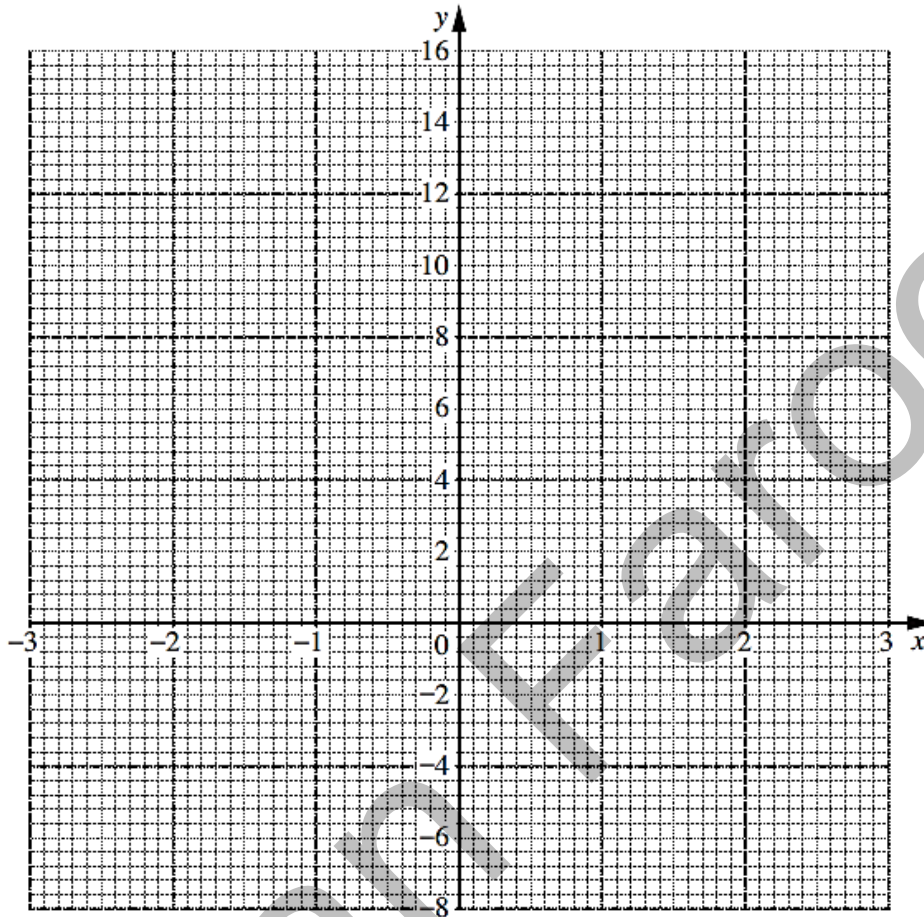
(c) Use your graph to

(i)   solve the equation $(2x-3)(x+2)=2$,   *Answer* ....[1]

(ii)   find the minimum value of y,   *Answer* ....[1]

(iii)   find the gradient of the curve at $(2\,,\,4)$.   *Answer* ....[2]

(d) (i)   Show that the x-coordinates of the points where $y=(2x-3)(x+2)$ and $y=1-2x$
would intersect are the solutions of the equation

$$2x^2+3x-7=0.$$

[1]

(ii)   Solve **algebraically** the equation $2x^2+3x-7=0$,   giving each answer correct to
2 decimal places.
        Answer $x=$ .... or ....[4]

12
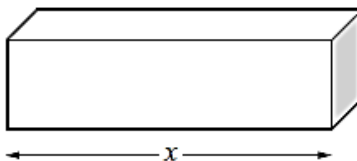
A cuboid has a square cross-section, shown shaded in the diagram.
The length of the cuboid is x cm.
The sum of the length of the cuboid and one of the sides of the square is 10 cm.

(a) Show that the volume of the cuboid, y cm^3, is given by $y = x^3 - 20x^2 + 100x$. [2]

(b) The table shows some values of x and the corresponding values of y for

$$y = x^3 - 20x^2 + 100x.$$

x	1	2	3	4	5	6	7	8	9
y	81	128	147	144	125	96			9

   (i) Complete the table. [1]

   (ii) On the grid opposite, plot the graph of $y = x^3 - 20x^2 + 100x$ for $1 \leqslant x \leqslant 9$. [3]

(c) Use your graph to find

   (i) the maximum volume of the cuboid, *Answer* .... cm^3 [1]

   (ii) the possible values of x when the volume of the cuboid is 120 cm^3.

   Answer $x =$ .... or ....[2]

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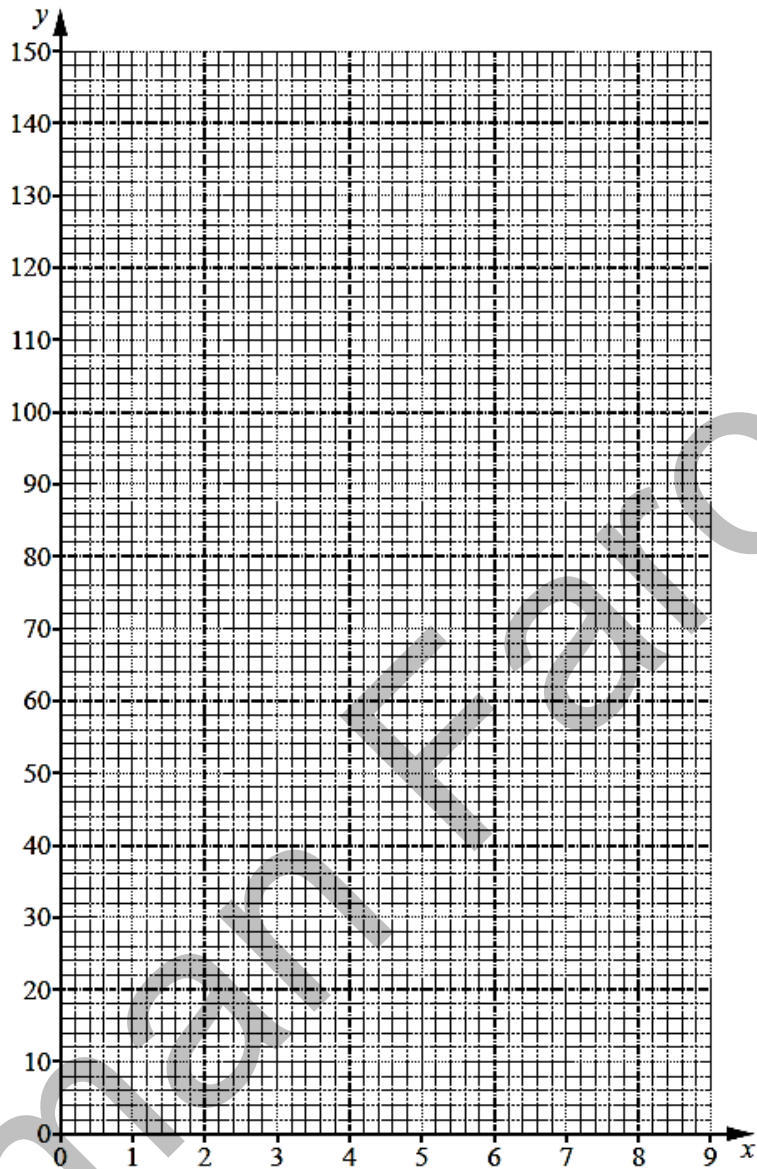
(d) [The volume of a sphere $= \frac{4}{3}\pi r^3$]

For this part of the question take π as 3.

A sphere has a radius of $\frac{1}{2}x$ cm.

By drawing a suitable graph on the grid, estimate the value of x when the sphere and the cuboid have the same volume.

Answer $x = $ ....[3]

Answers: **(b)(i)** 63, 32 **(c)(i)** 148 **(ii)** 1.8, 5.2 **(d)** 5.9 **J12/21/Q10**

13

(a) The table shows some values of x and the corresponding values of y for $y = \dfrac{6}{x^2}$.

x	−3	−2.5	−2	−1.5	−1	1	1.5	2	2.5	3
y		0.96	1.5	2.67	6	6	2.67	1.5	0.96	

(i) Complete the table. [1]

(ii) On the grid draw the graph of $y = \dfrac{6}{x^2}$ for $-3 \leqslant x \leqslant 3$.
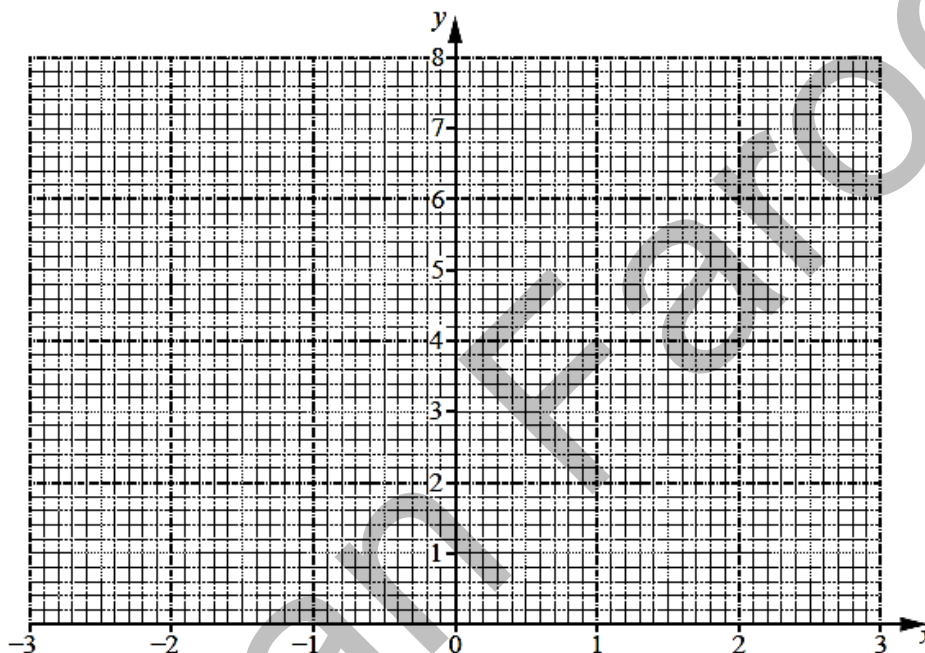
[2]

(iii) Use your graph to find the values of x when $y = 2$.

Answer $x =$ .... or .... [1]

(iv) By drawing a tangent, find the gradient of the curve when $x = 1.5$.

Answer .... [2]

(v) By drawing a suitable line on the grid, solve the equation $\dfrac{6}{x^2} = 2 - x$.

Answer $x =$ .... [2]

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(b) The graph shows a sketch of $y = 5a^x$.
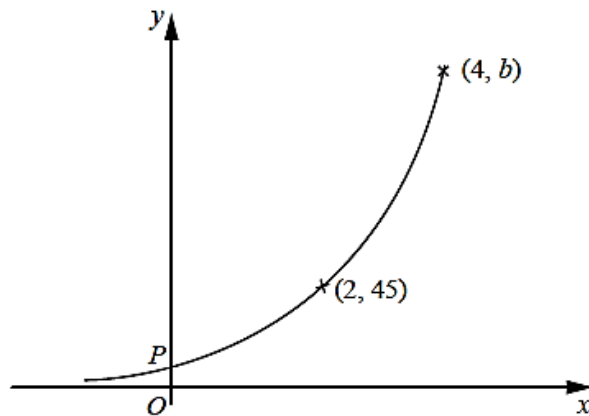
Two points on the curve are $(2, 45)$ and $(4, b)$.

(i) Find the values of a and b.

Answer $a =$ ....

$b =$ .... [2]

(ii) Find the coordinates of the point, P, where the graph crosses the y-axis.

Answer (.................... , ...) [1]

(iii) Find the gradient of the straight line joining the points P and $(2, 45)$.

Answer .... [1]

Answer. (a)(i) 0.66, 0.66 (iii) −1.7 to −1.8 and 1.7 to 1.8 (iv) −2.5 to 5 (v) −1.3 to −1.4 **J13/21/Q10**
(b)(i) $a = 3$, $b = 405$ (ii) (0, 5) (iii) 20

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14 The variables x and y are connected by the equation $y = \dfrac{x^3}{2} - 3x + 1$.

Some corresponding values of x and y are given in the table below.

x	−3	−2	−1	0	1	2	3
y		3	3.5	1	−1.5	−1	

(a) Complete the table. [2]

(b) On the grid below, plot the points from the table and join them with a smooth curve.

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[2]
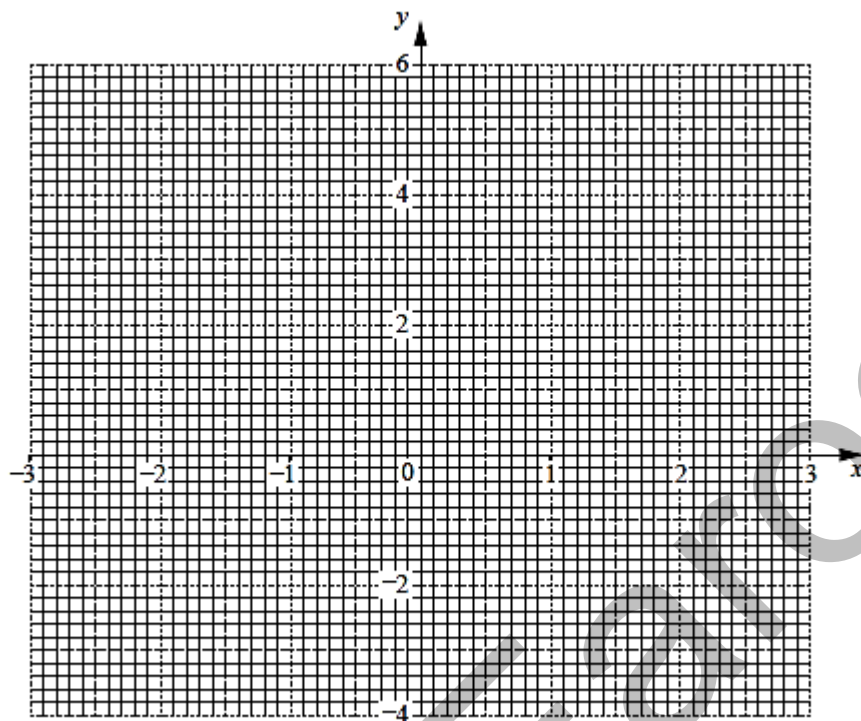
(c) Use your graph to solve the equation $\dfrac{x^3}{2} - 3x + 1 = 0$.

Answer .... [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(-2, 3)$.

Answer .... [2]

(e) The line AB intersects the curve at point P.
The coordinates of point A are $(0, 5)$.
The coordinates of point B are $(2, -3)$.

(i) Find the equation of line AB.　　Answer .... [2]

(ii) The x-coordinate of point P is a solution of the equation $\dfrac{x^3}{2} + Cx + D = 0$.

Find C and D.　　　　　　Answer　$C = $ ....

$D = $ .... [2]

Answers: **(a)** −3.5, 5.5　**(c)** $x = -2.7$ to −2.6, 0.3 to 0.4, 2.2 to 2.3　**(d)** 2 to 3　**(e)(i)** $y = -4x + 5$　　**J14/21/Q7**
(ii) $C = 1, D = -4$

15 $f(x) = x^3$

(a) Complete the following table.

x	−3	−2	−1	0	1	2	3
$f(x)$							

[1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-3 \leqslant x \leqslant 3$.
Using a scale of 2 cm to represent 10 units, draw a vertical y-axis for $-30 \leqslant y \leqslant 30$.
Using your axes, plot the points in the table and join them with a smooth curve.

Answer
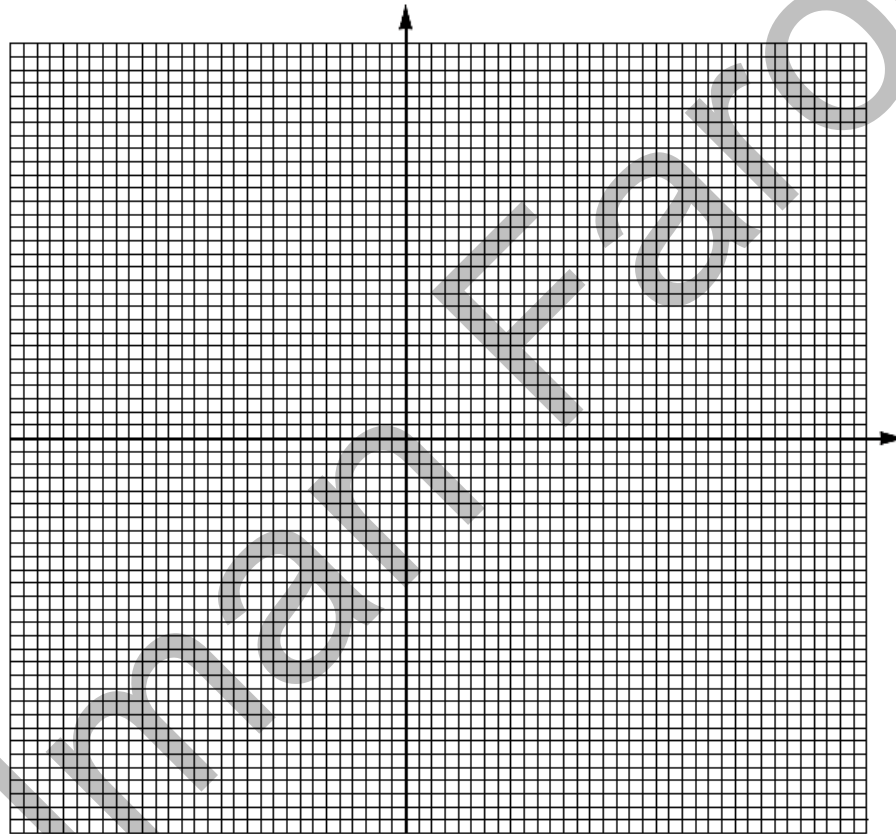
[2]

(c) (i) Use your graph to solve $f(x) = -15$.   *Answer* .... [1]

(ii) Use your graph to find a such that $f^{-1}(a) = 1.7$.   *Answer* .... [1]

(iii) Given that $f^{-1}(t) = u$, express t in terms of u.   *Answer* $t =$ .... [1]

(iv) By drawing a tangent to $\quad = (\quad)$, estimate the gradient of the curve when $x = 2$.

                                    Answer .... [2]

(d) **(i)** Using the same axes draw the line that represents the function $\quad g(x) = 5x + 3$.   [2]

     (ii) Hence find the three solutions of the equation $\quad f(x) = g(x)$.

                                 Answer $x =$ .... or .... or .... [2]

Answers: (a) –27 –8 –1 0 1 8 27 (c)(i) –2.4 to –2.6 (ii) 4 to 6 (iii) $t = u^3$ (iv) 10 to 13 d(ii) –1.95 to –1.7, –0.8 to –0.5, 2.4 to 2.6   **J15/21/Q9**

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16   Answer the whole of this question on a sheet of graph paper.

The table gives some values of x and the corresponding values of y, correct to one decimal place, where

$$y = 4x + \frac{25}{x^2} .$$

x	1	1.25	1.5	2	2.5	3	3.5	4
y	p	21.0	17.1	14.3	14.0	14.8	16.0	17.6

(a) Find the value of p.   [1]

(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 4$.
Using a scale of 4 cm to represent 10 units, draw a vertical y-axis for $0 \leqslant y \leqslant 30$.
On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c) Use your graph to find

     (i) a solution of $4x + \dfrac{25}{x^2} = 20,$ [1]

     (ii) the least value of $4x + \dfrac{25}{x^2}$.   [1]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$.   [2]

(e) On the axes used in part **(b)**, draw the graph of the straight line

$$y = 25 - 3x \quad \text{for values from } x = 0 \text{ to } x = 4.$$ [2]

(f) **(i)** Write down the x coordinates of the points at which the two graphs intersect.   [1]

     (ii) Find the equation, in the form $ax^3 + bx^2 + cx + d = 0$, which is satisfied by the
values of x found in part **(f)(i)**.   [1]

Answers: (a) 29; (c)(i) 1.28 to 1.33, (ii) 13.7 to 13.99; (d) –9 to –12; (f)(i)1.20 to 1.25 and 3.20 to 3.28,   **N01/2/Q9**
(ii) $7x^3 - 25x^2 + 25 = 0$.

17 Answer the whole of this question on a sheet of graph paper.

A man stood at the top of a tower.
He threw a ball vertically upwards.
The height, h metres, of the ball **above the top of the tower** at a time t seconds after it was thrown is given by the formula

$$h = 22t - 4.9t^2.$$

The table below shows some values of t and the corresponding values of h, correct to 1 decimal place.

t	0	1	2	2.5	3	4	5	6
h	0.0	17.1	24.4	24.4	21.9	9.6	−12.5	p

(a) Explain the significance of the value $h = -12.5$ when $t = 5$. [1]

(b) Find the value of p. [1]

(c) Using a scale of 2 cm to 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 2 cm to 10 metres, draw a vertical h-axis for $-50 \le h \le 30$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(d) Use your graph to find

   (i) the greatest height of the ball above the top of the tower, [1]

   (ii) the length of time for which the ball was more than 20 metres above the top of the tower. [2]

(e) (i) By drawing a tangent, find the gradient of the graph at $(4, 9.6)$. [2]

   (ii) Explain what your answer to (e)(i) tells you about the motion of the ball at $t = 4$. [1]

(f) The ball hit the ground 5.4 seconds after it was thrown.
Use your graph to find the height of the tower. [1]

Answers: **(b)** -44.4; **(d)(i)** 24.5 to 25.5 m, **(ii)** 1.90 to 2.00 s; **(e)(i)** -14 to −19; **(f)** 22 to 26 m.   **N02/2/Q10**

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18 Answer the whole of this question on a sheet of graph paper.

An open rectangular tank has a square base of side x metres.
The volume of the tank is 36 m³.

(a) (i) Find an expression, in terms of x, for the height of the tank. [1]

   (ii) Hence show that the total external surface area of the tank, A square metres, is given by

$$A = x^2 + \frac{144}{x}.$$ [1]

(b) The table below shows some values of x and the corresponding values of A, correct to 1 decimal place, where $A = x^2 + \dfrac{144}{x}$.

x	2	2.5	3	4	5	6	7	8
A	76.0	63.9	57.0	52.0	53.8	60.0	69.6	p

    (i)   Find the value of p.   [1]

    (ii) Using a scale of 2 cm to 1 metre, draw a horizontal x-axis for $2 \leqslant x \leqslant 8$.
        Using a scale of 2 cm to 10 m², draw a vertical A-axis for $40 \leqslant A \leqslant 90$.

        On your axes, plot the points given in the table and join them with a smooth curve.   [3]

    (iii) Use your graph to find
        (a) the values of x for which the surface area is 65 m²,   [2]
        (b) the gradient of the curve at $x = 6$,   [2]
        (c) the dimensions of the tank which has the least possible surface area.   [2]

Answers: **(a)** 36/x^2; **(b)(i)** 82; **(c)(iii)(a)** 2.40 to 2.46 and 6.52 to 6.60, **(b)** 7.25 to 7.75, **(c)** 4.00 to 4.40;   **N03/2/Q9**
1.85 to 2.25.

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19   **Answer the whole of this question on a sheet of graph paper.**

A solid cylinder of radius r centimetres and height h centimetres has a
volume of $100\,\pi$ cm³.
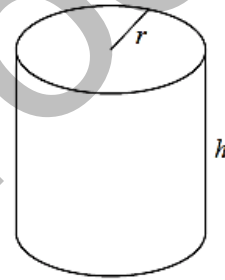
(a) **(i)** Show that $h = \dfrac{100}{r^2}$.   [1]

    (ii) The cylinder has a total surface area of πy square centimetres.

        Show that $y = 2r^2 + \dfrac{200}{r}$.   [1]

(b) The table below shows some values of r and the corresponding values of y, correct to the nearest whole number.

r	1	1.5	2	3	4	5	6
y	202	138	108	85	82	90	p

   (i) Find the value of p.   [1]

   (ii) Using a scale of 2 cm to represent 1 cm, draw a horizontal r-axis for $1 \leqslant r \leqslant 6$.
       Using a scale of 2 cm to represent 20 cm², draw a vertical y-axis for $70 \leqslant y \leqslant 220$.
       On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c) Use your graph to find the values of r for which $y = 100$.   [2]

(d) By drawing a tangent, find the gradient of the graph at the point where $r = 2$.   [2]

(e) Use your graph to find

   (i) the value of r for which y is least,   [1]

   (ii) the smallest possible value of the total surface area of the cylinder.   [1]

Answers: **(b)(i)** 105; **(c)** 2.20 to 2.27, 5.65 to 5.75; **(d)** –35 to –48; **(e)(i)** 3.6 to 3.8, **(ii)** 80π to 82π or   **N04/2/Q10**
251 to 257.

20 Answer the whole of this question on a sheet of graph paper.

The area of a rectangular garden, *ABCD*, is 100 m².
Inside the garden there is a rectangular lawn, *EFGH*,
whose sides are parallel to those of the garden.
EF is 4 m from *AB*.
FG, *GH* and *HE* are 1 m from *BC*, *CD* and *DA* respectively.
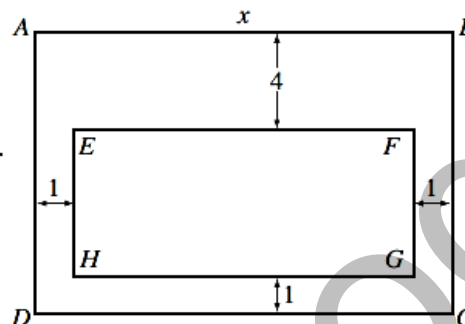
(a) Taking the length of *AB* to be *x* metres, write down expressions, in terms of *x*, for

   (i) *EF*,

   (ii) *BC*,

   (iii) *FG*.

   [2]

(b) Hence show that the area, *y* square metres, of the lawn, *EFGH* is given by

$$y = 110 - 5x - \frac{200}{x}.$$

   [1]

(c) The table below shows some values of *x* and the corresponding values of *y*, correct to 1 decimal place, where

$$y = 110 - 5x - \frac{200}{x}.$$

x	4	5	6	7	8	9	10
y	*p*	45.0	46.7	46.4	45.0	42.8	40.0

   Find the value of *p*.   [1]

(d) Using a scale of 2 cm to 1 metre, draw a horizontal *x*-axis for $4 \leqslant x \leqslant 10$.
   Using a scale of 2 cm to 2 square metres, draw a vertical *y*-axis for $40 \leqslant y \leqslant 48$.
   On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(e) By drawing a tangent, find the gradient of the curve where *x* = 8.   [2]

(f) Use your graph to find

   (i) the range of values of *x* for which the area of the lawn is at least 44 m²,   [2]

   (ii) the value of *x* for which the area of the lawn is greatest.   [1]

Answers: **(a)(i)** *x* − 2, **(ii)** 100/*x*, **(iii)** [100/*x*] − 5; **(c)** 40; **(e)** −1.60 to −2.00; **(f)(i)** {4.65 to 4.80} to {8.45 to 8.55}, **(ii)** 6.20 to 6.40.   **N05/2/Q10**

21   Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^2}{5} + \frac{5}{x}.$$

The table below shows some values of x and the corresponding values of y correct to 1 decimal place.

x	1	1.5	2	3	4	5	6
y	5.2	3.8	3.3	3.5	4.5	6.0	p

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \leqslant x \leqslant 6$ and a vertical y-axis for $0 \leqslant y \leqslant 9$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the values of x in the range $1 \leqslant x \leqslant 6$ for which $\frac{x^2}{5} + \frac{5}{x} - 4 = 0$. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(4, 4.5)$. [2]

(e) (i) On the same axes, draw the graph of $y = \frac{1}{2}x + 3$. [2]

(ii) Write down the x coordinates of the points at which the two graphs intersect. [1]

(iii) Find the equation, in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in part (e)(ii). [1]

Answer: (a) 8 or 8.03   (c) 1.35 to 1.45 and 3.55 to 3.70   (d) 1.20 to 1.40   N06/2/Q10

(e)(ii) 1.45 to 1.55 and 4.55 to 4.65   (iii) $2x^3 - 5x^2 - 30x + 50 = 0$

635

22 **Answer the whole of this question on a sheet of graph paper.**

A stone was thrown from the top of a vertical cliff. Its position during the flight is represented by the equation $y = 24 + 10x - x^2$, where y metres is the height of the stone above the sea and x metres is the horizontal distance from the cliff.

(a) Solve the equation $0 = 24 + 10x - x^2$. [2]

(b) The table shows some values of x and the corresponding values of y.

x	0	2	4	6	8	10
y	24	40	48	48	40	24

   (i) Using a scale of 1 cm to represent 1 metre, draw a horizontal x-axis for $0 \leqslant x \leqslant 14$.
Using a scale of 2 cm to represent 10 metres, draw a vertical y-axis for $0 \leqslant y \leqslant 50$.

      On your axes, plot the points from the table and join them with a smooth curve. [3]

   (ii) Use your answer to part **(a)** to complete the graph which represents the flight of the stone. [1]

   (iii) Find the height of the stone above the sea when its horizontal distance from the cliff was 7 m. [1]

   (iv) Use your graph to find how far the stone travelled horizontally while it was 6 m or more above the top of the cliff. [2]

(c) It is given that $24 + 10x - x^2 = p - (x-5)^2$.

   (i) Find the value of p. [1]

   (ii) **Hence** find

     (a) the greatest height of the stone above the sea, [1]

     (b) the horizontal distance from the cliff when the stone was at its greatest height. [1]

Answers: **(a)** 12 or −2, **(b)(i)** plotting all six points correctly and joining with a smooth curve, **(ii)** curve drawn to (12,0), **(iii)** 45 m, **(iv)** 8.5 to 8.9 m, **(c)(i)** 49, **(ii)(a)** 49 m, **(b)** 5 m.   **N07/2/Q8**

23 **Answer the whole of this question on a sheet of graph paper.**

The number of bacteria in a colony doubles every half hour.
The colony starts with 50 bacteria.
The table below shows the number of bacteria in the colony after t hours.

Time (t hours)	0	0.5	1	1.5	2	2.5	3	3.5
Number of bacteria (y)	50	100	200	400	800	1600	3200	6400

(a) Using a scale of 4 cm to represent 1 hour, draw a horizontal t-axis for $0 \leqslant t \leqslant 4$.
Using a scale of 2 cm to represent 1000 bacteria, draw a vertical y-axis for $0 \leqslant y \leqslant 7000$.

On your axes, plot the points given in the table, and join them with a smooth curve. [3]

(b) Use your graph to find the number of bacteria in the colony when $t = 2.75$. [1]

(c) **(i)** By drawing a tangent, find the gradient of the graph when $t = 2.5$. [2]

(ii) State briefly what this gradient represents. [1]

(d) The number of bacteria in another colony is given by the equation

$$y = 6500 - 1000t.$$

(i) On the same axes, draw a graph to represent the number of bacteria in this colony. [2]

(ii) Find the value of t when the number of bacteria in each colony is the same. [1]

(e) Given that the equation of the first graph is $y = ka^t$, find the value of

(i) k, [1]

(ii) a. [1]

Answers: **(a)** All points plotted and a smooth curve drawn, **(b)** 2200 to 2400, **(c)(i)** 1800 to 2800, **(ii)** Rate of change of the number of bacteria per hour, **(d)(i)** Correct, ruled straight line, cutting the curve, **(ii)** 3.025 to 3.075, **(e)(i)** 50, **(ii)** 4. **N08/2/Q10**

24   Answer the whole of this question on a sheet of graph paper.

(a) The variables x and y are connected by the equation

$$y = 4x^3 - 18x^2 + 20x.$$

The table below shows some values of x and the corresponding values of y.

x	0	0.5	1	1.5	2	2.5	3	3.5
y	0	6	6	3	0	0	6	p

   (i)   Calculate the value of p.   [1]

   (ii) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 4$.
         Using a scale of 1 cm to represent 2 units, draw a vertical y-axis for $-4 \leqslant y \leqslant 24$.
         On your axes, plot the points given in the table and join them with a smooth curve.   [3]

   (iii) Using your graph, find the values of x when $y = 4$.   [2]

(b) A rectangular card is 5 cm long and 4 cm wide.
     As shown in the diagram, a square of side x centimetres is cut off from each corner.
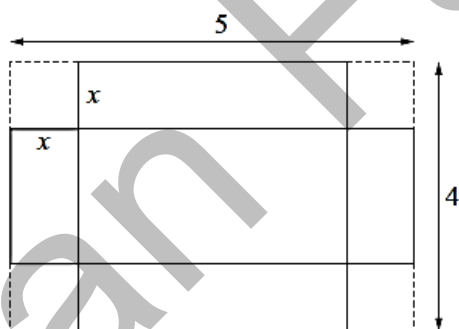
The card is then folded to make an open box of height x centimetres.

   (i)   Write down expressions, in terms of x, for the length and width of the box.   [1]

   (ii) Show that the volume, V cubic centimetres, of the box is given by the equation

$$V = 4x^3 - 18x^2 + 20x.$$ [2]

   (iii) Which value of x found in (a)(iii) **cannot** be the height of a box with a volume of $4\,\text{cm}^3$? [1]

   (iv) Using the graph drawn in part (a)(ii), find

      (a) the greatest possible volume of a box made from this card,   [1]

      (b) the height of the box with the greatest volume.   [1]

Answers: (a)(i) $p = 21$ (ii) plotting all 8 points correctly, and joining with a smooth curve (iii) $x = 0.2$ **N09/2/Q8**
to 0.35, 1.3 to 1.4 and 2.8 to 2.95 (b)(i) $5 - 2x$ and $4 - 2x$ (ii) $V = x(5 - 2x)(4 - 2x)$ leading to
$V = 4x^3 - 18x^2 + 20x$ (iii) $x = 2.8$ to 2.95 (iv)(a) their maximum value of y between $x = 0$ and 2
(b) 0.7 to 0.8 cm

25 Answer the WHOLE of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = x^2 - 1 .$$

(a) Copy and complete the following table of values. [2]

x	−4	−3	−2	−1	0	1	2	3	4
y									

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-4 \leqslant x \leqslant 4$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-2 \leqslant y \leqslant 16$.

On your axes, plot the points from your table and join them with a smooth curve. [3]

(c) It is given that $f(x) = \dfrac{x+7}{2}$.

    (i) Using the same axes, draw the graph of $y = f(x)$. [2]

    (ii) Find $f^{-1}(3)$. [2]

    (iii) (a) Write down the x coordinates of the two points where the graphs intersect. [1]

          (b) Find the equation that is satisfied by these two values of x.
          Express your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers. [2]

Answers: (a) 15, 8, 3, 0, −1, 0, 3, 8, 15 (ii) −1 (iii)(a) −1.9 and 2.4 (b) $2x^2 - x - 9 = 0$ N10/21/Q8

26 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^3}{10} - \frac{x}{2}.$$

The table below shows some corresponding values of x and y.

x	0	1	2	3	4	4.5
y	0	−0.4	−0.2	1.2	4.4	p

(a) Calculate p. [1]

(b) Using a scale of 2 cm to 1 unit on each axis, draw a horizontal x-axis for $0 \leqslant x \leqslant 5$ and a vertical y-axis for $-1 \leqslant y \leqslant 7$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $\frac{x^3}{10} - \frac{x}{2} = 0.3$ for values of x in the range $0 \leqslant x \leqslant 5$. [1]

(d) (i) Draw the chord joining the two points $(0, 0)$ and $(3, 1.2)$ and calculate its gradient. [1]

    (ii) Draw a tangent at the point where the gradient of the curve is equal to the gradient of the chord. [1]

(e) (i) On the same axes, draw the graph of the straight line $y = -x + 6$. [2]

    (ii) Write down the x coordinate of the point where the line crosses the curve. [1]

    (iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$.

        Find A and B. [2]

Answers: (a) 6.9   (c) 2.5   (d)(i) 0.4   (e)(ii) 3.5   (iii) $A = 5$ and $B = -60$ **N10/22/Q8**

27   The variables x and y are connected by the equation $y = 2x - \dfrac{5}{2x}$.

The table below shows some values of x and the corresponding values of y.
The values of y are correct to 1 decimal place where appropriate.

x	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	−9.5	−4	−1.8	−0.5	0.5	1.3	2.1	2.8

(a) On the grid, plot the points given in the table and join them with a smooth curve.
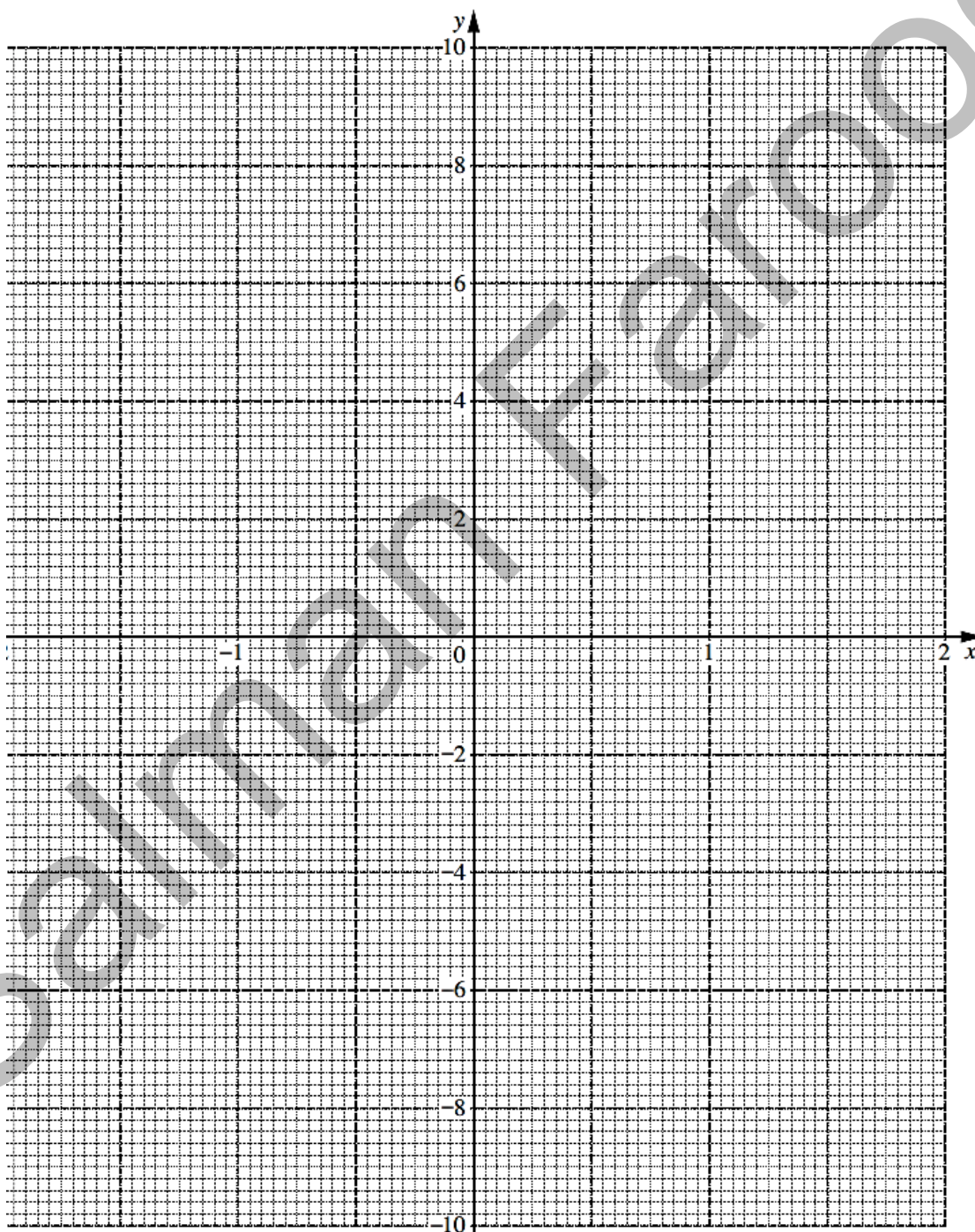
[2]

(b) By drawing a tangent, find the gradient of the curve at the point (0.75, −1.8).

Answer .... [2]

(c) The line $y = 2 - x$ intersects the curve $y = 2x - \dfrac{5}{2x}$ at the point P.

  (i) On the grid, draw the graph of the straight line $y = 2 - x$. [2]

  (ii) Write down the x coordinate of P.   *Answer* .... [1]

  (iii) This value of x is a solution of the equation $6x^2 - Bx - C = 0$.

    Find B and C.   *Answer* $B = $ ....

                  $C = $ .... [3]

(d) Let $f(x) = 2x - \dfrac{5}{2x}$.

  (i) Given that $f(a) = b$, show that $f(-a) = -b$. [1]

  (ii) **Hence**, using the table on the previous page, draw the graph of $y = 2x - \dfrac{5}{2x}$ [1]
    for $-2 \leqslant x \leqslant -0.25$.

Answers: (b) 5.5 – 7.5 (c)(ii) 1.3 (iii) $B = 4$, $C = 5$ N11/21/Q8

642

28 Two companies, A and B, were started 10 years ago.
Initial investments of $25 or multiples of $25 could be made when Company A started business.

(a) The table shows the value of an initial investment of $25 at the end of each of the
next 10 years.

Number of years (x)	0	1	2	3	4	5	6	7	8	9	10
Value in dollars (y)	25	28	31	35	39	44	49	55	62	69	78

(i) Calculate the value of an initial investment of $500 after 8 years.

Answer $.................................. [1]

(ii) On the grid, plot the points given in the table and join them with a smooth curve.
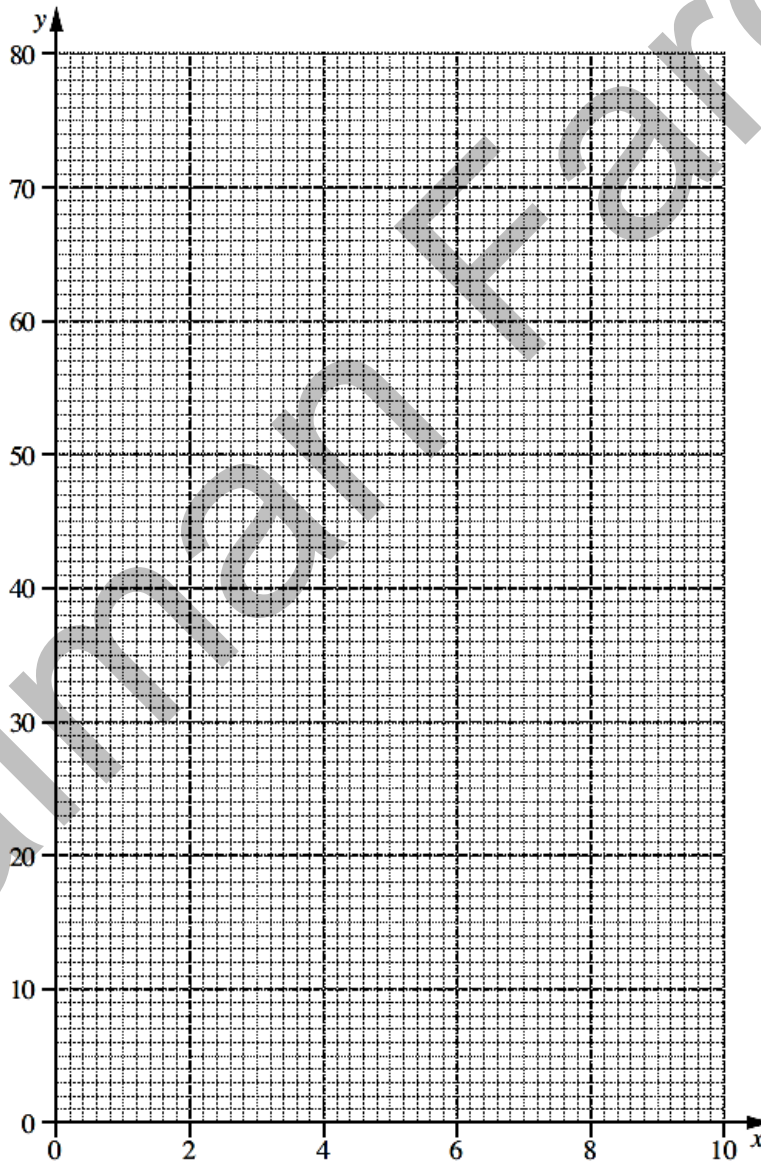
[2]

(iii) Using your graph, find x when the value of an initial investment of $100 had increased to $168.

Answer .... [1]

(b) An initial investment of $25 was made when company B started business.
The value, y dollars, after x years, is given by the equation $y = 3.75x + 25$.

(i) Calculate the value of an initial investment of $500 after 8 years.

Answer $.................................. [1]

(ii) On the grid, draw the graph of $y = 3.75x + 25$. [2]

(c) Using your graphs, find the value of x when an initial investment of $25 had increased to the same value in each company.

Answer .... [1]

(d) (i) By drawing a tangent to the graph representing an investment in company A, find the rate of increase of this investment when $x = 7$.

Answer .... [2]

(ii) State the rate of increase of an investment in company B.

Answer .... [1]

(iii) By drawing another tangent to the graph representing an investment in company A, find the value of x when the rates of increase of investments in each company were the same.

Answer .... [1]

Answers: **(a)(i)** 1240 **(iii)** 4.6 **(b)(i)** 1100 **(c)** 4.8 **(d)(i)** $6 \le$ gradient ≤ 7 **(ii)** 3.75 dollars per year **(iii)** 2 **N11/22/Q8**

8   The variables x and y are connected by the equation

$$y = x^3 - 2x^2 + 1.$$

The table below shows some values of x and the corresponding values of y, correct to 1 decimal place where appropriate.

x	-1	-0.5	0	0.5	1	1.5	2	2.5
y	-2	0.4	1	0.6	0	-0.1	1	p

(a) Calculate p. Give your answer correct to 1 decimal place.

Answer $p = $ ....[1]

(b) On the graph paper opposite, using a scale of 2 cm to represent 1 unit on both axes,
     draw a horizontal x-axis for $-2 \leqslant x \leqslant 3$ and
     draw a vertical y-axis for $-3 \leqslant y \leqslant 5$.
     On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c) Use your graph to find all the solutions of $x^3 - 2x^2 + 1 = 0$.

Answer $x = $ ....[2]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = -0.3$.

Answer   ....[2]

(e) The equation $x^3 - 2x^2 - x + 2 = 0$ can be solved by finding the intersection of the graphs
     of $y = x^3 - 2x^2 + 1$ and the straight line $y = x + a$.

   (i)   Find the value of a.

Answer $a = $ ....[1]

   (ii) Hence solve the equation $x^3 - 2x^2 - x + 2 = 0$.

Answer $x = $ ....[3]

Answer: (a) 4.1 (c) correct readings from graph (d) 1 to 2 (e)(i) -1 (ii) -1, 1, 2
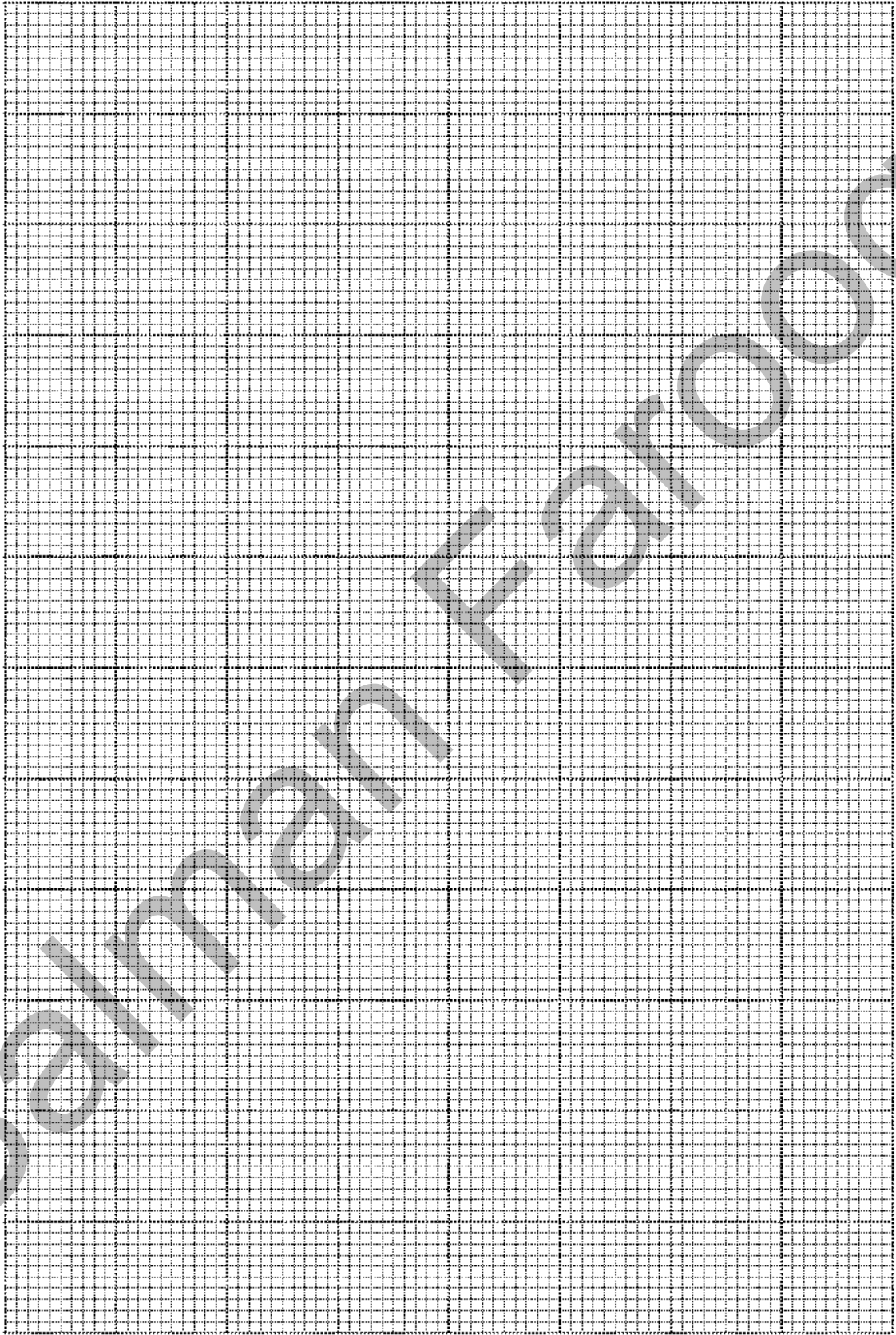
The number of bacteria in a colony **trebles** every hour.
The colony starts with 50 bacteria.
The table below shows the number of bacteria (y) in the colony after t hours.

Time (t hours)	0	1	2	2.5	3	3.5	4
Number of bacteria (y)	50	150	450	780	1350	2340	

(a) Complete the table. [1]

(b) On the grid on the opposite page plot the points in the table, and join them with a
     smooth curve. [3]

(c) Use your graph to find the number of bacteria in the colony when $t = 3.2$.

Answer ....[1]

(d) (i) By drawing a tangent, estimate the gradient of the curve when $t = 2.5$.

Answer ....[2]

     (ii) What does this gradient represent?

     Answer ....

     .... [1]

(e) Given that the equation of the graph is $y = ka^t$, find k and a.

Answer k = ... a = ... [1]

(f) The number of bacteria in another colony is given by the equation $y = 500 + 500t$.

     (i) On the same axes, draw a graph to represent the number of bacteria in this colony.

[2]

     (ii) State the value of t when the number of bacteria in each colony is the same.

Answer ....[1]
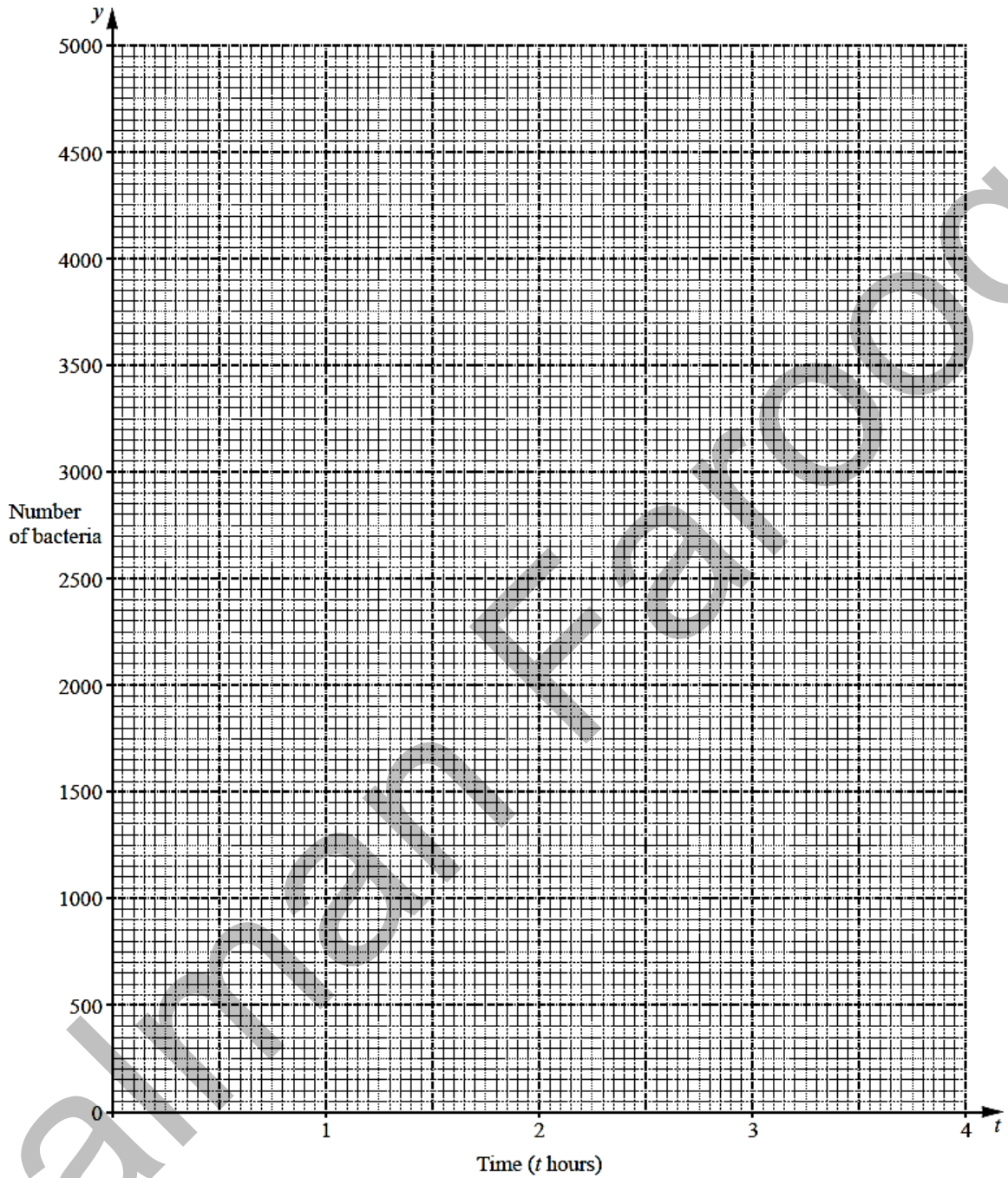
Answer. (a) 4050 (c) 1700 (d)(i) 870 (ii) Rate of increase (e) $k = 50$ $a = 3$ (f)(ii) 3.45 **N13/21/Q9**

31

Hendrik travels by plane from London to Bangkok.
When it is 04 00 local time in London it is 10 00 local time in Bangkok.

(a) The flight takes 11 hours and 15 minutes.

If he leaves London at 21 50 local time, what is the local time in Bangkok when he arrives?

Answer .... [2]

(b) On his return journey, Hendrik leaves Bangkok at 07 45 local time and arrives back in London on the same day at 13 40 local time.

How long was his return flight?   *Answer* .... hours .... minutes [2]

(c) The graph opposite shows the exchange rate between British Pounds (£) and Thai Baht (THB) on the day Hendrik arrives in Bangkok.

(i) Use the graph to estimate the cost in British Pounds of an item costing 13 000 THB.

Answer £ .... [1]

(ii) The exchange rate can be written as £1 = k THB.

Find k.   **5**   *Answer* .... [1]
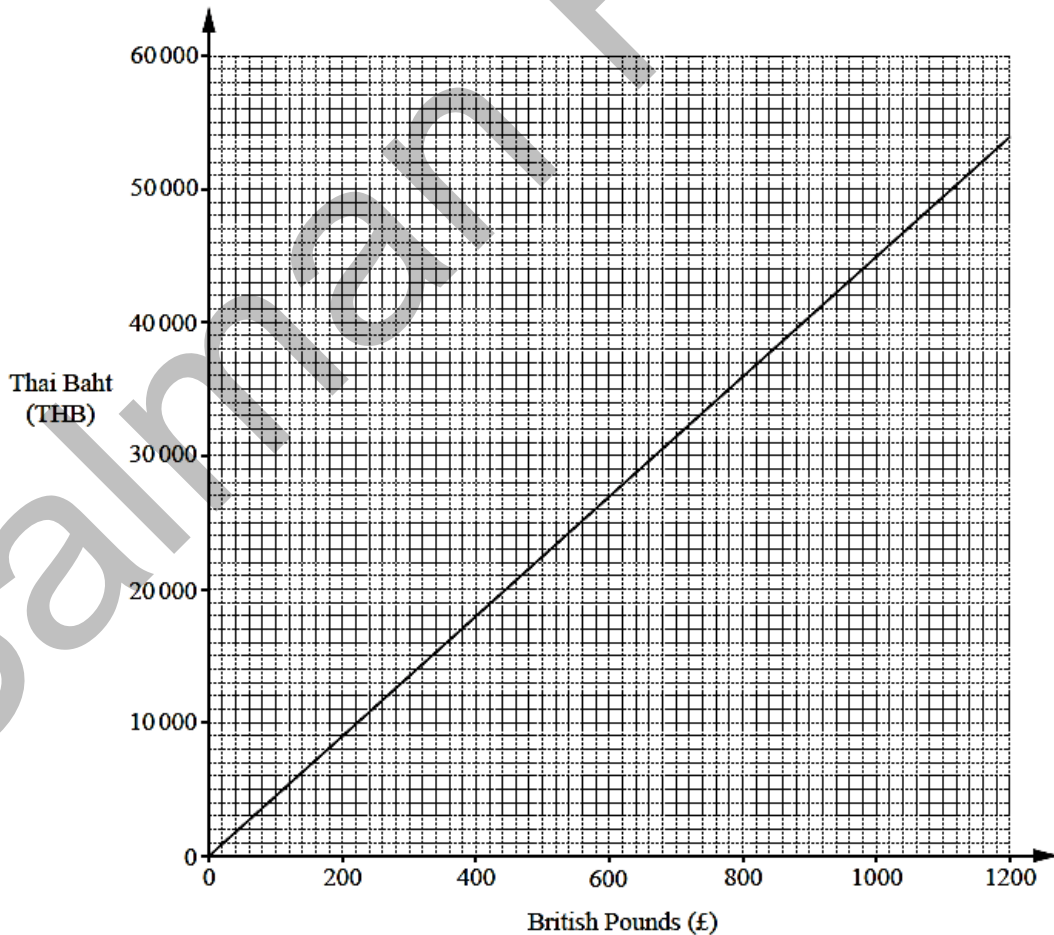
(d) The cost of flights from London to Bangkok is shown in the table below.
For this cost, passengers are allowed to take luggage up to the weight shown.
Passengers taking more than this weight of luggage pay an excess charge at the rate shown.

	Cost of flight	Weight of luggage included	Charge per extra 1 kg
Business Class	£1932	30 kg	£24
Economy Class	£683	23 kg	£24

Calculate the total cost of Hendrik flying Economy Class from London to Bangkok with luggage weighing 29 kg.

Answer £ .... [2]

Answers: **(a)** 15 05; **(b)** 11 hours 55 minutes; **(c)(i)** 290; **(ii)** 45; **(d)** 827.   **N14/21/Q2**

32

(a) (i) Complete the table and hence draw the graph of $y = x^2 - 2x - 8$.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
y		7	0	-5	-8	-9	-8	-5	0	7	

[3]
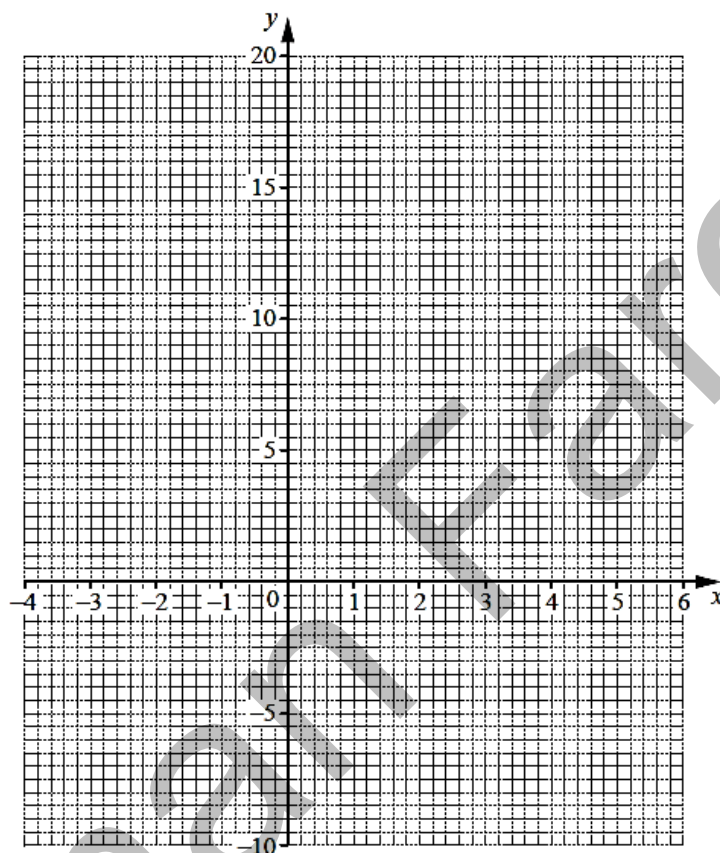
(ii) Use your graph to solve $x^2 - 2x - 8 = 2$.

Answer $x =$ .... or .... [2]

(iii) By drawing a tangent, estimate the gradient of the curve at $(3, -5)$.

Answer .... [2]

(b) Solve algebraically $x^2 + x - 9 = 0$, giving your answers correct to 2 decimal places.

Answer $x =$ .... or .... [3]

(c) The x-coordinates of the intersection of the line L and the curve $y = x^2 - 2x - 8$ are the solutions of the equation $x^2 + x - 9 = 0$.

Find the equation of the line L.

Answer .... [2]

33   [Curved surface area of a cone $= \pi r l$]
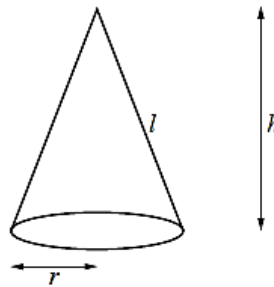
The diagram shows a solid cone with radius r cm, height h cm and slant height l cm.
Suleman makes some solid cones.
The slant height of each of his cones is 4 cm more than its radius.
Use $\pi = 3$ throughout this question.

(a) Show that the total surface area, A cm^2, of each of Suleman's cones is given by $A = 6r(r + 2)$.

[2]

(b) Complete the table for $A = 6r(r + 2)$.

r	0	1	2	3	4	5	6
A	0	18			144	210	288

[1]

(c) On the grid opposite, draw the graph of $A = 6r(r + 2)$.   [2]

(d) Find an expression for h in terms of r.   *Answer* $h = $ .... [2]

652
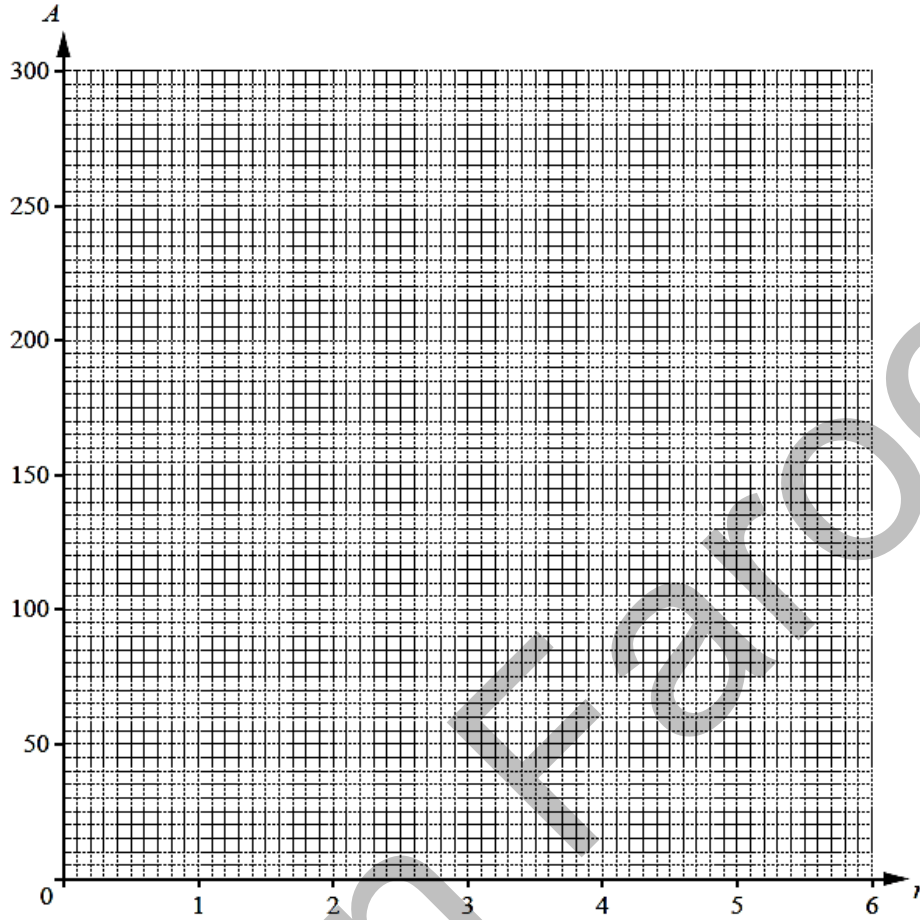
(e) The height of one of Suleman's cones is 12 cm.
Calculate its radius.

Answer .... cm [2]

(f) Another of Suleman's cones has a surface area of 200 cm^2.

(i) Use your graph to find the radius of this cone.

Answer .... cm [1]

(ii) This cone is placed in a box of height p cm, where p is an integer.
Find the smallest possible value of p.

Answer p = ... [2]
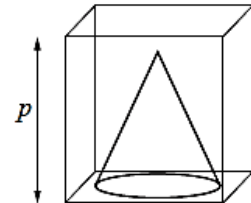
Answers: **(a)** $A = 6r(r+2)$ correctly derived; **(b)** 48, 90; **(c)** Correct curve; **(d)** $\sqrt{8r+16}$; **(e)** 16; **(f)(i)** 4.9; **(ii)** 8   **N15/21/Q8**

34

$$y = \frac{3}{5} \times 2^x$$

The table shows some values of x and the corresponding values of y, correct to one decimal place where necessary.

x	-1.5	-1	0	1	2	2.5	3	3.5	4
y	p	0.3	0.6	1.2	2.4	3.4	4.8	6.8	9.6

(a) Calculate p.   *Answer* .... [1]

(b) On the grid,
   - using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leqslant x \leqslant 4$,
   - using a scale of 1 cm to 1 unit, draw a vertical y-axis for $0 \leqslant y \leqslant 10$,
   - plot the points from the table and join them with a smooth curve.   [3]
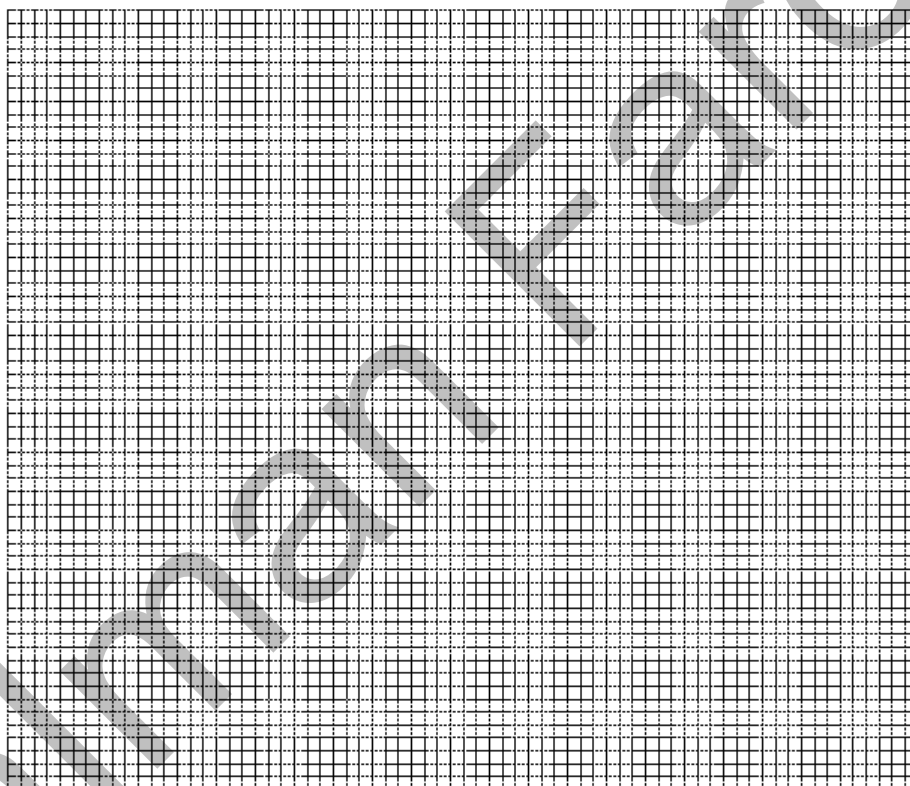
(c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$.

Answer .... [2]

(d) (i) On the same grid, draw the straight line that passes through $(-0.4, 0)$ and $(2, 3.6)$.

[1]

(ii) Find the equation of this line in the form $y = mx + c$.

Answer .... [2]

(iii) Write down the x-coordinates of the points where the line intersects the curve.

Answer $x =$ .... and $x =$ .... [1]

(iv) These x-coordinates satisfy the equation

$$2^x = Ax + B.$$

Find the values of A and B. *Answer* $A =$ .... $B =$ .... [2]

Answers: (a) 0.2 (b) correct curve (c) 2.2 to 2.5 (d)(i) correct line (ii) $y = 1.5x + 0.6$ (iii) 0 and 3.1 **N16/21/Q8**
(iv) $A = 2.5$, $B = 1$

35

(a) (i) The points $(4, -3)$ and $(0, 5)$ lie on the line L.

Find the equation of line L.   *Answer* .... [2]

(ii) The line M is parallel to line L and passes through the point $(-2, 3)$.

Find the equation of line M.   *Answer* .... [2]

(b) The table below shows some values of x and the corresponding values of y for $y = x + \dfrac{3}{x} - 3$.

x	0.5	1	1.5	2	3	4	5	6
y	3.5	1	0.5	0.5	1	1.75	2.6	

(i) Complete the table.   [1]

(ii) Using a scale of $2\,\text{cm}$ to 1 unit on both axes, draw a horizontal x-axis for $0 \leqslant x \leqslant 7$ and a vertical y-axis for $0 \leqslant y \leqslant 4$.

Draw the graph of $y = x + \dfrac{3}{x} - 3$ for $0.5 \leqslant x \leqslant 6$.
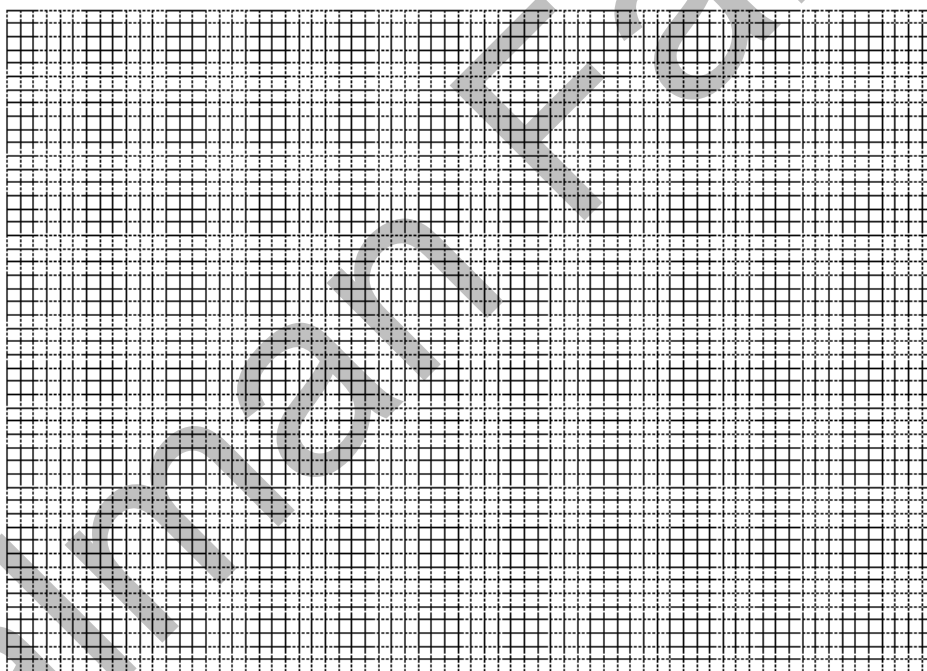
[3]

(iii) By drawing a tangent, estimate the gradient of the curve at $(1, 1)$.

Answer .... [2]

(iv) Use your graph to solve the equation $x + \dfrac{3}{x} = 5$.   *Answer* $x =$ .... or .... [2]

Probability Paper 1

1 A fair five-sided spinner is numbered using the prime numbers 2, 3, 5, 7 and 11.
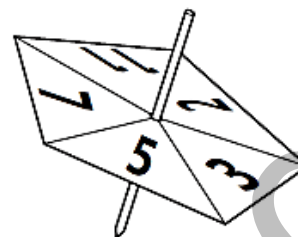
(a) In a game, players spin it twice and add the two numbers obtained.

   (i) Complete the possibility diagram.

   Answer (a)(i)

+	2	3	5	7	11
2	4	5			
3					
5			10	12	
7		12			
11					

[1]

   (ii) Find the probability that the total of the two numbers is

      (a) a prime number,   *Answer (a)(ii)(a)* ....[1]

      (b) a perfect square.

                             (b) ....[1]

(b) In another game, players spin it twice and multiply the two numbers obtained.
   Without drawing another possibility diagram, write down the probability that this product is a prime number.

                             Answer (b) ....[1]

Answer: (a)(i)

–	–	7	9	13
5	6	8	10	14
7	8	–	–	16
9	10	–	14	18
13	14	16	18	22

(a)(ii)(a) $\frac{6}{25}$ (a)(ii)(b) $\frac{1}{5}$ (b) 0   **J06/1/Q21**

2   A bag contains 6 red counters and 4 blue counters.
Two counters are taken from the bag at random, without replacement.

(a)   Complete the tree diagram below that represents these events.

Answer (a)
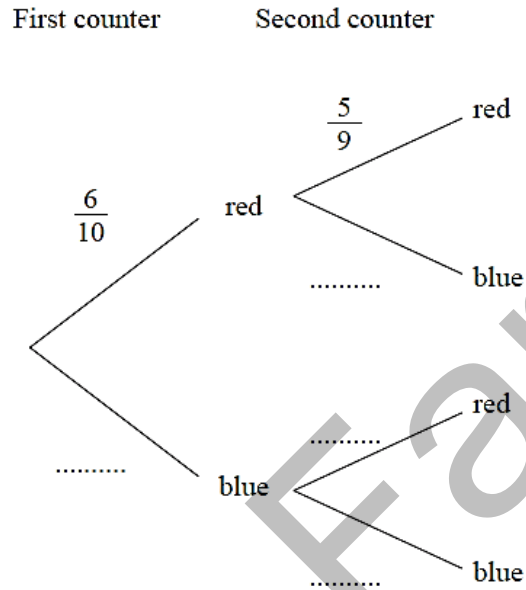
First counter   Second counter

$\frac{6}{10}$ red

$\frac{5}{9}$ red

....   blue

....   blue

....   red

....   blue

[1]

(b)   Expressing your answer as a fraction in its simplest form, calculate the probability that both counters are the same colour.

Answer (b) ....[2]

Answer: (a) $\frac{4}{10}, \frac{4}{9}, \frac{6}{9}, \frac{3}{9}$,   (b) $\frac{7}{15}$

J10/12/Q16

3 Kelvin chooses a pair of trousers and a shirt to wear for college.
He chooses grey trousers or black trousers.
He chooses a red shirt or a yellow shirt.

The probability that he chooses grey trousers is $\frac{1}{3}$.

The probability that he chooses a red shirt is $\frac{4}{5}$.
His choice of shirt is independent of his choice of trousers.

(a) Complete the tree diagram.

Trousers   **Shirt**

.... ——— Red

Grey

.... ——— Yellow

.... ——— Red

.... ——— Black

.... ——— Yellow
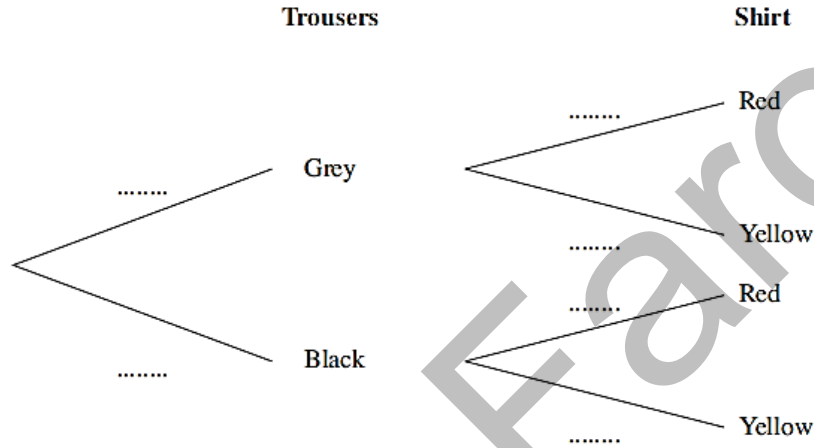
[2]

(b) What is the probability that Kelvin chooses grey trousers and a red shirt?

Answer .... [1]

(c) What is the probability that Kelvin does not choose either black trousers or a red shirt?

Answer .... [2]

Answers: **(a)** Correct tree diagram **(b)** $\frac{4}{15}$ **(c)** $\frac{1}{15}$ **J11/11/Q21**

4 Sachin and Zaheer play a game of tennis and a game of badminton.
The results of the games are independent and the games cannot be drawn.
The probability that Sachin wins the game of tennis is $\frac{3}{4}$.

The probability that Zaheer wins the game of badminton is $\frac{3}{5}$.

(a) What is the probability that Sachin wins both games?   Answer ....[1]

(b) What is the probability that Zaheer wins just one of the games?

Answer ....[2]

Answer: **(a)** $\frac{3}{10}$ **(b)** $\frac{11}{20}$ **J12/11/Q14**

5

On a plate there are ten biscuits.
Four of the biscuits are round and six of the biscuits are square.
Sabah chooses a biscuit at random from the plate and eats it.
She then chooses another biscuit at random from the plate.
The tree diagram shows the possible outcomes and some of their probabilities.

First biscuit   Second biscuit
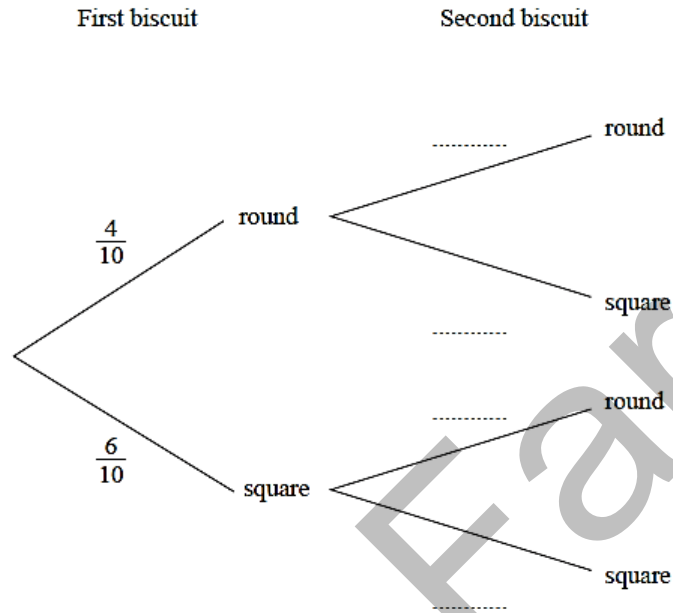
(a) Complete the tree diagram.   [2]

(b) Calculate the probability that Sabah chooses

    (i) two round biscuits,   *Answer* .... [1]

    (ii) one round biscuit and one square biscuit.   *Answer* .... [2]

Answers: (a) $\dfrac{3}{9}, \dfrac{6}{9}, \dfrac{4}{9}, \dfrac{5}{9}$ (b)(i) $\dfrac{12}{90}$ (ii) $\dfrac{48}{90}$ **J14/11/Q24**

6 A bag contains 5 balls, of which 3 are red and 2 are blue.

One ball is taken, at random, from the bag and is not replaced.
If this ball is red, another ball is taken, at random, from the bag and is not replaced.
This process is repeated until a blue ball is taken from the bag.

Part of the tree diagram that represents these outcomes is drawn below.

First ball   Second ball

$\frac{3}{5}$ ——— red $\frac{2}{4}$ ——— red

$\frac{2}{5}$ ➤ blue $\frac{2}{4}$ ➤ blue
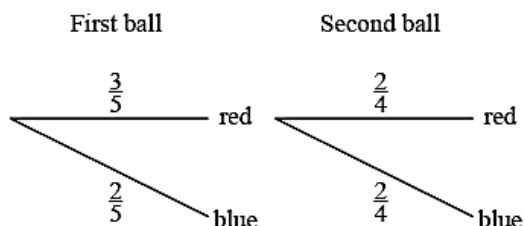
(a) Complete the tree diagram.   [2]

(b) Expressing each answer as a fraction, find the probability that

    (i) the second ball taken is blue,   *Answer* .... [1]

    (ii) a blue ball is the second, or the third, ball taken.   *Answer* .... [2]

Answers: (a) $\frac{1}{3}$, $\frac{2}{3}$, 0, 1 (b)(i) $\frac{3}{10}$ (ii) $\frac{1}{2}$

7

Box 1   Box 2
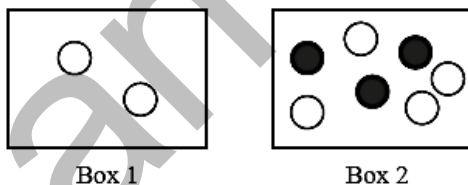
Box 1 contains 2 white balls. Box 2 contains 4 white balls and 3 black balls.

(a) Ann chooses, at random, one ball from each box.

    (i) Find the probability that these balls are both black.

                                            Answer .... [1]

    (ii) Find the probability that these balls have different colours.

                                            Answer .... [1]

(b) From the original contents of **Box 2**, Belle chooses, at random, two balls without replacement.

    Find the probability that these balls are both white.   *Answer* .... [1]

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(c) Carla chooses one of the boxes at random.
With the original box contents, she then chooses, at random, one ball from this box.

Find the probability that the ball is white.

Answer .... [2]

Answers: (a)(i) 0 (a)(ii) $\frac{3}{7}$ (b) $\frac{2}{7}$ (c) $\frac{11}{14}$

8 A bag contains 1 red, 1 blue and 3 green balls.
Two balls are taken from the bag, at random, without replacement.
The tree diagram that represents these events is drawn below.
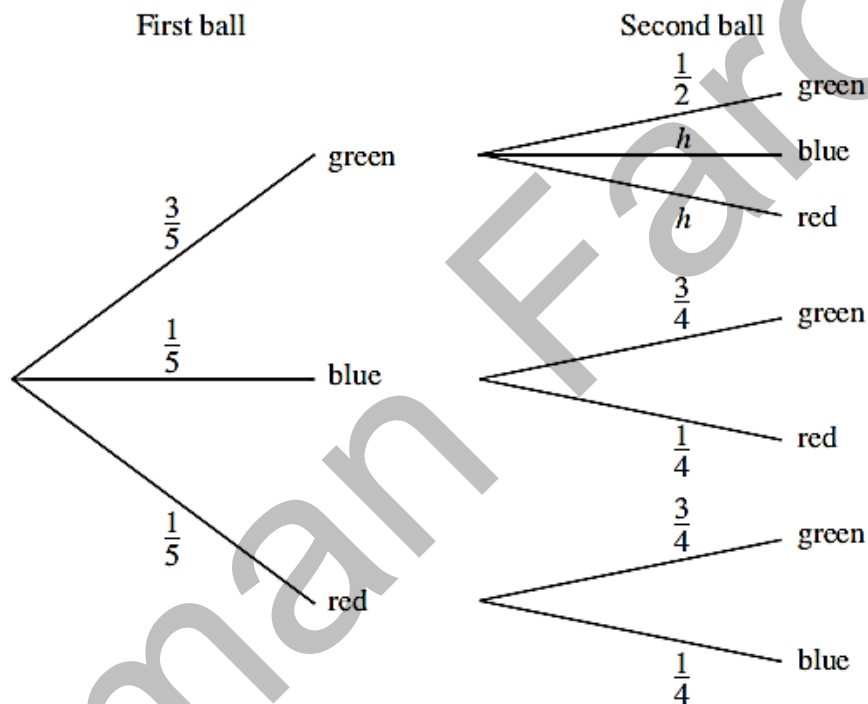
(a) Write down the value of h.

Answer (a) $h =$ .... [1]

(b) Expressing each answer in its simplest form, calculate the probability that

    (i) both balls are green,   (b)(i) .... [1]

    (ii) both balls are blue,   (ii) .... [1]

    (iii) neither ball is green.   (iii) .... [1]

Answer : (a) $h = \frac{1}{4}$ (b)(i) $\frac{3}{10}$ (ii) 0   (iii) $\frac{1}{10}$

9

A bag contains red, green and yellow pegs.
A peg is taken at random from the bag.
The probability that it is red is 0.35 and the probability that it is green is 0.4.

(a) Find the probability that it is

    (i) yellow,   *Answer (a)(i)* ....[1]

    (ii) not red.   (ii) ....[1]

(b) Originally there were 16 green pegs in the bag.
    Find the total number of pegs.   *(b)* ....[1]

Answers: **(a) (i)** 0.25,   **(ii)** 0.65,   **(b)** 40.   **N08/1/Q8**

10

In a group of 8 students there are 5 boys and 3 girls.
Two students are chosen at random.
The tree diagram shows the possible outcomes and their probabilities.

Answer (a)

First student   Second student
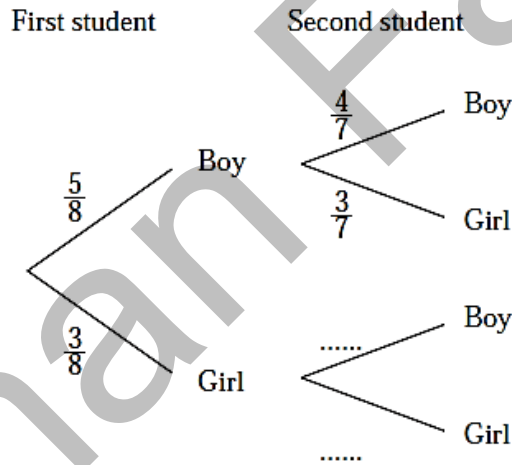
(a) Complete the tree diagram.   [1]

(b) Expressing each answer as a fraction in its lowest terms, find the probability that

    (i) two boys are chosen,   *Answer (b)(i)* ....[1]

    (ii) at least one boy is chosen.   *Answer (b)(ii)* ....[2]

Answers: **(a)** $\frac{5}{7}, \frac{2}{7}$ correctly placed   **(b)(i)** $\frac{5}{14}$ **(ii)** $\frac{25}{28}$ **N09/1/Q21**

11 Two bags contain beads.
The first bag contains 2 black and 3 white beads.
The second bag contains 2 black, 1 white and 2 red beads.
A bead is taken, at random, from each bag.
The tree diagram that represents these outcomes is drawn below.

First bag   Second bag

                                      $\frac{2}{5}$ black
                                      $\frac{1}{5}$
                              black $<$ white
               $\frac{2}{5}$
                                      $\frac{2}{5}$ red

                                      $\frac{2}{5}$ black
               $\frac{3}{5}$ $\frac{1}{5}$
                              white $<$ white
                                      $\frac{2}{5}$ red

Giving each answer in its simplest form, find the probability that

(a) both beads are black,   *Answer* (a) ....[1]

(b) both beads are red,   *Answer* (b) ....[1]

(c) exactly one bead is black.   *Answer* (c) ....[2]

Answers: (a) 4/25 (b) 0 (c) 12/25   **N10/11/Q19**

665

12 A bag contains 7 balls, 6 of which are green and 1 is red.
Two balls are taken from the bag, at random, without replacement.
The tree diagram that represents these events is drawn below.

First ball   Second ball
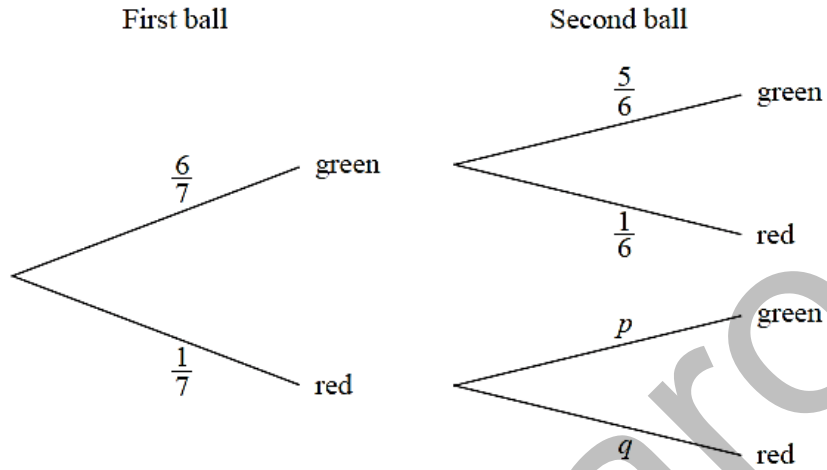
(a) Find the values of p and q. *Answer* (a) $p = $ .... $q = $ ....[1]

(b) Expressing each answer as a fraction in its simplest form, find the probability that

(i) both balls are green, *Answer* (b)(i) ....[1]

(ii) the two balls have different colours. *Answer* (b)(ii) ....[1]

Answer: (a)(i) $p = 1$, $q = 0$ (b)(i) $\dfrac{5}{7}$ (ii) $\dfrac{2}{7}$

13

A bag contains 1 red and 3 blue balls.
Two balls are taken from the bag, at random, without replacement.
The tree diagram that represents all the outcomes is shown below.

First ball   Second ball
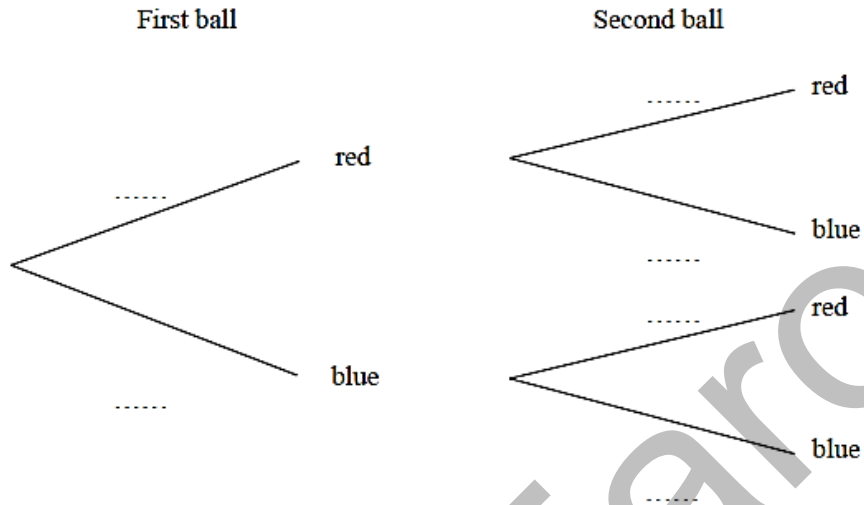
(a) Write the appropriate probability on each branch.   [2]

(b) Find the probability that the second ball taken is red.   *Answer*   ....[1]

Answers: (a) $\left(\frac{1}{4} \text{ and } \frac{3}{4}\right)$, (0 and 1) and $\left(\frac{1}{3} \text{ and } \frac{2}{3}\right)$ (b) $\frac{1}{4}$

N12/11/Q12

14 Two bags contain beads.
The first bag contains 2 white, 2 red and 3 black beads.
The second bag contains 3 white and 2 black beads.
One bead is taken, at random, from each bag.
The tree diagram is shown below.

First bead   Second bead

$$\begin{array}{cc}
\text{white} & \frac{3}{5}\ \text{white} \\
& \frac{2}{5}\ \text{black}
\end{array}$$
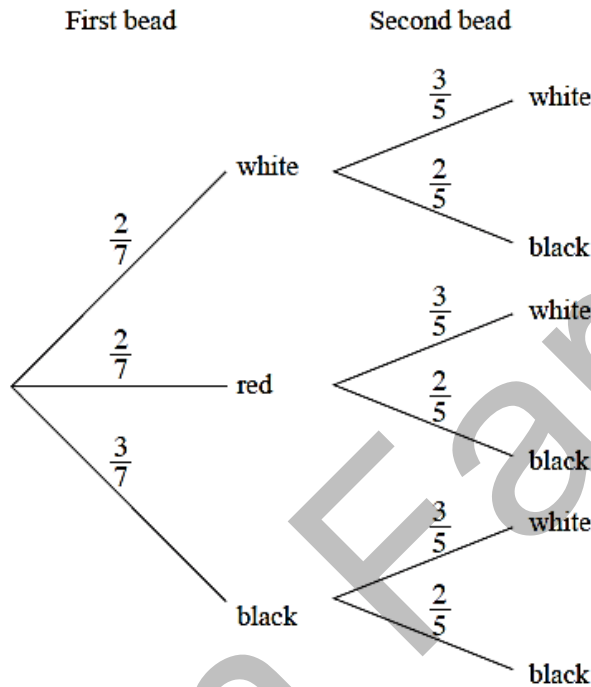
Find the probability that

(a) both beads are white,   *Answer* ....[1]

(b) both beads are red,   *Answer* ....[1]

(c) exactly one bead is black.   *Answer* ....[2]

Answers: (a) $\frac{6}{35}$ (b) 0 (c) $\frac{17}{35}$

---

15 Paul takes examinations in Maths and Physics.
The probability that he passes Maths is 0.7 .
The probability that he passes Physics is 0.6 .
The results in each subject are independent of each other.   *Answer* .... [2]

Calculate the probability that he passes Maths and does **not** pass Physics.

Answer: 0.28

Probability Paper 2

1   (b)   An ordinary unbiased die has faces numbered 1, 2, 3, 4, 5 and 6.
Sarah and Terry each threw this die once.
Expressing each answer as a fraction **in its lowest terms**, find the probability that

     (i)   Sarah threw a 7,   [1]

     (ii)   they both threw a 6,   [1]

     (iii)   neither threw an even number,   [1]

     (iv)   Sarah threw exactly four more than Terry.   [1]

(b)(i) 0, (ii) $\dfrac{1}{36}$, (iii) $\dfrac{1}{4}$, (iv) $\dfrac{1}{18}$.

                                                J03/2/Q5b

2   **(b)**   A bag contained 5 Red and 2 Blue beads.
Chris took 3 beads, at random, and without replacement, from the bag.
The probability tree shows the possible outcomes and their probabilities.
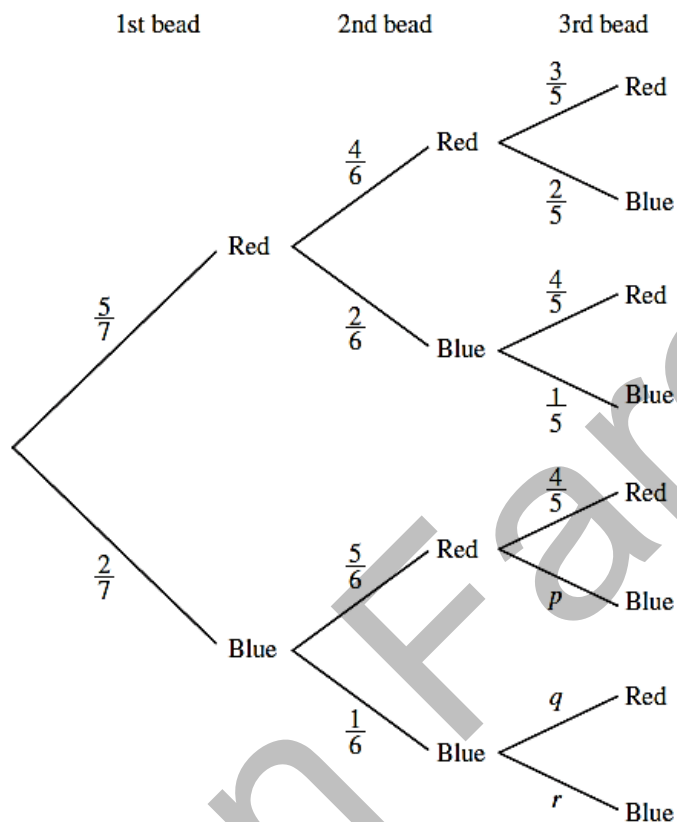
(i)   Write down the values of p, q and r.   [2]

(ii)   Expressing each answer as a fraction in its lowest terms, find the probability that

  (a)   three Red beads were taken,   [1]

  (b)   the first bead was Red, the second Blue and the third Red,   [1]

  (c)   two of the beads were Red and one was Blue.   [2]

(b)(i) $\frac{1}{5}$, 1, 0, (ii) $\frac{2}{7}$, $\frac{4}{21}$, $\frac{4}{7}$.   J05/2/Q5b

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3   **(b)**   Tina has two fair, normal 6-sided dice. One is red and the other is blue.
She throws both of them once.
You may find it helpful to draw a possibility diagram to answer the following questions.

Find, as a fraction in its lowest terms, the probability that

  (i)   the red die shows a 2 and the blue die does **not** show a 2,   [1]

  (ii)   the sum of the two numbers shown is equal to 5,   [1]

  (iii)   one die shows a 3 and the other shows an even number.   [2]

(b)(i) $\dfrac{5}{36}$,   (ii) $\dfrac{1}{9}$,   (iii) $\dfrac{1}{6}$;

4   (a)   Ben travels to work by bus, by bicycle or by car.
The probability that he travels by bus on any day is 0.5.
The probability that he travels by bicycle on any day is 0.2.
The tree diagram below shows some of the probabilities of the possible journeys on Monday and Tuesday.
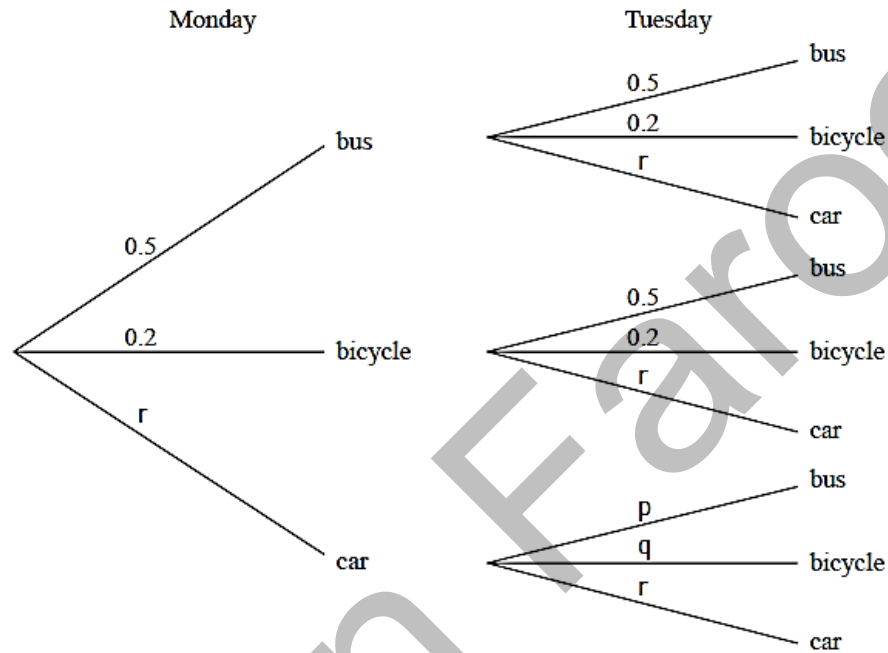
(i)   Find the values of p, q and r.   [2]

(ii)   Find the probability that Ben travels to work

    (a)   by bus on both Monday and Tuesday,   [1]

    (b)   by bus on one day and by bicycle on the other day.   [2]

Answers: (a)(i) $p = 0.5$, $q = 0.2$ and $r = 0.3$ (ii)(a)   0.25   (ii)(b)   0.2   **J10/21/Q5a**

5
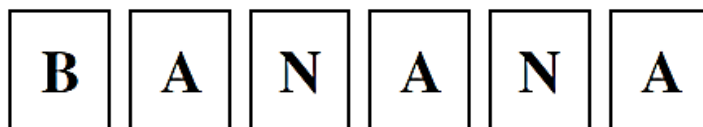
$$\boxed{B}\ \boxed{A}\ \boxed{N}\ \boxed{A}\ \boxed{N}\ \boxed{A}$$

The letters spelling the word BANANA are written on six tiles.

(a) Find the probability that a tile chosen at random has the letter N on it.
Give your answer as a fraction in its simplest form.

Answer ....[1]

(b) The six tiles are placed in a bag.
Three tiles are chosen at random without replacement.
The first is placed in Position 1, the second in Position 2 and the third in Position 3.

_____   _____   _____

Position 1   Position 2   Position 3

  (i) Find the probability that the three tiles spell BAN.
Give your answer as a fraction in its simplest form.

Answer ....[2]

(ii) The tiles are now replaced and the process is repeated.

Find the probability that the three tiles spell either ANN or ANA.
Give your answer as a fraction in its simplest form.

Answer ....[2]

J11/22/Q3

Answers: (a) $\frac{1}{3}$ (b) (i) $\frac{1}{20}$ (ii) $\frac{3}{20}$

6

(a) A box of chocolates contains 10 milk chocolates and 2 plain chocolates.
Sacha eats 3 chocolates chosen at random from the box.
The tree diagram shows the possible outcomes and their probabilities.
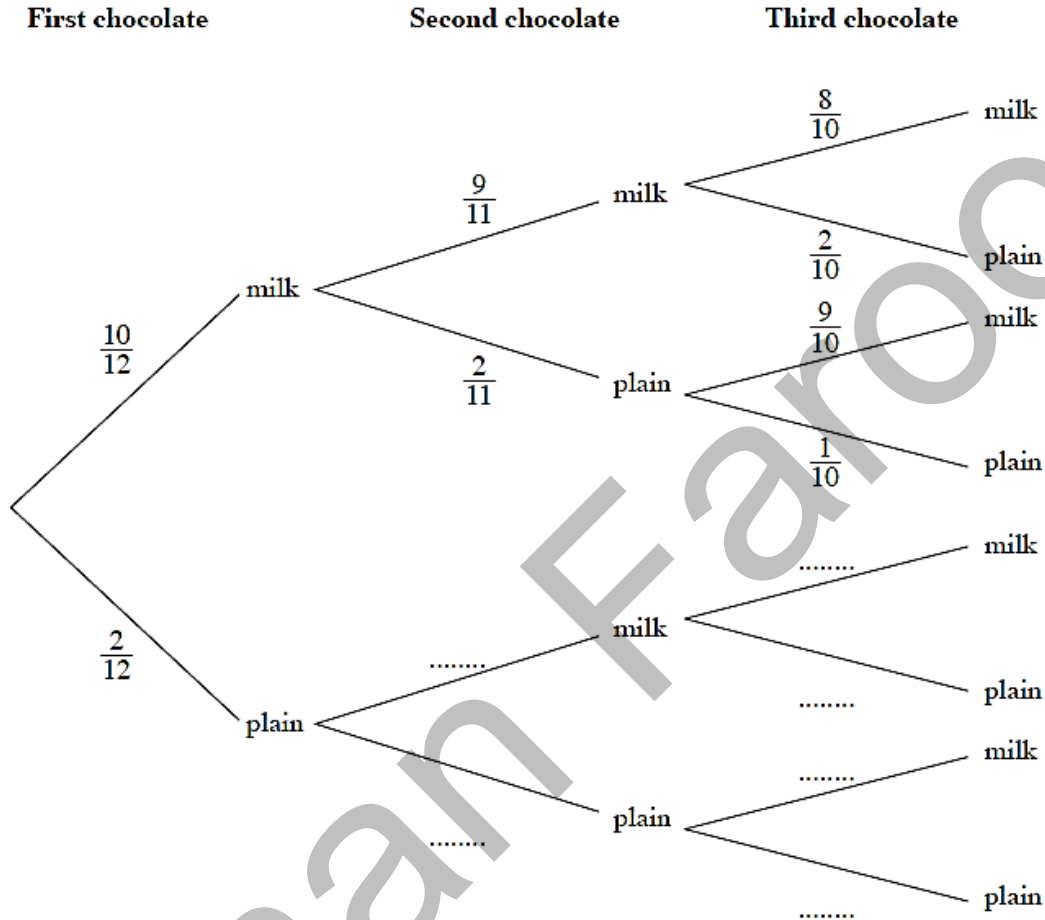
| First chocolate | Second chocolate | Third chocolate |

(i) Complete the tree diagram. [2]

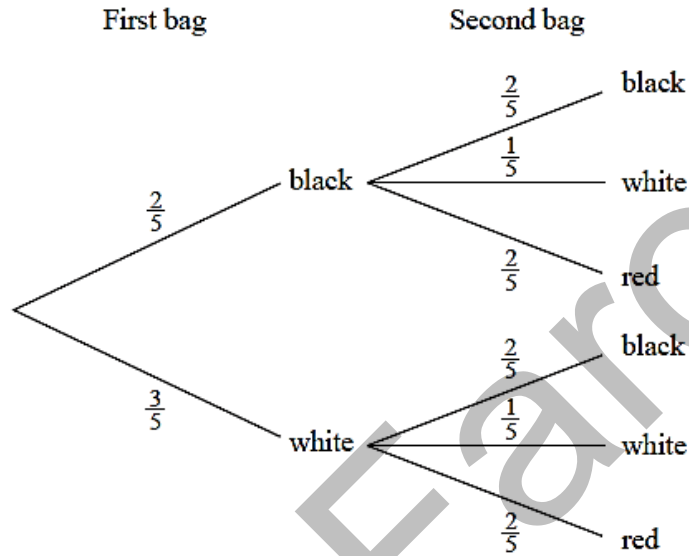(ii) Expressing each answer as a fraction in its lowest terms, find the probability that Sacha

(a) eats 3 milk chocolates, *Answer* .... [1]

(b) eats 2 milk chocolates and 1 plain chocolate in any order.

Answer .... [2]

Answer: (a)(i) 10/11,1/11,9/10,1/10,10/10,0/10 (ii)(a) 6/11 **J13/21/Q4**

7 (a) A bag contains five balls, numbered 1, 2, 3, 4 and 5.
Another bag contains six balls, numbered 1, 2, 3, 4, 5 and 6.

One ball is drawn at random from each bag.

Find the probability that

(i) one ball is numbered 1 and the other 6, [1]

(ii) both balls have an odd number, [1]

(iii) both balls have the same number, [2]

(iv) the sum of the numbers on the balls is 9. [2]

8 A fair five-sided spinner is numbered 1, 3, 5, 7 and 9.
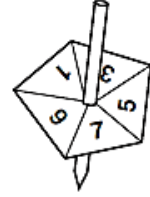
(a) Maria spins it once. Find the probability that the number obtained is

(i) 7,   Answer .... [1]

(ii) an odd number.   Answer .... [1]

(b) Pedro spins it twice and adds the two numbers obtained.
Some of the results are shown in the possibility diagram below.

+	1	3	5	7	9
1	2	4			
3					12
5					
7				14	
9					

(i) Complete the possibility diagram. [2]

(ii) Find the probability that the sum of the two numbers is

(a) odd,   Answer .... [1]

(b) 14 or more.   Answer .... [1]

(c) Katrina spins it three times.

Calculate the probability that the three numbers obtained are the same.
Express your answer as a fraction in its lowest terms. *Answer* .... [2]

Answers: (a)(i) $\frac{1}{5}$ (ii) 1 (b)(ii) 0 (iii) $\frac{6}{25}$ (c) $\frac{1}{25}$

9

(b) A bag contains 12 discs.
There are 8 blue and 4 red discs.

A disc is picked out at random and not replaced.
A second disc is then picked out at random and not replaced.

The tree diagram below shows the possible outcomes and one of their probabilities.

First disc   Second disc

            ....   Blue   ....   Blue

                                    ....   Red

                                    ....   Blue

            ....   Red $\frac{3}{11}$ Red
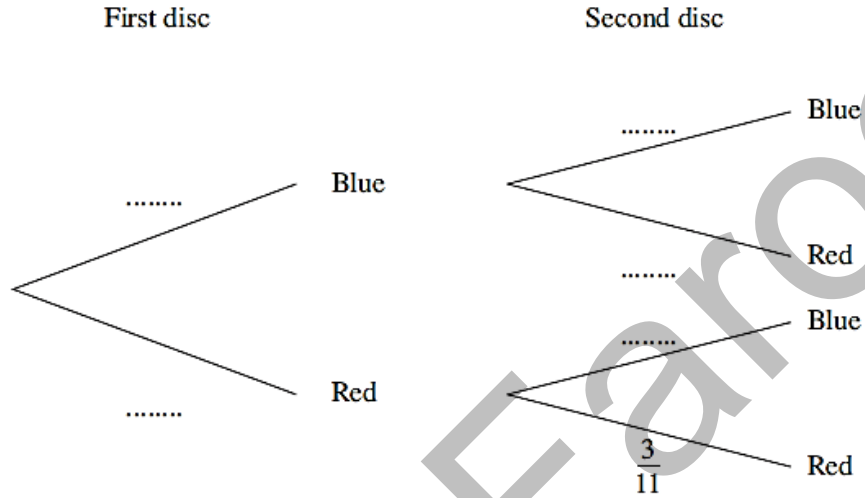
(i) Complete the tree diagram.   [2]

(ii) Expressing each of your answers as a fraction **in its lowest terms**, calculate the probability that

     (a) both discs are red,   *Answer*   .... [1]

     (b) at least one disc is blue.   *Answer*   .... [2]

(iii) A third disc is picked out at random.
Calculate the probability that all three discs are red.

                                           Answer   .... [1]

Answers: (a)(ii) 43 (iii) 18 (iv) 26 (b)(i) $\frac{8}{12}$ and $\frac{4}{12}$ for the first disc, $\frac{7}{11}, \frac{4}{11}$ and $\frac{8}{11}$ for the second disc   **N11/22/Q11**

(ii)(a) $\frac{1}{11}$ (b) $\frac{10}{11}$ (iii) $\frac{1}{55}$

10   (b) A bag contains 5 red counters, 6 blue counters and 1 green counter.

   (i)   Complete the pie chart to represent this data.

Red
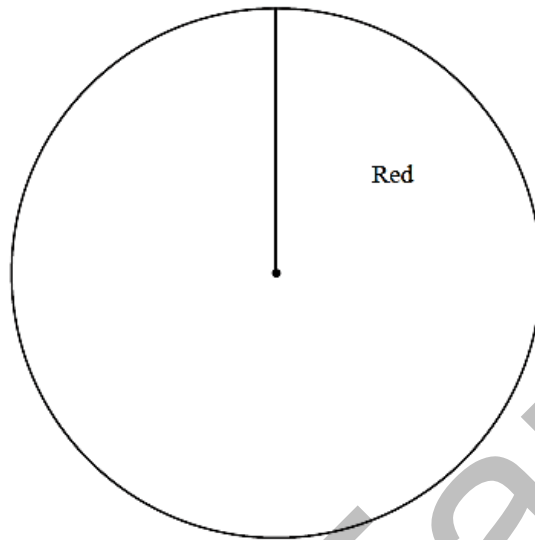
[2]

   (ii) Ahmed takes a counter at random from the bag.
         Find the probability that the counter is red.   *Answer* .... [1]

   (iii) Simeon takes two counters at random from the bag of twelve counters.
         He places them next to each other on a table.

         Find the probability that the two counters are different colours.

                                                          Answer .... [3]

---

11 **(b)** A bag contains R red beads and B blue beads.
Two beads are chosen, at random, without replacement.
The tree diagram shows the possible outcomes and their probabilities.

First bead   Second bead

$$\frac{11}{19} \quad \text{red}$$

$$\frac{3}{5} \quad \text{red}$$

$$\frac{8}{19} \quad \text{blue}$$

$$\frac{12}{19} \quad \text{red}$$

$$\frac{2}{5} \quad \text{blue}$$

$$\frac{7}{19} \quad \text{blue}$$
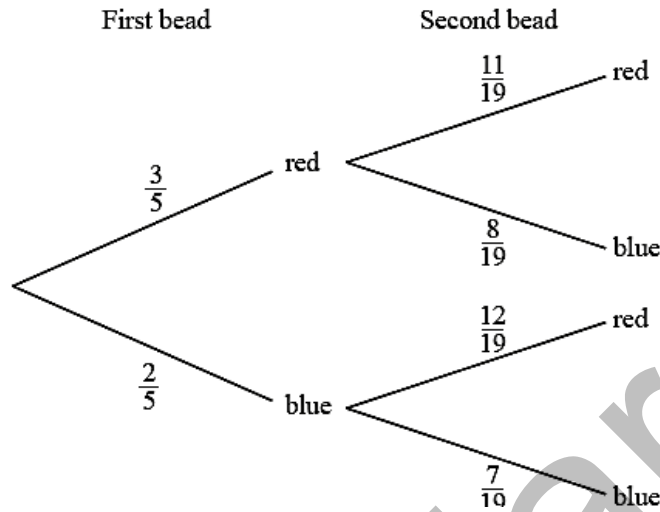
(i)   Calculate the probability that both beads are red.

Answer .... [1]

(ii) Calculate the probability that the two beads are different colours.

Answer .... [2]

(iii) What is the value of R?

Answer .... [1]

(iv) Of the red beads, half have a yellow spot.

Calculate the probability that, of the two chosen beads, **neither** has a yellow spot.

Answer .... [2]

Answers: (a)(i) correct histogram (ii) 39.4 (b)(i) $\frac{33}{95}$ (ii) $\frac{48}{95}$ (iii) 12 (iv) $\frac{91}{190}$ **N16/21/Q10**

12   Adam has a bag containing 9 balls, numbered from 1 to 9.

(a)   Adam takes a ball at random from the bag and replaces it.

Find the probability that the ball has an odd number.   *Answer* .... [1]

(b)   Adam takes a ball from the 9 balls in the bag, notes the number and replaces it.
He then takes a second ball from the bag, notes the number and replaces it.

(i)   Work out the probability that both numbers are odd.   *Answer* .... [1]

(ii)   Work out the probability that one number is odd and the other is even.

Answer .... [2]

(c)   Adam now takes two balls from the 9 balls in the bag, **without replacement**.

Work out the probability that the two numbers are either both odd or both even.

Answer .... [3]

N17/21/Q4

Statistics Paper 1

1 Mr. Smith asked the children in his class 'What is your favourite colour?'
Their replies are given below.

Green	Blue	Green	Yellow	Blue
Green	Red	Blue	Green	Blue
Yellow	Green	Yellow	Blue	Yellow
Blue	Blue	Green	Blue	Yellow
Green	Blue	Green	Yellow	Blue

(a) By making tally marks, or otherwise, obtain the frequency distribution of the colours.

Answer (a)

Colour	Frequency
Green	
Blue	
Red	
Yellow	

[1]

(b) State the mode of this distribution.   *Answer* (b) .... [1]

2 The cumulative frequency curve shows the distribution of the times of 300 competitors in a women's marathon race.
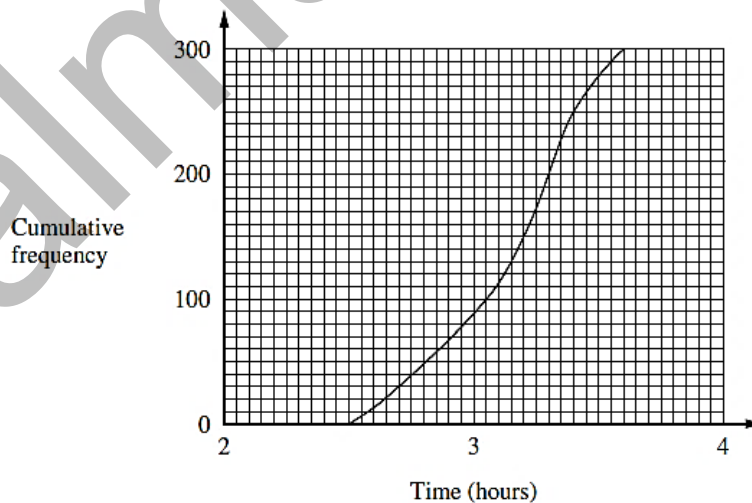
Use the curve to answer the following questions.

(a) The race was won by Tegla.   *Answer* (a) .... h .... min [1]
   Find her time, giving your answer in hours **and** minutes.

(b) Find the median time in hours **and** minutes.   *(b)* .... h .... min [1]

(c) The qualifying time for the Olympic Games was achieved by ten percent of the runners.
   The race began at 11.30.
   At what time did the last qualifying athlete finish the race?
   Express your answer using the 24 hour clock.   *(c)* .... [2]

3   The numbers of goals scored in 20 football matches were

5 0 5 4 1 0 5 5 1 3
4 5 0 0 5 5 3 2 5 4

(a) (i)   Complete the table in the answer space.

   (ii)   Using the axes in the answer space, represent the information as a bar chart.

(b) State the median.   *Answer* (b) .... [1]

(c) Calculate the mean number of goals.   *(c)*... [2]

Answer (a)(i)

Number of goals		Frequency
0		
1		
2		
3		
4		
5		

Answer (a)(ii)   [1]

682

Answer (a)(ii)   [1]

Frequency
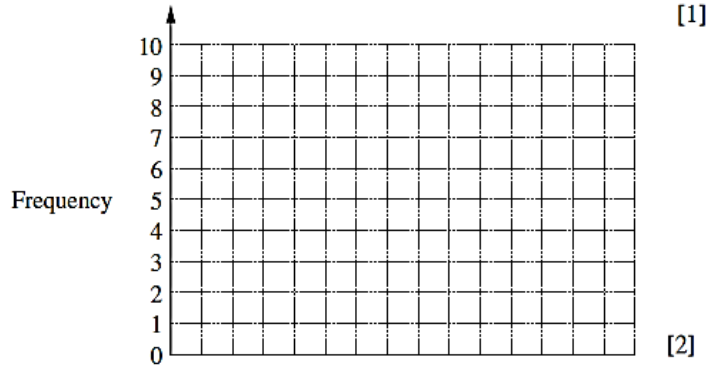
Answers: **(a)** 4, 2, 1, 2, 3, 8; **(b)** 4; **(c)** 3.1.   J03/1/Q25

4   Some children were asked how many television programmes they had watched on the previous day. The table shows the results.

Number of programmes watched	0	1	2	3
Number of children	7	3	1	y

(a)   If the median is 2, find the value of y.   *Answer (a)* .... [1]

(b)   If the median is 1, find the greatest possible value of y.   *(b)* .... [1]

Answers: **(a)** 10; **(b)** 8.   J04/1/Q9

5   One hundred children were asked how far they could swim.
The results are summarised in the table.

Distance (d metres)	0 < d ≤ 100	100 < d ≤ 200	200 < d ≤ 400
Number of children	30	50	20

(a)   The histogram in the answer space represents part of this information.
Complete the histogram.

683

Frequency density

Distance (*d* metres) [1]
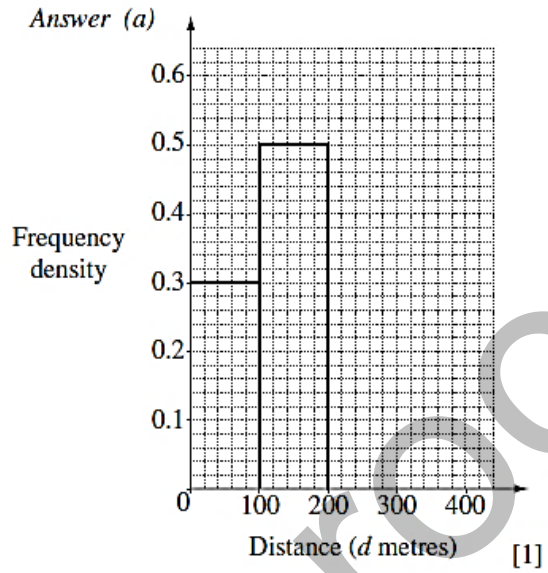
(b) A pie chart is drawn to represent the three groups of children.
Calculate the angle of the sector that represents the group of 20 children.

Answer (b) .... [1]

Answers: **(a)** Rectangle of width 200 and height 0.1; **(b)** 72°. **J05/1/Q11**

---

6 The cumulative frequency curve shows the distribution of the masses of 100 people.

Cumulative frequency

Mass (*m* kg)
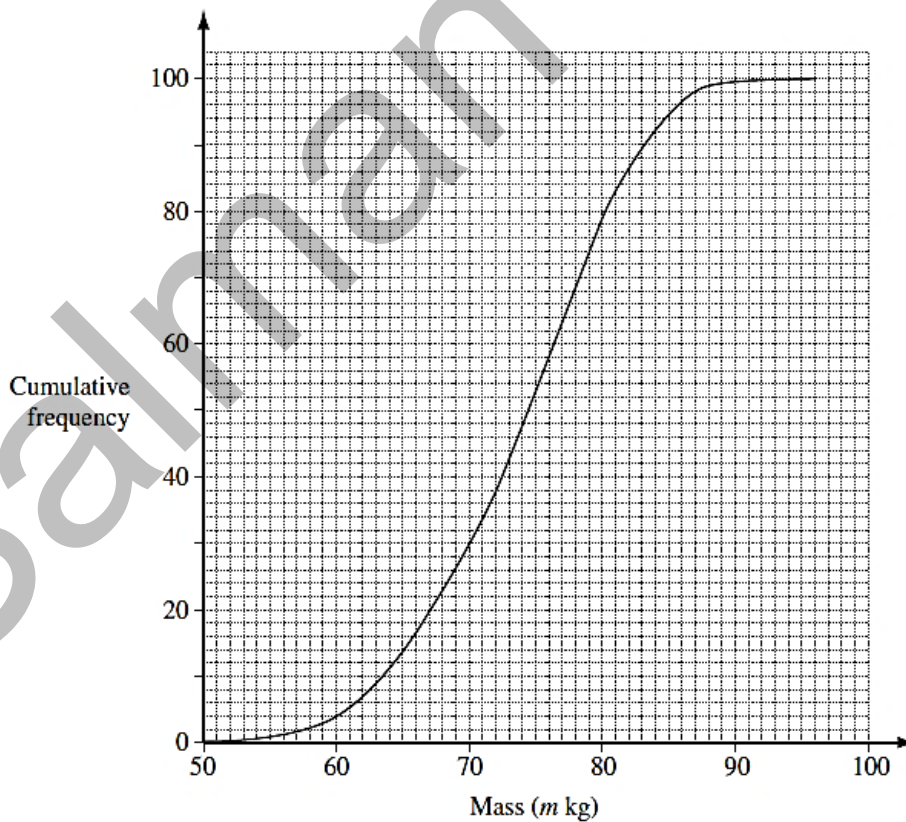
Find

(a) the median,

(b) the upper quartile,

(c) the number of people with masses in the range $65 < m \leqslant 72$.

Answer (a) ....kg [1]

(b) ....kg [1]

(c) .... [1]

Answers: **(a)** 74.4 to 74.7 kg; **(b)** 79.1 to 79.4 kg; **(c)** 23 to 25.

7

The diagram shows a gauge for measuring the water level in a reservoir.
Readings, in metres, taken over a certain period were as follows:

$-2.3, \ -1.6, \ -0.4, \ 0.1, \ -0.5, \ 0.3, \ -1.2.$

For these readings

(a) find the difference, in metres, between the highest and lowest levels,

(b) find the median,   Answer (a) .... m [1]

(c) calculate the mean.   (b) .... m [1]

(c) .... m [2]
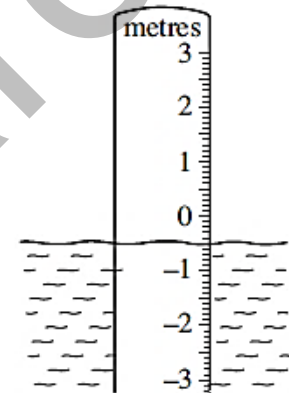
Answer: **(a)** 2.6 m   **(b)** −0.5 m   **(c)** −0.8 m

---

8   On a certain stretch of road, the speeds of some cars were recorded.
The results are summarised in the table.
Part of the corresponding histogram is shown alongside.

Speed (x km/h)	Frequency
$25 < x \le 45$	q
$45 < x \le 55$	30
$55 < x \le 65$	p
$65 < x \le 95$	12

(a) Find the value of

   (i) p,
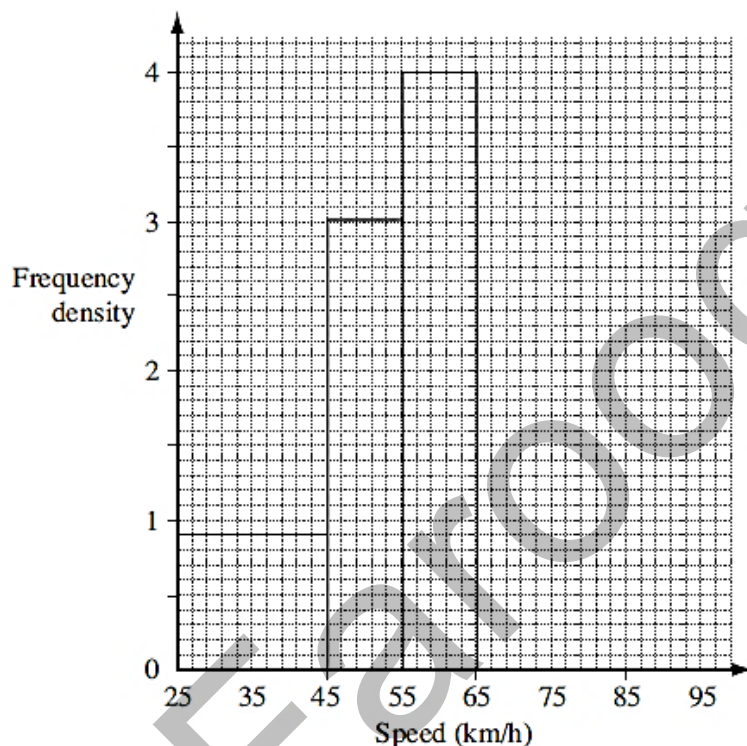
   (ii) q.

(b) Complete the histogram.

Answer (a)(i) $p =$ ....[1]

(ii) $q =$ ....[1]

[1]

Answer: **(a)(i)** 40   **(ii)** 18   **(b)** Rectangle of width 30, height 0.4   **J07/1/Q11**

---

9   The heights of 40 children were measured.
The results are summarised in the table below.

Height ($h$cm)	$105 < h \le 115$	$115 < h \le 125$	$125 < h \le 135$	$135 < h \le 145$
Frequency	5	10	20	5

(a) (i) Identify the modal class.

   (ii) Calculate an estimate of the mean height.

Answer (a)(i) ....[1]

(ii) ....cm [3]

(b) The cumulative frequency curve representing this information is shown below.
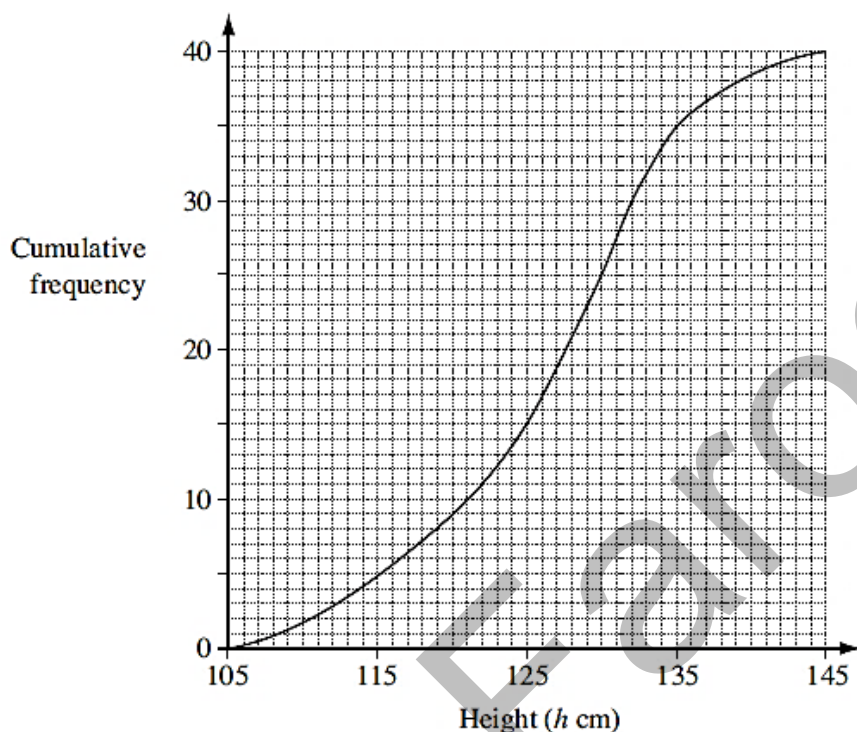
Use the curve to find

(i) the interquartile range,

(ii) the number of children whose heights are in the range 120 cm to 130 cm.

Answer (b)(i) ....cm [2]

(ii) ....[1]

Answer. **(a)(i)** $125 < h \leq 135$ **(ii)** 126.25 **(b)(i)** 11 **(ii)** 16

J07/1/Q25

---

10 **(a)** Fifty students were asked how many books they each took to school on Monday. The results are summarised in the table below.

Number of books	0	1	2	3	4	5	6	7
Frequency	10	11	8	3	6	7	4	1

(i) Write down the median.

(ii) Calculate the mean number of books.

687

(iii) What is the probability that two students, chosen at random, both took 5 books to school?

Give your answer as a fraction in its simplest form.

Answer *(a)*(i) ....[1]

(ii) ....[3]

(iii) ....[2]

(b) The fifty students were also asked how long they each took to travel to school. The results are summarised in the table below.

Time of travel (*t* minutes)	$4 \leqslant t < 6$	$6 \leqslant t < 8$	$8 \leqslant t < 10$	$10 \leqslant t < 12$
Frequency	21	11	13	5

Draw a frequency polygon on the grid below to illustrate this data.

Answer (b)

Time (*t* minutes)
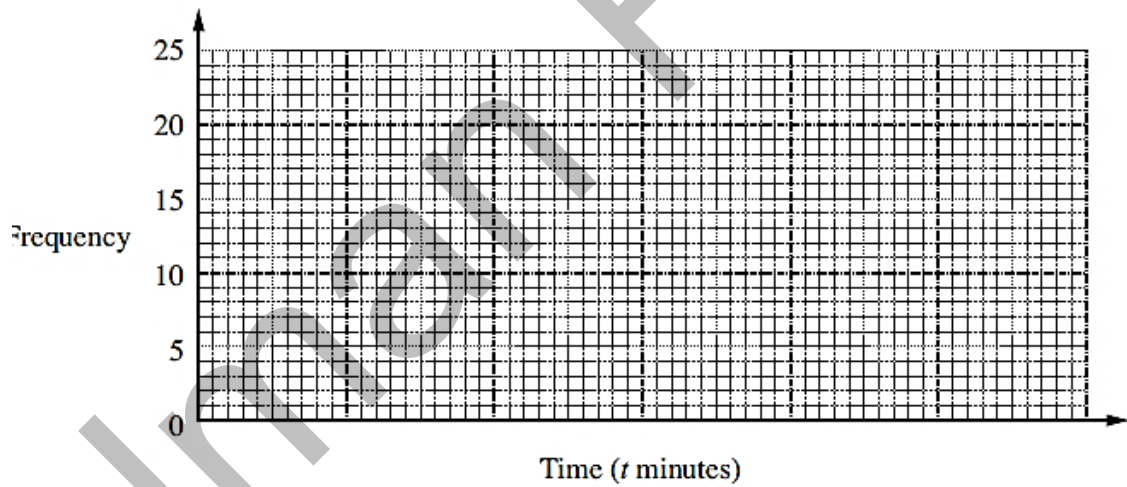
[2]

Answers: **(a)(i)** 2 (ii) 2.52 (iii) $\dfrac{3}{175}$ **J08/1/Q24**

688

11
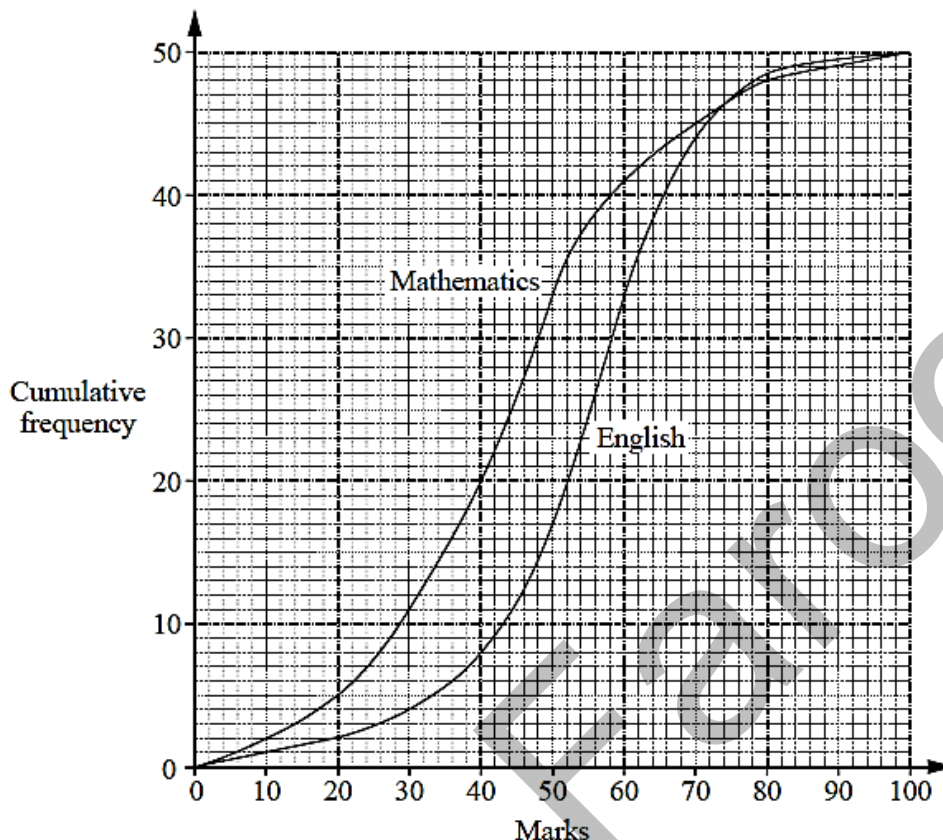
Fifty students each took a Mathematics and an English test. The distributions of their marks are shown in the cumulative frequency graph.

(a) Use the graph

    (i) to estimate the median mark in the English test,

               Answer *(a)*(i) ....[1]

    (ii) to estimate the 20th percentile mark in the Mathematics test.

               Answer *(a)*(ii) ....[1]

(b) State, **with a reason**, which test the students found more difficult.   . [1]

Answer: **(a)(i)** 54 to 56   **(ii)** 28 to 30   **(b)** Mathematics because the median for Mathematics is   **J09/1/Q9**
lower than the median for English.

12   Sara carries out a survey of the colours of cars in a car park.
She draws a pie chart to represent her results.

(a) There are 7 red cars.
The angle representing the red cars is 40°.

Calculate the total number of cars in the car park.   *Answer* *(a)* ....[1]

(b) Sara's pie chart is a circle with circumference 28 cm.

Find, in terms of π, the diameter of the circle.　　*Answer* (b) .... cm [1]

Answer: (a) 63　　(b) 28 / π　　　　　　　　　　　　　　　　　　　**J10/11/Q3**

13　Ida keeps a record of time spent on the internet each day.
Her results are summarised in the table.

Time (t minutes)	Frequency
$0 \leq t < 10$	4
$10 \leq t < 30$	20
$30 \leq t < 60$	39
$60 \leq t < 100$	32
$100 \leq t < 120$	6

On the axes below, draw a histogram to show these results.
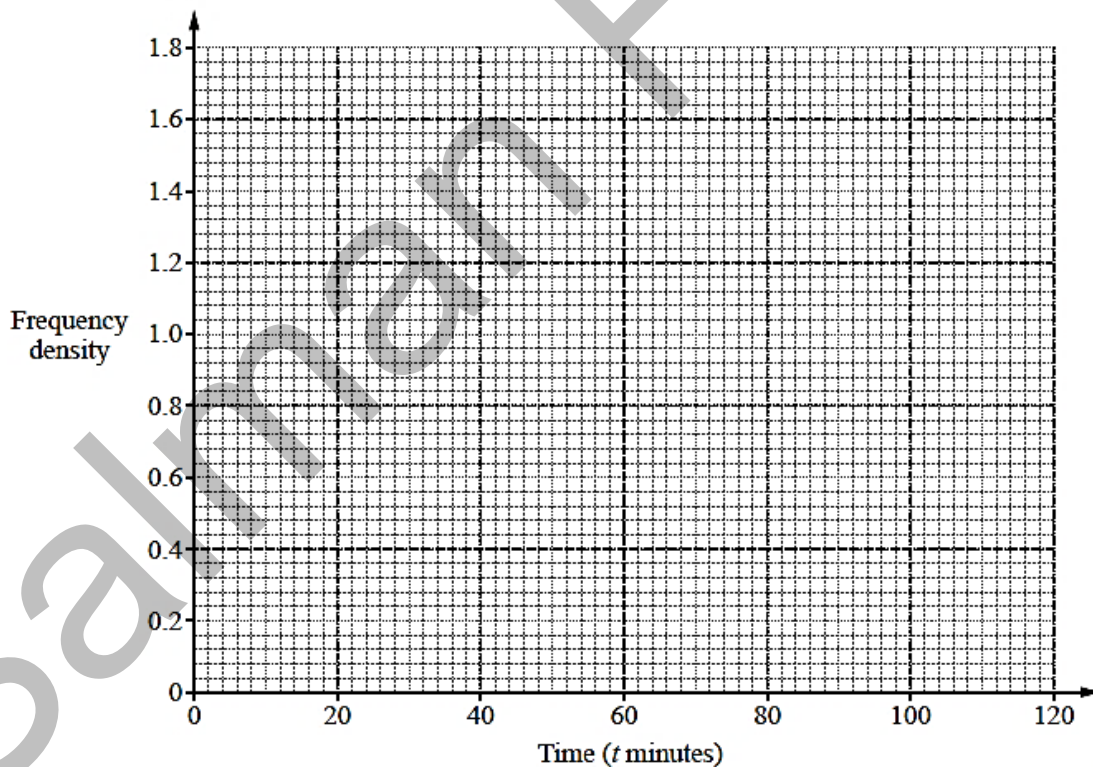
[3]

Answer: correct histogram　　　　　　　　　　　　　　　　　　　**J10/11/Q14**

14 Dai played three games of cricket.
His mean score was 9 runs.
His median score was 8 runs.
His highest score was 7 runs more than his lowest score.

(a) Find the number of runs he scored in each of the three games.

Answer (a)........... , ... , ... [3]

(b) Dai batted in a fourth game.
The mean of his four scores was 11 runs.

Find the number of runs that Dai scored in the fourth game.

Answer (b) .... [1]

Answers: (a) 6, 8, 13   (b)   17   **J10/11/Q16**

15   The graph shows the cumulative frequency curve for the ages of 60 employees.
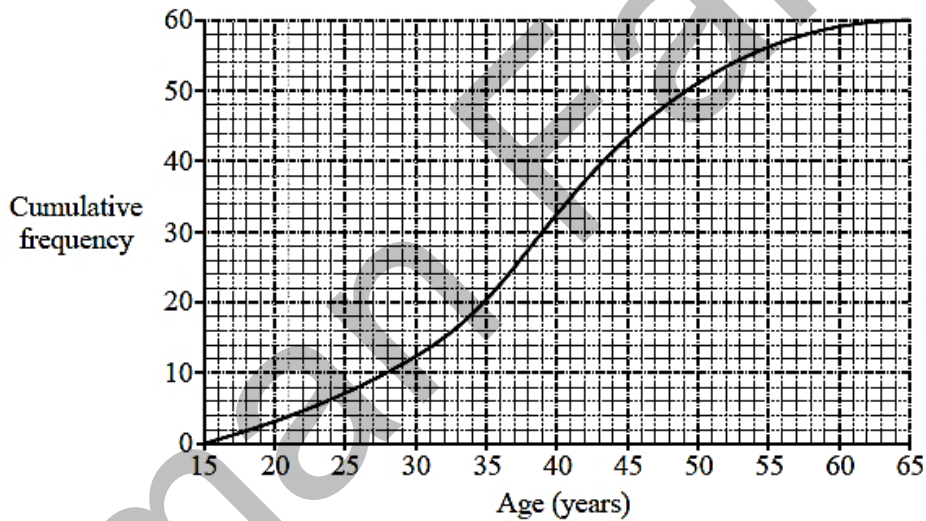
Use the graph to estimate

(a) the median,   Answer (a) .... [1]

(b) the interquartile range,   Answer (b) .... [2]

(c) the number of employees aged over 50.   Answer (c) .... [1]

Answer: (a)   39   (b)   14   (c)   9   **J10/12/Q20**

691

16 Jamil recorded the number of text messages sent by the students in his class on one day.
The results are shown in the bar chart.
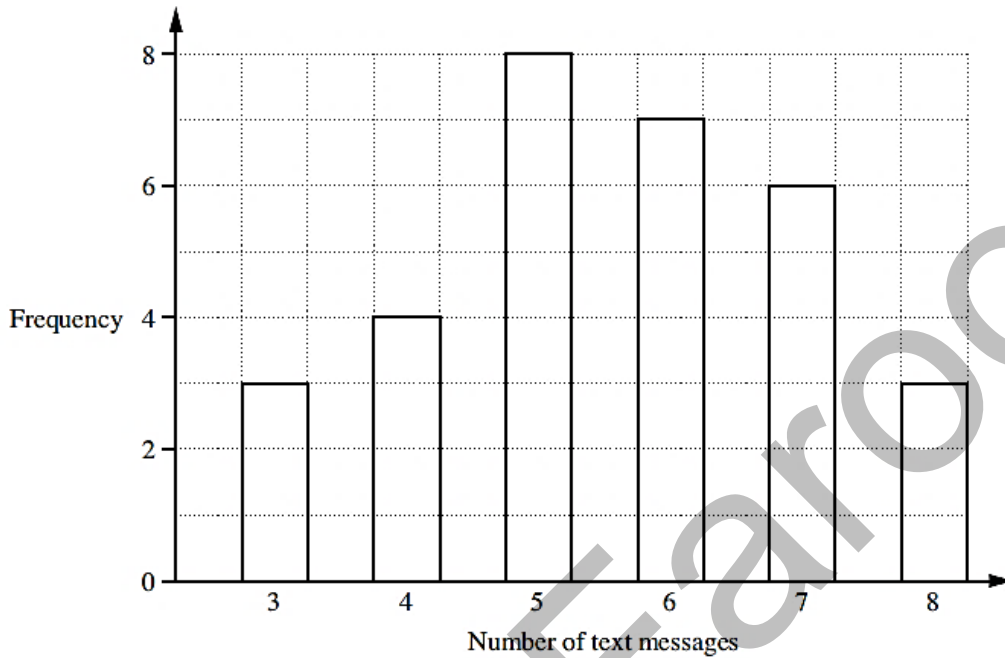
Use the bar chart to find

(a) the number of students in Jamil's class,

Answer .... [1]

(b) the median number of text messages sent,

Answer .... [1]

(c) the modal number of text messages sent.

Answer .... [1]

Answers: (a) 31 (b) 6 (c) 5

17 The table shows the distribution of the number of complete lengths swum by a group of swimmers.

Number of complete lengths (n)	$0 < n \leqslant 20$	$20 < n \leqslant 40$	$40 < n \leqslant 60$	$60 < n \leqslant 80$
Frequency	5	20	10	5

(a) Find the modal class.

Answer .... [1]

(b) Calculate an estimate of the mean.

Answer .... [3]

Answer: (a) $20 < n \leqslant 40$ (b) 37.5

18

A group of 100 students was asked how many minutes each spent talking on their mobile phone during one day.
The histogram summarises this information.
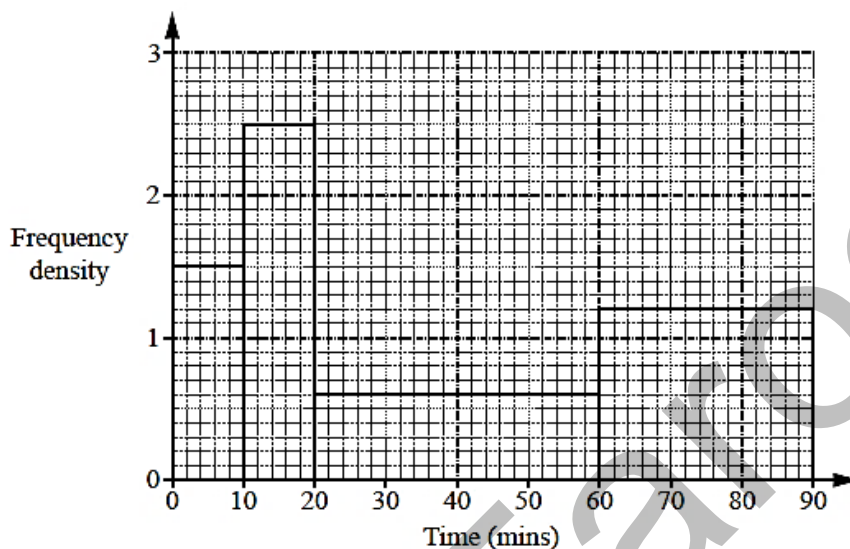
(a) Use the histogram to

    (i) find the number of students who spent between 0 and 10 minutes talking on their mobile phone,

                                   Answer ....[1]

    (ii) estimate the number of students who spent between 25 and 65 minutes talking on their

  mobile phone.   *Answer* ....[2]

(b) A pie chart is drawn to represent the information shown in the histogram.

  Calculate the angle of the sector that represents the students who spent between 0 and 10 minutes talking on their mobile phone.

                                     Answer ....[1]

Answer: **(a) (i)** 15 **(ii)** 27 **(b)** 54

J12/11/Q21

693

19 Eighty cyclists were each asked the distance (in kilometres) they cycled last week.

Distance (kilometres)
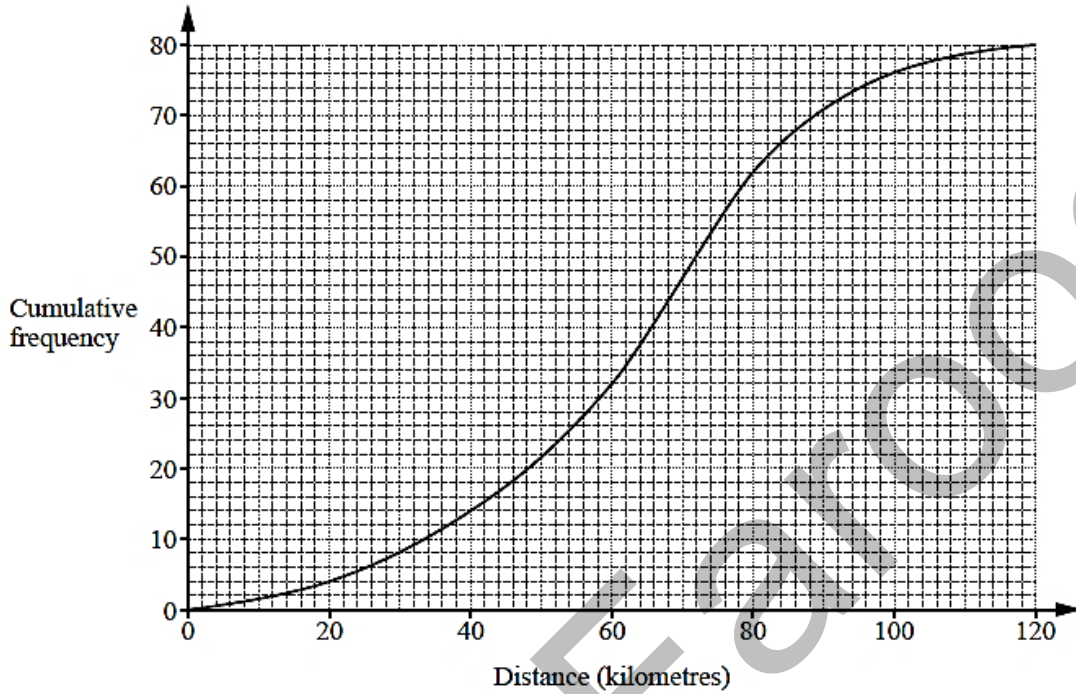
The cumulative frequency diagram represents the results.

Use the graph to estimate

(a) the number of cyclists who cycled between 60 and 80 kilometres,

Answer .... [1]

(b) the median distance cycled,

Answer .... km [1]

(c) the interquartile range for the distance cycled.

Answer .... km [2]

Answers: (a) 30 (b) 66 (c) 30 — J13/11/Q18

20 The table shows the ages of guests at a party.

Age (y years)	$10 \leqslant y < 20$	$20 \leqslant y < 40$	$40 \leqslant y < 45$	$45 \leqslant y < 50$	$50 \leqslant y < 65$
Frequency	p	20	8	q	18

The histogram represents some of this information.

Age (y years)

(a) Use the histogram to find the value of

(i) p,

Answer p = ... [1]

(ii) q.
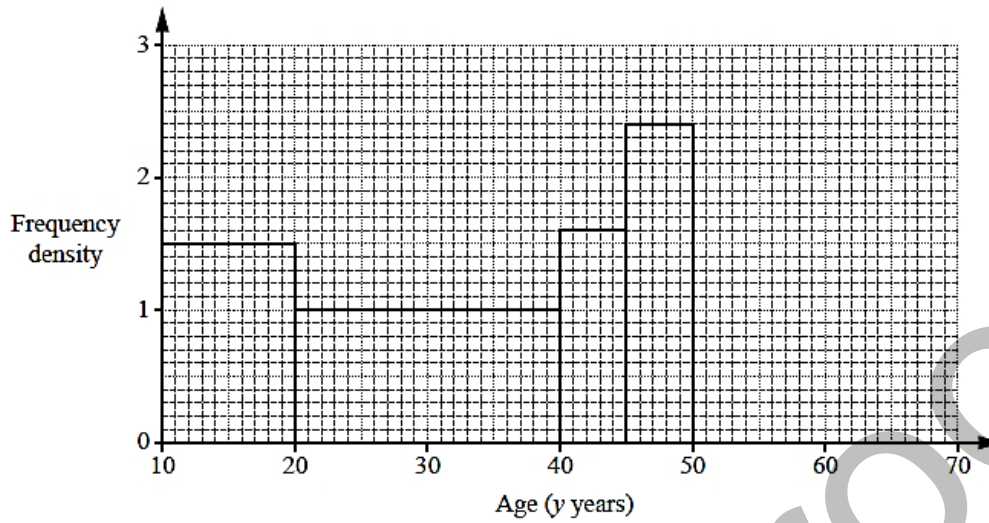
Answer q = ... [1]

(b) Complete the histogram. [1]

Answers: (a)(i) 15 (ii) 12 (b) column from 50 to 65 with frequency density 1.2.   **J14/11/Q14**

21 The times taken by each member of a group of people to run one kilometre were recorded.
The results are shown in the table.

Time (t minutes)	$2 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 6$	$6 < t \leqslant 7$	$7 < t \leqslant 8$
Frequency	0	4	5	3	1	0

On the grid below, draw a frequency polygon to represent these results.

Time (t minutes)
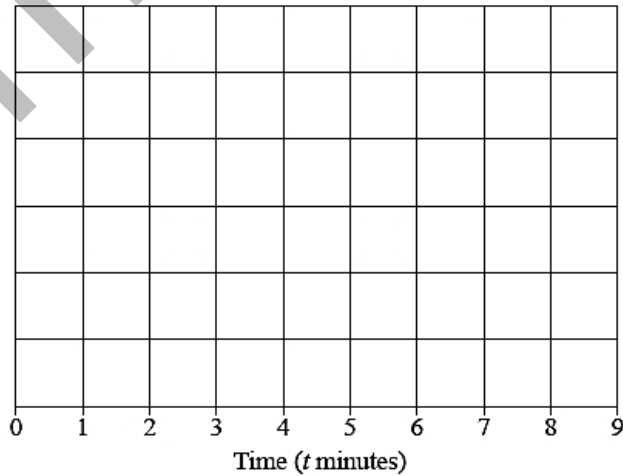
[2]

22
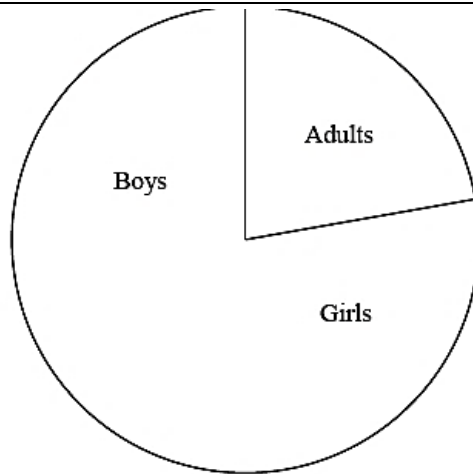
A pie chart is used to illustrate the numbers of adults, girls and boys in a group of people.
The angles for the adults and girls are 80° and 120° respectively.
The diagram shows part of the pie chart.

(a) Complete the pie chart. [1]

(b) Express the ratio of numbers of   adults : girls : boys   in the form $a : b : c$, where
   a, b and c are the smallest possible whole numbers.

   Answer .... : .... : .... [1]

(c) There are 6 more girls than adults.

   Calculate the number of people in the whole group.   *Answer* .... [1]

Answers: **(b)** 2 : 3 : 4 **(c)** 54

23

Stella walks to a park.   *Answer* .... [2]

For 4 minutes she walks at a rate of 80 steps per minute.
For 1 minute she walks at a rate of 120 steps per minute.

Find the mean number of steps per minute she takes.

Answer: 88

24

Henri did a survey of the lengths of the leaves on a plant.
The results are summarised in the table.

Length (x cm)	$1 < x \leqslant 3$	$3 < x \leqslant 4$	$4 < x \leqslant 5$	$5 < x \leqslant 8$
Frequency	6	8	10	6

(a) When asked to draw a histogram to illustrate the results, Henri drew the following diagram.
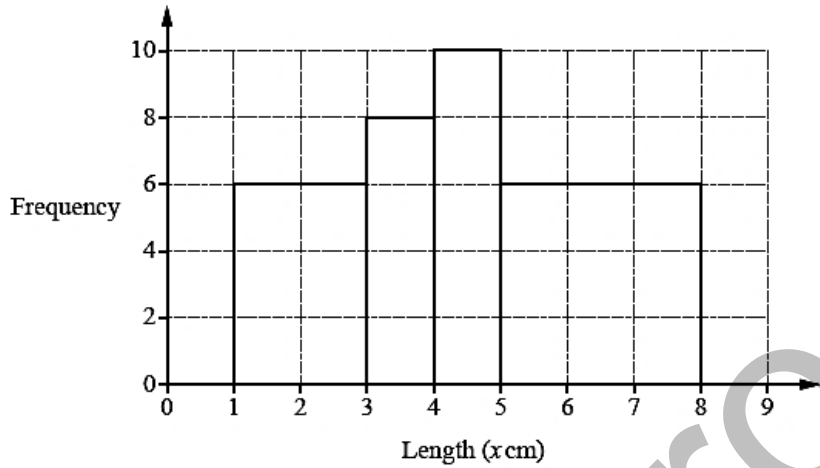
Explain why this diagram is incorrect.

....

.... [1]

(b) On the grid below, draw a correct histogram for Henri's results.
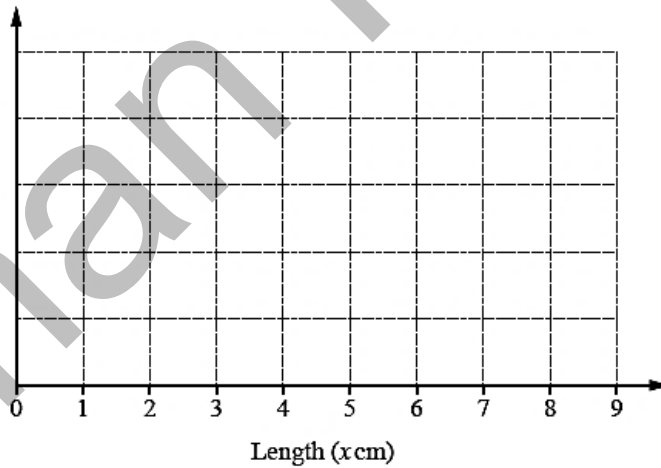
Length (*x* cm)

[3]

J16/11/Q18

25

The number of goals scored in each of 50 football matches was recorded.
The results are given in the table.

Number of goals scored	0	1	2	3	4	5	6
Frequency	16	11	9	7	6	0	1

For these results, find

(a) the mode,

Answer .... [1]

(b) the median,

Answer .... [1]

(c) the mean.

Answer .... [2]

Answers: **(a)** 0 **(b)** 1 **(c)** 1.6

J16/11/Q20

26

The ages of guests at a family party were recorded.
The results are summarised in the table.

Age (b years)	$5 < b \leqslant 10$	$10 < b \leqslant 20$	$20 < b \leqslant 30$	$30 < b \leqslant 50$
Frequency	p	18	14	q

The histogram below shows some of these results.
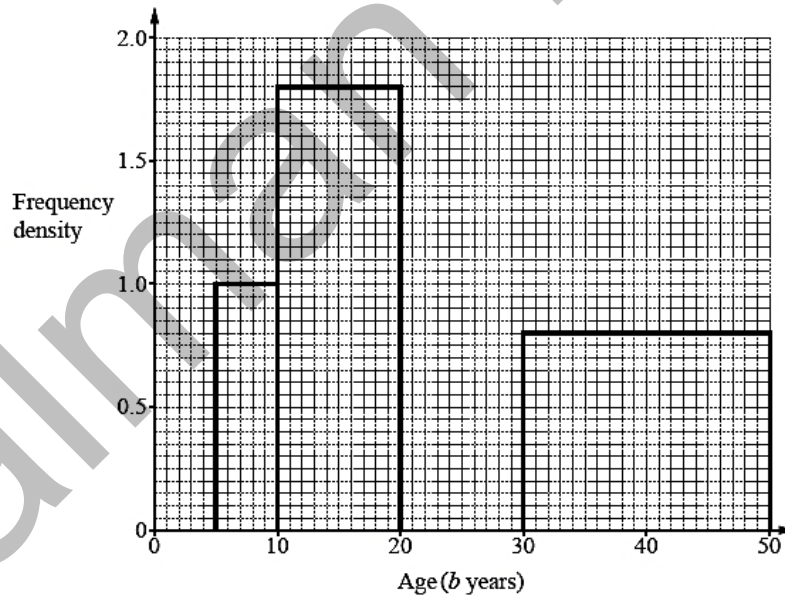
(a) Use the histogram to find the value of

(i) p,

Answer $p = $ .... [1]

(ii) q.

Answer $q = $ .... [1]

(b) Complete the histogram. [1]

698

27
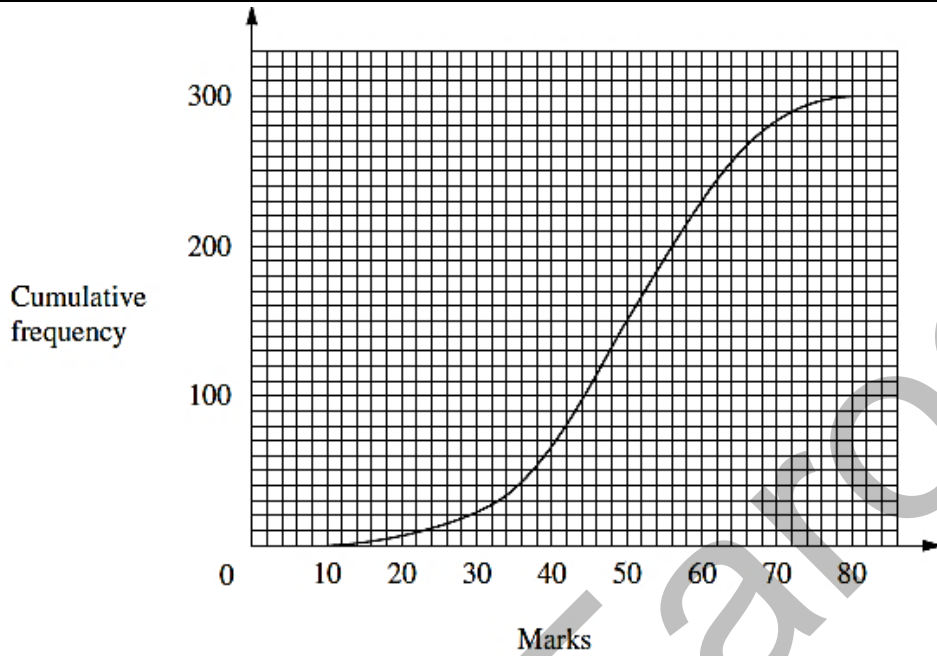
The graph is a cumulative frequency curve showing the marks gained by 300 candidates in an examination.

(a) Use the curve to estimate

    (i) the median mark,

    (ii) the number of candidates who gained 33 marks or less.

(b) It is given that 70 candidates achieved a grade A.
Use the curve to estimate the smallest mark required for a grade A.

Answer (*a*) (i) .... [1]

    (ii) .... [1]

    (*b*) .... [1]

28 (a) An article in a newspaper reported that the number of crimes had been reduced by half from 1991 to 2001. The article contained the bar chart shown here.
Explain why this bar chart might be considered misleading.

Number of crimes committed

(b)

Frequency density
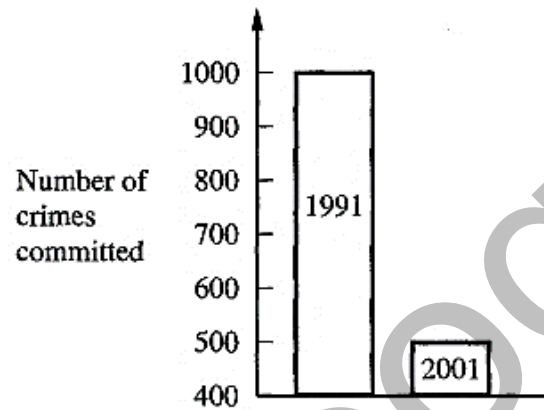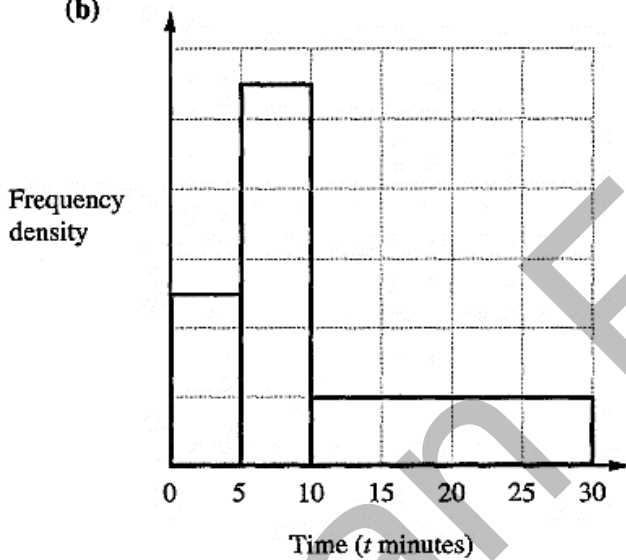
Time (*t* minutes)

The histogram alongside shows the distribution of times taken by a group of students to travel to school.
11 students took at least 5 but less than 10 minutes.
Complete the table in the answer space.

Answer (b)

Time (*t* minutes)	Number of students
$0 \leqslant t < 5$	
$5 \leqslant t < 10$	11
$10 \leqslant t < 30$	

[2]

Answers: (a) Since the axis starts at 400, the bars are not proportional to the crimes; (b) 5, 8.   N02/1/Q8

29 The table below shows the number of children living in the houses on a road.

Number of Children	0	1	2	3	4	5	6
Number of Houses	3	7	5	4	0	0	1

Find

(a) the modal number of children,

(b) the median number of children,

(c) the mean number of children.

Answers: (a) 1; (b) 1.5; (c) 1.75.   N02/1/Q15

700

30

Cumulative frequency

Marks
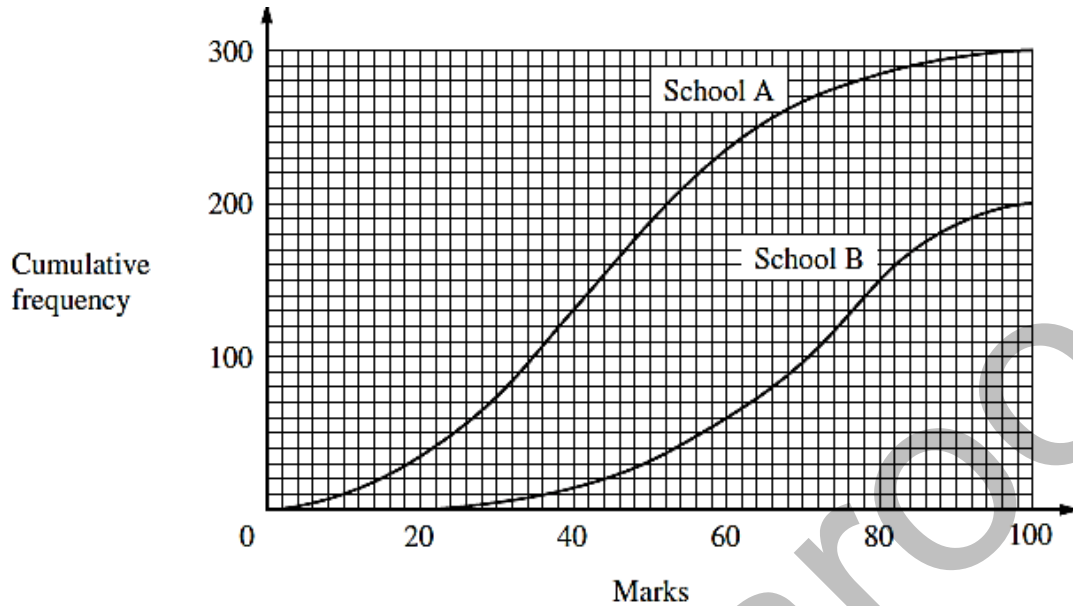
All the students from two schools, A and B, take the same examination paper.
The cumulative frequency curves show the results for the two schools.

(a) Estimate the median mark of the students from school A.

(b) Estimate the percentage of the students from school B who gained more than 80 marks.

(c) State, with a reason, which school achieved the better results.

Answers: **(a)** 43 to 44; **(b)** 25%; **(c)** medians or other proportions compared.

---

31 The temperatures, at noon, on five days were

$$-2\,°C, \quad -1\,°C, \quad 1\,°C, \quad -2\,°C, \quad 5\,°C.$$

(a) Find the median temperature.

(b) Calculate the mean temperature.

(c) The temperature, at noon, on another day was $x\,°C$.
The mean temperature for the six days was $1.5\,°C$.
Find the value of x.

Answers: **(a)** −1; **(b)** 0:2; **(c)** 8.

32   The lengths of 40 nails were measured.
Their lengths, in centimetres, are summarised in the table below.

Length (l cm)	Frequency
$0 < l \leqslant 4$	14
$4 < l \leqslant 8$	18
$8 < l \leqslant 16$	8

(a)   On the axes in the answer space, draw the histogram which represents this information.

Answer (a)

Frequency density

Length (*l*cm)

[2]
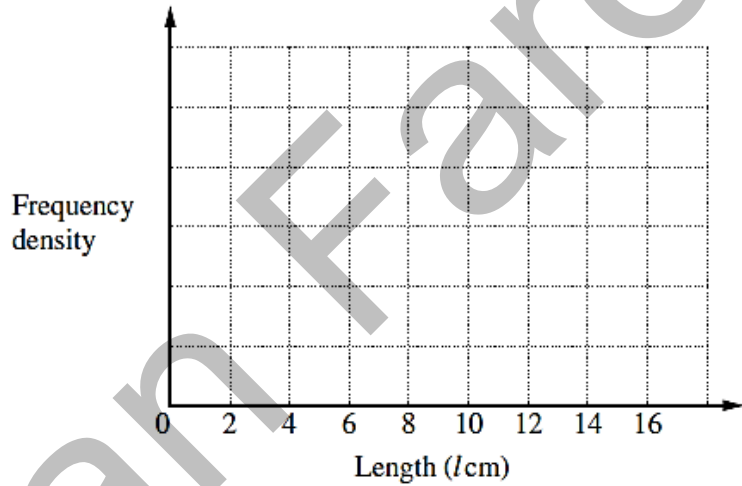
(b)   Calculate an estimate of the mean length of the nails.

Answer (b) ....cm [2]

Answer: **(b)** 5.8.

33
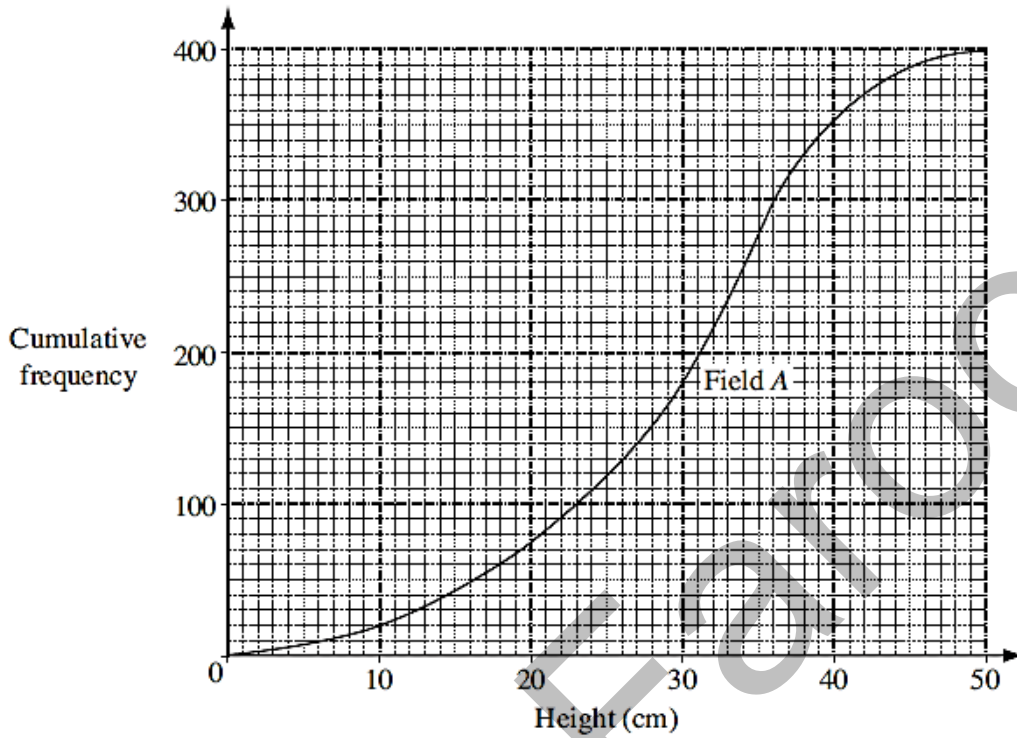
The diagram above is the cumulative frequency curve for the heights of 400 plants which were grown in Field *A*.

Use the graph to find

(a) the number of plants that grew to a height of **more** than 30 cm,

(b) the interquartile range.

Answer (a) .... [1]

(b) .... [1]

(c) Another 400 plants were grown in Field *B*.
The cumulative frequency distribution of the heights of these plants is shown in the table.

Height (*h* cm)	$h \leqslant 10$	$h \leqslant 15$	$h \leqslant 20$	$h \leqslant 25$	$h \leqslant 30$	$h \leqslant 35$	$h \leqslant 40$	$h \leqslant 50$
Cumulative frequency	35	75	130	200	280	330	370	400

On the same axes as for Field *A*, draw the cumulative frequency curve for the plants grown in Field *B*. [2]

(d) By comparing the two curves, state, with a reason, which Field produced the taller plants.

Answer (d) Field .... produced the taller plants because ....

.... [1]

Answers: (a) 220; (b) 13; (d) *A* with a reason.

34

The temperature at the bottom of a mountain was 8°C.
The temperature at the top was –26°C.
Find

(a) the difference between the two temperatures,

Answer (a) ....°C [1]

(b) the mean of the two temperatures.

(b) ....°C [1]

Answer: (a) 34° C; (b) –9° C.

35

In an experiment, the heights of some plants were measured.
The table below summarises the results.

Height (*h* cm)	$2 < h \leqslant 3$	$3 < h \leqslant 4$	$4 < h \leqslant 5$	$5 < h \leqslant 8$
Frequency	15	25	20	15

Complete the histogram which represents this information.

Answer

Frequency density (vertical axis, values 0, 5, 10, 15, 20, 25)

Height (h cm) (horizontal axis, values 1 to 9)
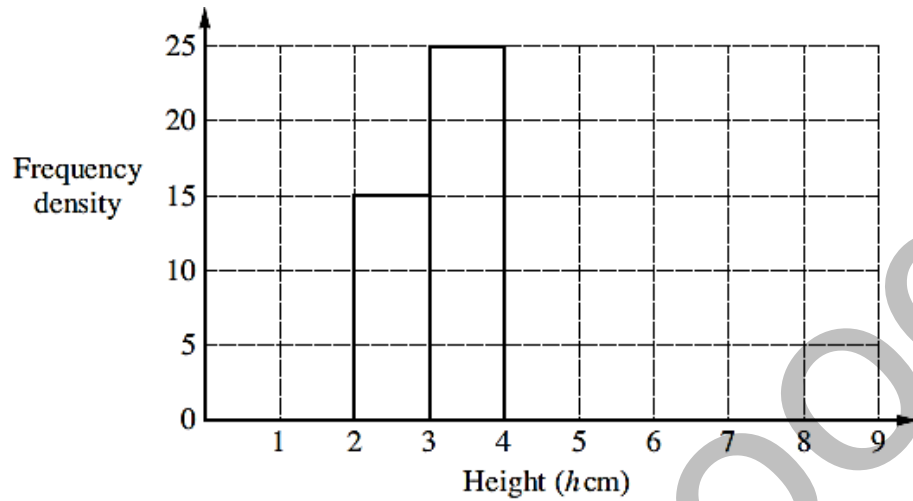
[2]

Answer :   rectangle from 4 to 5, height 20 ; rectangle from 5 to 8, height 5   **N07/1/Q7**

---

36   The number of items bought by 10 customers at a local store is shown below.

   6   7   5   9   10   7   18   10   7   9

(a)   State the mode of this distribution.   *Answer (a)* ....[1]

(b)   Find the median number of items bought.   (b) ....[1]

Answers: **(a)** 7,   **(b)** 8.   **N08/1/Q5**

37

(a) The graph shows the cumulative frequency curve for the playing times of the individual tracks on Andrew's MP3 player.

Use the graph to find

(i) the median,

(ii) the interquartile range.

Playing time (minutes)
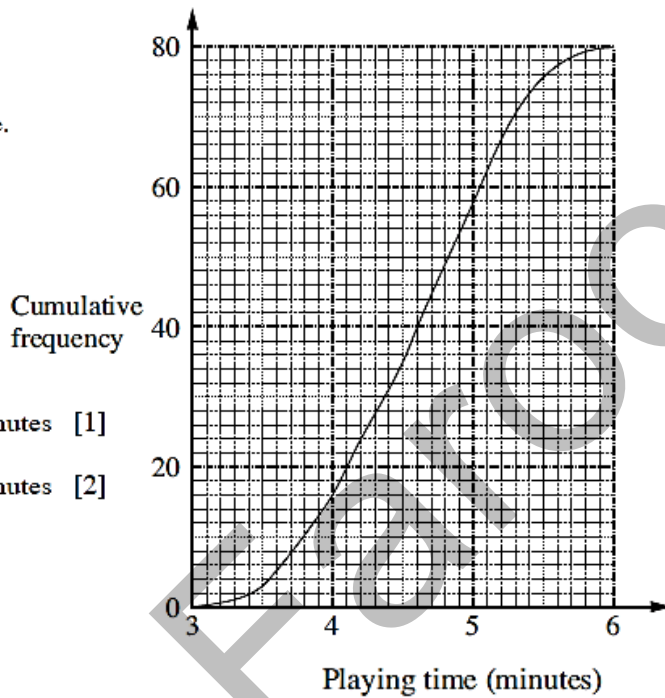
Answer (a)(i) .... minutes [1]

(ii) .... minutes [2]

(b) The table summarises the playing times of each of the 100 tracks on Tom's MP3 player.

Playing time (t minutes)	Frequency
$2.5 < t \leq 3.5$	5
$3.5 < t \leq 4.5$	30
$4.5 < t \leq 5.5$	50
$5.5 < t \leq 6.5$	15

Answer (b) .... minutes [3]

Calculate an estimate of the mean playing time of the individual tracks.

Answers: **(a)(i)** 4.55 to 4.65 minutes, **(ii)** 0.9 to 1 minutes, **(b)** 4.75 minutes. **N08/1/Q23**

706

38 The colours of the cars which passed a house were noted.
The results are shown in the pie chart below.
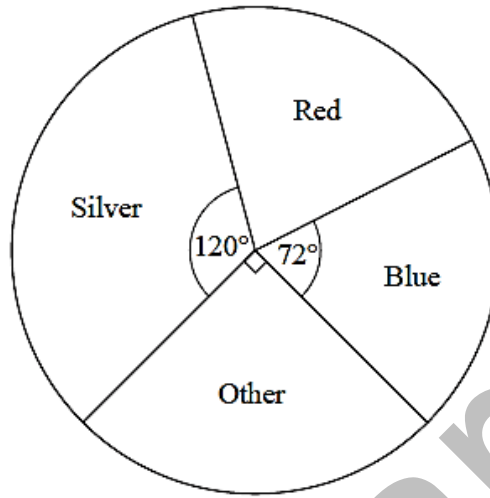
There were 12 blue cars.

How many cars

(a) passed the house,

Answer (a) ....[1]

(b) were red?

Answer (b) ....[2]

Answers: (a) 60   (b) 13   **N09/1/Q8**

39 The table below shows the number of pets owned by 20 families.

Number of pets	0	1	2	3	4	5	6	7
Number of families	2	5	3	2	4	1	1	2

Find

(a) the modal number of pets,

Answer (a) ....[1]

(b) the mean number of pets.

Answer (b) ....[2]

Answers: (a) 1   (b) 2.9   **N09/1/Q11**

40 The table below shows the distribution of the length, in metres, of cars in a car park.

Length (x metres)	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 2.75$	$2.75 \leqslant x < 3$	$3 \leqslant x < 3.5$	$3.5 \leqslant x < 4.5$
Number of cars	3	5	p	8	4

(a) Use the histogram in the answer space to find p.   *Answer (a)* $p =$ ....[1]

(b) Complete the histogram.

Length (x metres)

[2]
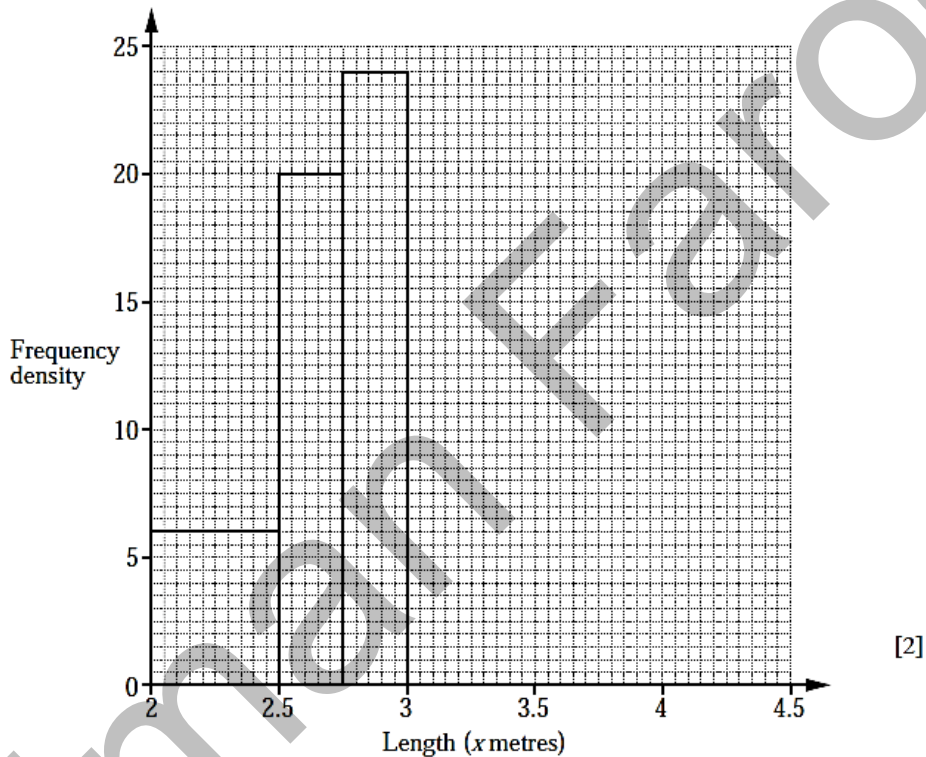
Answers: **(a)** 6   **(b)** rectangle, base 3 to 3.5, height 16; rectangle, base 3.5 to 4.5, height 4   **N09/1/Q17**

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41 The temperatures, in °C, at midnight on 12 consecutive days were

$$-1, \quad 0, \quad -4, \quad 1, \quad 2, \quad -2 \quad -1, \quad -3, \quad 1, \quad 2, \quad 3, \quad 2.$$

(a) Find the mode of these temperatures.   *Answer (a)* ....°C [1]

(b) Find the median of these temperatures.   *Answer (b)* ....°C [1]

Answers: **(a)** 2 **(b)** ½   **N10/11/Q10**

708

42   The grouped frequency table below shows the times taken for 70 students to solve a problem.

Time (t minutes)	$0 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 6$	$6 < t \leqslant 8$
Number of students	24	12	16	10	8

(a) Complete the cumulative frequency table for this information.

Answer (a)

Time (t minutes)	$t \leqslant 3$	$t \leqslant 4$	$t \leqslant 5$	$t \leqslant 6$	$t \leqslant 8$
Number of students	24				

[1]

(b) In which group of the frequency table does the 40th percentile lie?

Answer (b) ....[1]

(c) Complete the frequency density table for this information.

Answer (c)   [2]

Time (t minutes)	$0 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 6$	$6 < t \leqslant 8$
Frequency density	8	12	16		

Answers: (a) 36, 52, 62, 70 (b) $3 < t \leqslant 4$ (c) 10, 4   **N10/11/Q22**

---

43   The temperatures, in °C, at midnight on 10 consecutive days were

$$4, \ 1, \ 0, \ -2, \ -1, \ -3, \ 1, \ -2, \ 3, \ -1.$$

(a) Find the difference between the highest and the lowest temperature.

Answer (a) ....°C [1]

(b) How many of these temperatures are within 2.5°C of 1°C?

Answer (b) ....[1]

Answer. (a) 7 (b) 6   **N10/12/Q4**

44 The times taken for 200 children to run 100 m were recorded.
The cumulative frequency curve summarises the results.
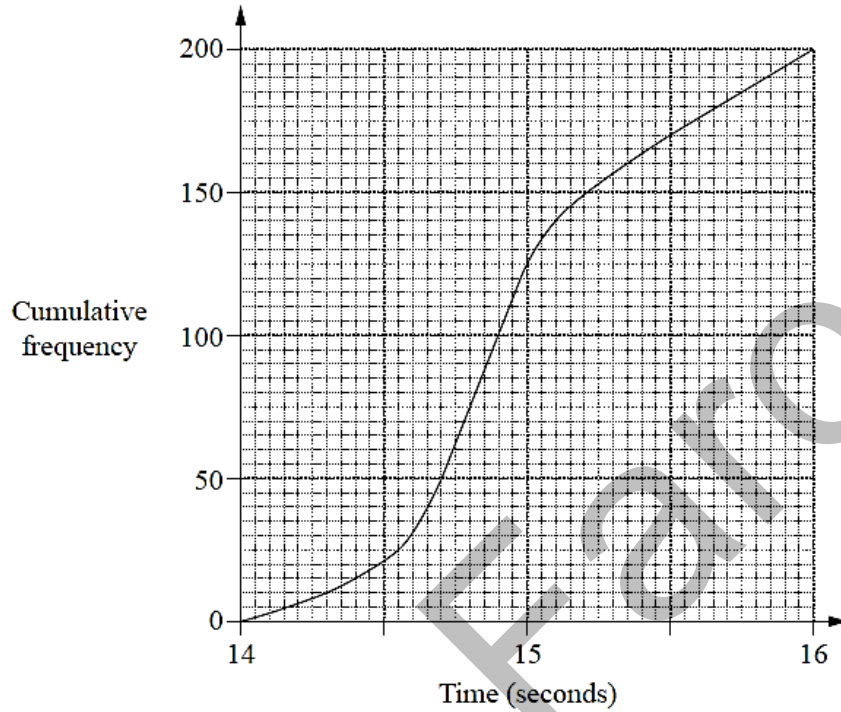
Use the curve to find

(a) the lower quartile,

Answer *(a)* .... seconds [1]

(b) the number of children who took at least 15.5 seconds.

Answer *(b)* ....[2]

Answer: **(a)** 14.7 **(b)** 30   **N10/12/Q18**

45 The table shows the results when a 6-sided die was thrown 50 times.

Score	1	2	3	4	5	6
Frequency	7	7	6	9	11	10

(a) Write down the modal score.   *Answer* .... [1]

(b) Calculate the mean score.   *Answer* .... [2]

Answers: **(a)** 5 **(b)** 3.8   **N11/11/Q14**

46 In a survey, some people were asked which of three songs, labelled A, B and C, they liked best.
The diagram shows part of a pie chart illustrating the results.
The angle of the sector that represents the people who liked C best is 168°.

(a) Complete the pie chart. [1]
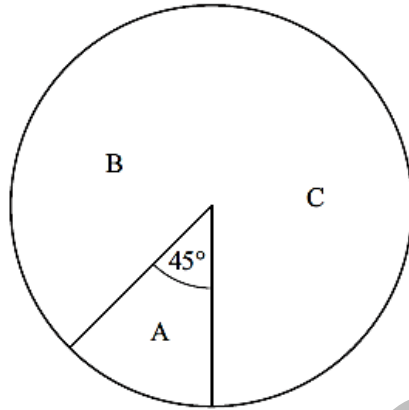
(b) Expressing your answer in its lowest terms, find the fraction of people in the survey who liked C best.

Answer .... [1]

(c) Given that 30 people liked A best, calculate the number of people in the survey.

Answer .... [1]

Answer: (b) $\frac{7}{15}$ (c) 240

47 The distribution of the lengths of time taken by an engineer to repair some washing machines is given in the table.

Time (t hours)	$1 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 8$
Frequency	k	5	4	3

The histogram represents some of this information.
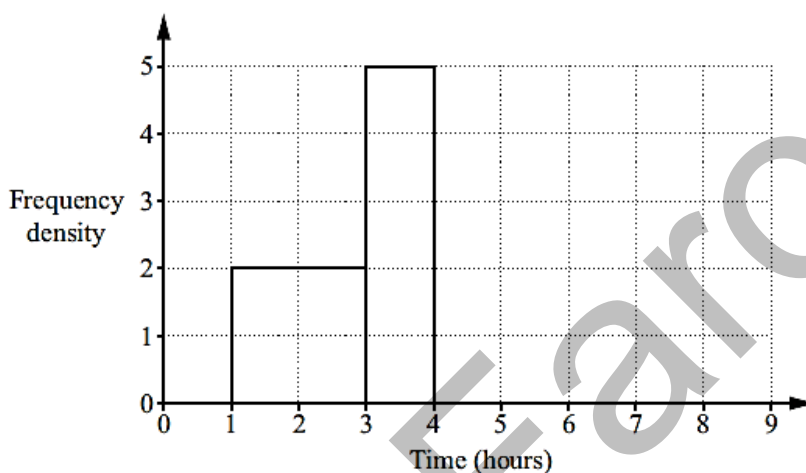
(a) Find k.   *Answer* $k =$ .... [1]

(b) Complete the histogram.   [2]

Answer. (a) 4   **N11/12/Q16**

48 The table shows the number of goals scored by 40 football teams during one weekend.

Number of goals	0	1	2	3	4	5	6
Number of teams	16	6	6	6	4	0	2

Find

(a) the mode,   *Answer* .... [1]

(b) the median,   *Answer* .... [1]

(c) the mean.   *Answer* .... [2]

Answer. (a) 0 (b) 1 (c) 1.6   **N11/12/Q24**

49 The mean mass of Ali, Ben and Carl is 40 kg.
The mass of Dan is 48 kg.

Find the mean mass of the four boys.   *Answer* .... kg [2]

Answer. 42   **N12/11/Q8**

712

50 In an experiment, 4 dice are thrown and the number of Fives is recorded.
The experiment is repeated 12 times.
The table shows the results.

Number of Fives	0	1	2	3	4
Frequency	1	2	3	5	1

For this distribution of Fives,

(a) write down the mode, *Answer* ....[1]

(b) find the median. *Answer* ....[1]

Answers: (a) 3 (b) 2.5

51 The lengths of the leaves of a plant were measured.
The results are shown in the table.

Length (x centimetres)	$1 < x \leqslant 3$	$3 < x \leqslant 4$	$4 < x \leqslant 5$	$5 < x \leqslant 7$	$7 < x \leqslant 10$
Frequency	8	5	6	12	12
Frequency density					

(a) Complete the table to show the frequency densities. [2]

(b) One leaf is chosen at random.

Find an estimate of the probability that this leaf is more than 6 cm long.

Answer ....[1]

Answers: (a) All of 4, 5, 6, 6, 4 (b) $\dfrac{18}{43}$

52

(a) An ordinary die is thrown 15 times.
These are the numbers thrown.

4   5   3   2   2   5   6   1   6   3   5   2   5   1   3

  (i) Find the mode.   *Answer* ....[1]

  (ii) Find the median.   *Answer* ....[1]

(b) -20 -8 x

The mean of these three numbers is -5.

Find x.   *Answer* $x =$ ....[1]

Answers: (a) (i) 5 (ii) 3 (b) 13   **N13/11/Q16**

53 Each member of a group of 100 people was asked how long they spent at a gym one afternoon. The results are summarised in the **cumulative** frequency table below.

Time (t mins)	$t \leqslant 20$	$t \leqslant 40$	$t \leqslant 60$	$t \leqslant 90$	$t \leqslant 120$
Cumulative frequency	6	20	46	88	100

(a) How many people spent between 60 and 90 minutes at the gym?

Answer .... [1]

(b) On the grid below, draw the cumulative frequency curve to represent the information in the table.
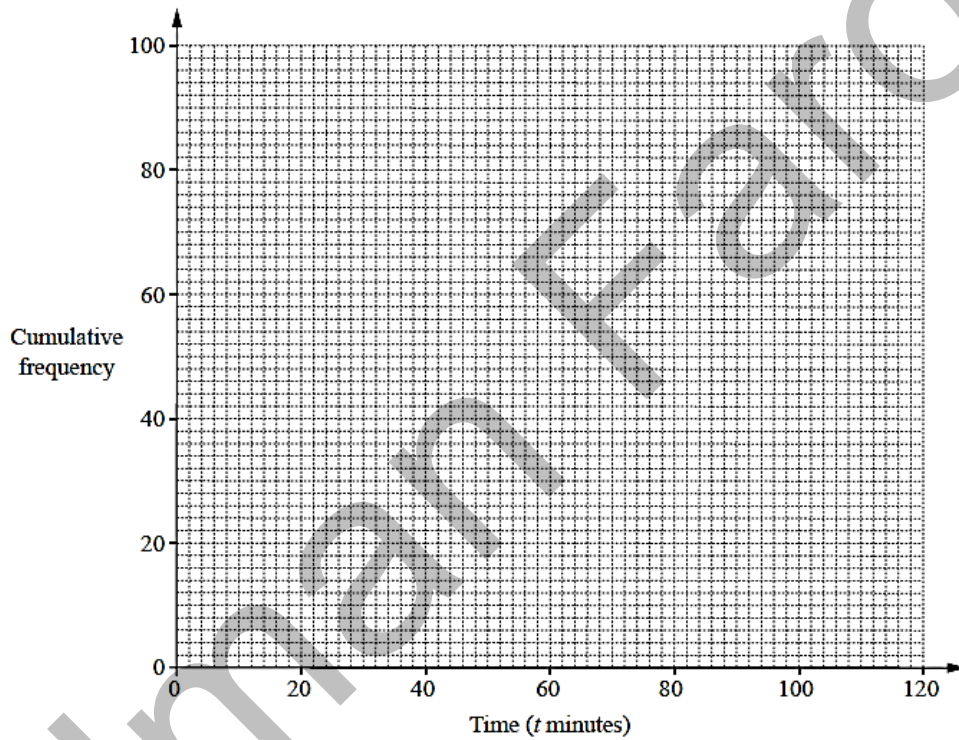
[2]

(c) Use your cumulative frequency curve to estimate

(i) the median time spent at the gym, Answer .... minutes [1]

(ii) the number of people who spent between 50 and 80 minutes at the gym.

Answer .... [2]

Answers: (a) 42; (b) Correct plots at 20, 40, 60, 90, 120 and cumulative frequency curve drawn; (c)(i) 62 to 64 inclusive, (ii) 41 to 46 inclusive. **N14/11/Q22**

54 A group of five numbers has a mean of 3.8 and a median of 3. The numbers 3 and 6 are added to the group.

(a) Find the mean of the seven numbers.

Answer .... [2]

(b) Find the median of the seven numbers.

Answer .... [1]

Answers: **(a)** 4 **(b)** 3   **N15/11/Q12**

55 Each member of a group of 50 people was asked how many films they watched in a month. The results are shown in the table below.

Numbers of films watched	Frequency
0	5
1	12
2	13
3	15
4	4
5	1

(a) Find the mode.

Answer .... [1]

(b) Calculate the mean number of films watched in a month.

Answer .... [2]

Answers: **(a)** 3 **(b)** 2.08   **N15/11/Q13**

56 The table shows the masses of different fruits sold at a market stall on one day.

Fruit	Apples	Pears	Oranges	Bananas	Total
Mass (kg)	30	10	18	32	90

(a) Complete the pie chart to illustrate the data.
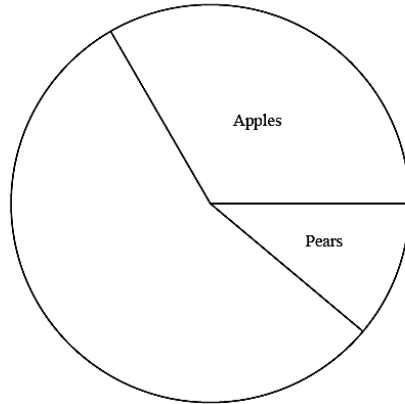
(b) The stallholder buys apples for 60 cents per kilogram.
She sells them all for 72 cents per kilogram.

Calculate her percentage profit.

Answer ....% [2]

57 The cumulative frequency graph shows information about the reaction times of 60 people.
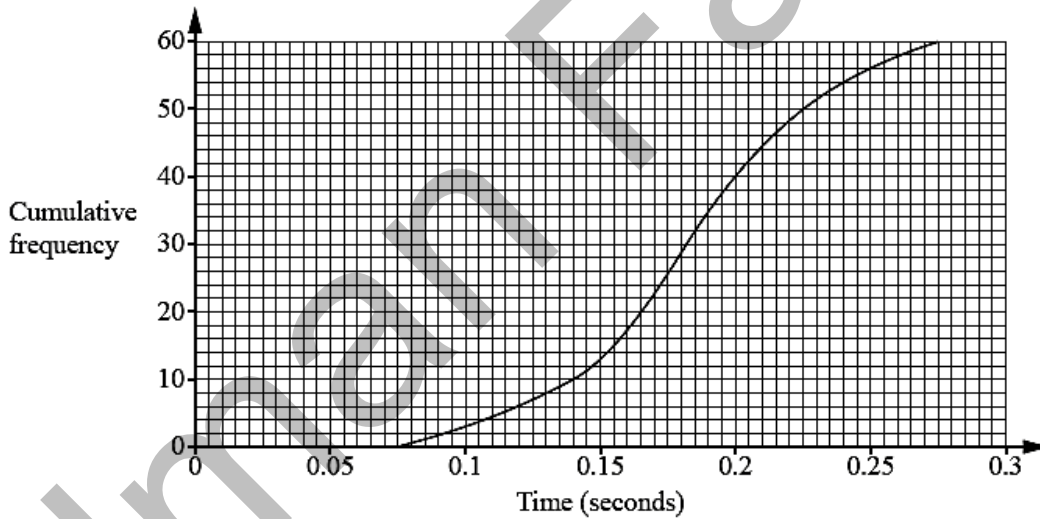
Use the graph to estimate

(a) the lower quartile,

Answer .... [1]

(b) the number of people who have a reaction time of more than 0.2 seconds.

Answer .... [1]

58 During one day, the temperature, in °C, was recorded every 2 hours.
The twelve results are given below.

$$-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 4 \quad 5 \quad 4 \quad 2 \quad 0 \quad -2 \quad -2$$

For these results, find

(a) the median,

Answer .... °C [1]

(b) the mean,

Answer .... °C [1]

(c) the difference between the highest and the lowest of these temperatures.

Answers: (a) 0.5 (b) $\frac{2}{3}$ (c) 8

59 One week the temperatures, in degrees Celsius, at midnight were recorded.
The results are given below.

$$-1 \quad -3 \quad 2 \quad 5 \quad -2 \quad 1 \quad -2$$

Use these results to find

(a) the mode,

Answer .... [1]

(b) the median,

Answer .... [1]

(c) the mean.

Answer .... [1]

Answers: (a) −2 (b) −1, (c) 0

60

In a survey, some people were asked which of three types of tea, labelled X, Y and Z, they preferred.
The diagram shows part of a pie chart that illustrates the results.
The angle of the sector that represents the people who preferred Y is 168°.

(a) Complete the pie chart.
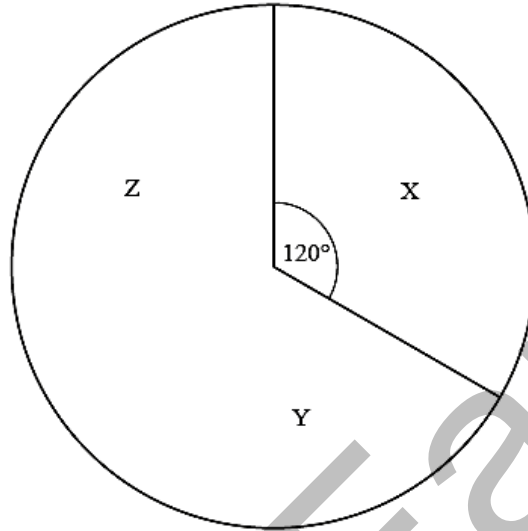
[1]

(b) Find the fraction of people who preferred Y.
Express your answer in its simplest form.

Answer .... [1]

(c) Given that 80 people preferred X, calculate the number of people in the survey.

Answer .... [1]

Answers: **(a)** Correct line **(b)** $\dfrac{7}{15}$ **(c)** 240

N17/11/Q13

61 The lengths of 90 leaves of a plant were measured.
The results are given in the table.
The diagram shows part of the cumulative frequency curve.

Length (t cm)	$0 < t \leqslant 1$	$1 < t \leqslant 2$	$2 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 6$
Frequency	7	17	29	24	10	3
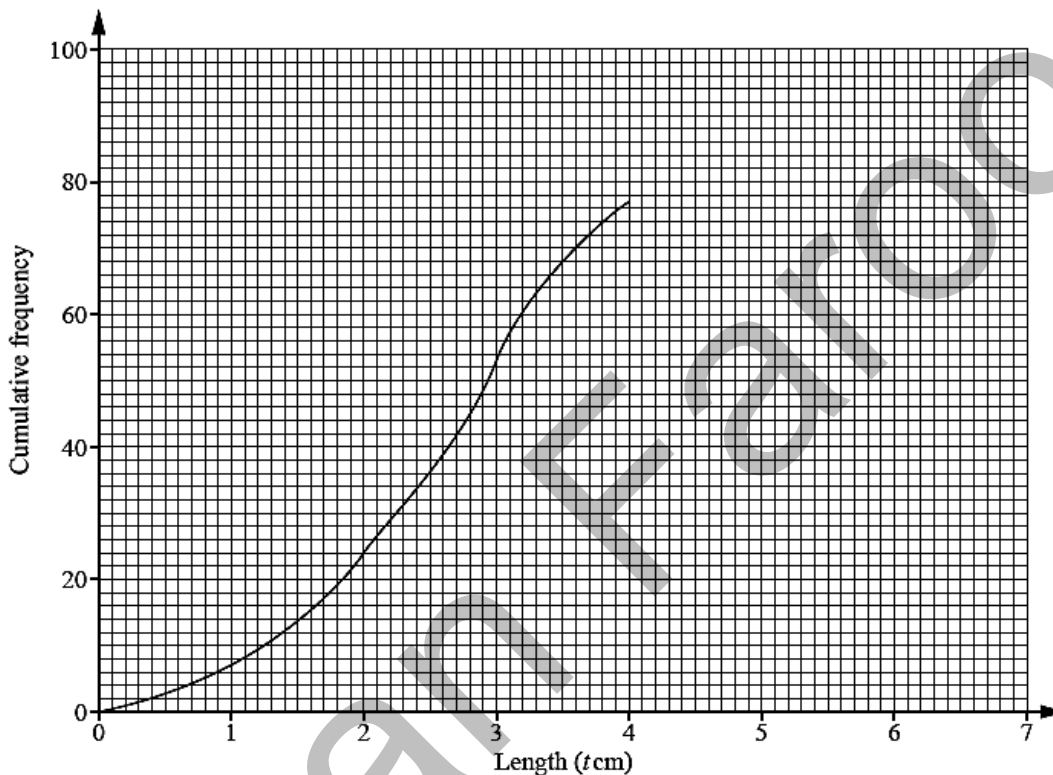
(a) Complete the cumulative frequency curve. [1]

(b) Use the curve to estimate

  (i) the median,

  Answer .... cm [1]

  (ii) the number of leaves with a length less than 3.5 cm.

  Answer .... [1]

Answers: (a) Curve from (4,77) to (6, 90) via (5, 87) (b)(i) 2.8 (c) 67 or 68   **N17/11/Q17**

Statistics Paper 2

1   **Answer the whole of this question on a sheet of graph paper.**

The masses of 80 parcels sent out by a garden centre are given in the table below.

Mass (m kilograms)	$0 < m \leqslant 2$	$2 < m \leqslant 4$	$4 < m \leqslant 6$	$6 < m \leqslant 10$	$10 < m \leqslant 15$
Frequency	12	18	20	20	10

(a) Using a scale of 1 cm to represent 1 kg, draw a horizontal axis for $0 \leqslant m \leqslant 15$.
Choose a suitable scale for the vertical axis and draw a histogram to represent this data.   [3]

(b) Estimate the number of parcels which had a mass greater than 9 kg.   [1]

(c) Calculate an estimate of the mean mass.   [3]

(d) One parcel was chosen at random and not replaced.
A second parcel was chosen at random from the remainder.
Giving each answer as a fraction in its lowest terms, find the probability that

   (i) both parcels were chosen from the $6 < m \leqslant 10$ group,   [1]

   (ii) one parcel was chosen from the $6 < m \leqslant 10$ group and the other parcel was **not** chosen from the $6 < m \leqslant 10$ group.   [2]

J02/2/Q6

2   ⁵   (a) One hundred and sixty students took an examination.
The table shows the marks needed for each grade.
The cumulative frequency curve shows the distribution of their marks.

Grade A	$70 < $ mark
Grade B	$55 < $ mark $\leqslant 70$
Grade C	$40 < $ mark $\leqslant 55$
Grade D	$20 < $ mark $\leqslant 40$
Grade U	mark $\leqslant 20$

721
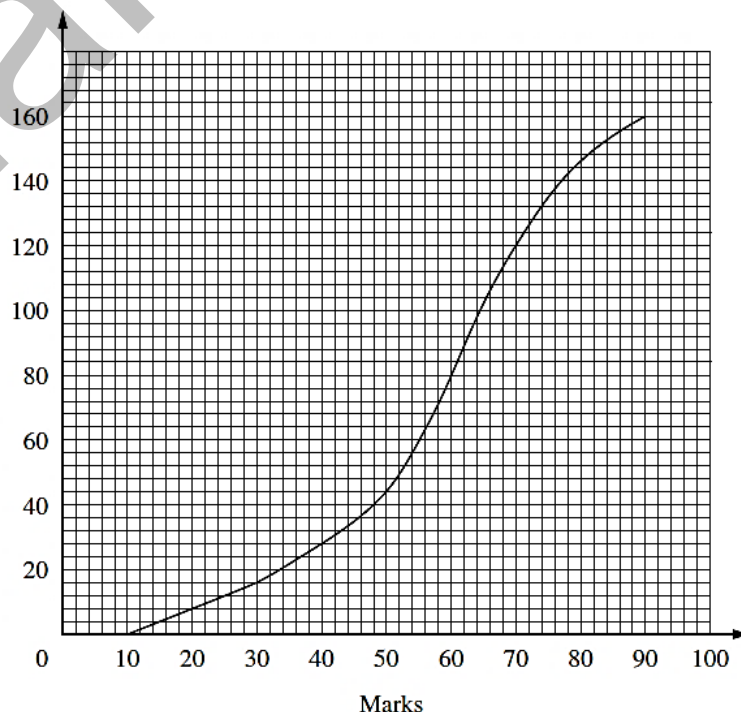
(i) Use the graph to estimate

    (a) the median,   [1]

    (b) the interquartile range,   [2]

    (c) the number of students who were awarded a Grade C.   [2]

(ii) A pie chart was drawn to illustrate the grades awarded to the students.

Calculate the angle of the sector which represented the number of students who were awarded a Grade C.   [2]

Answers: **(a)(i)(a)** 60, **(b)** 22, **(c)** 32, **(ii)** 72°

3

10 **Answer the whole of this question on a sheet of graph paper.**

The ages of a sample of 40 students were recorded.
The results are given in the table below.

Age (x years)	$8 < x \leqslant 10$	$10 < x \leqslant 11$	$11 < x \leqslant 12$	$12 < x \leqslant 14$	$14 < x \leqslant 16$	$16 < x \leqslant 19$
Frequency	7	8	6	10	3	6

(a) Using a scale of 1 cm to represent 1 year, draw a horizontal axis for ages from 8 to 19 years.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 8 units.
On your axes, draw a histogram to illustrate the distribution of ages.   [3]

(b) In which interval does the median lie?   [1]

(c) Calculate an estimate of the mean age of the students.   [3]

(d) Calculate an estimate of the number of students who were under 13 years old.   [1]

(e) One student is chosen at random from this sample of 40 students.

Write down the probability that this student is

    (i) under 8,   [1]

    (ii) over 16.   [1]

(f) A second student is now chosen at random from the remaining 39 students.

Calculate the probability that one student is over 16 and the other is not over 16.
Give your answer as a fraction in its lowest terms.   [2]

Answers: **(a)** Widths 2, 1, 1, 2, 2, 3, Heights $3\frac{1}{2}$, 8, 6, 5, $1\frac{1}{2}$, 2; **(b)** $11 < x \leqslant 12$; **(c)** 12.4; **(d)** 26;

(e)(i) 0, **(ii)** $\frac{3}{20}$; **(f)** $\frac{17}{65}$.

4 (a) Sweet packets contain sweets of different colours.
The number of yellow sweets in each of 25 packets was recorded.
The table below shows the results.

Number of yellow sweets	0	1	2	3	4	5
Frequency	8	5	5	4	2	1

For this distribution,

(i) write down the mode,   [1]

(ii) write down the median,   [1]

(iii) calculate the mean.   [2]

Answers: **(a)(i)** 0, **(ii)** 1, **(iii)** 1.6;   J05/2/Q5

5 (a) The lengths of 120 leaves were measured.
The cumulative frequency graph shows the distribution of their lengths.
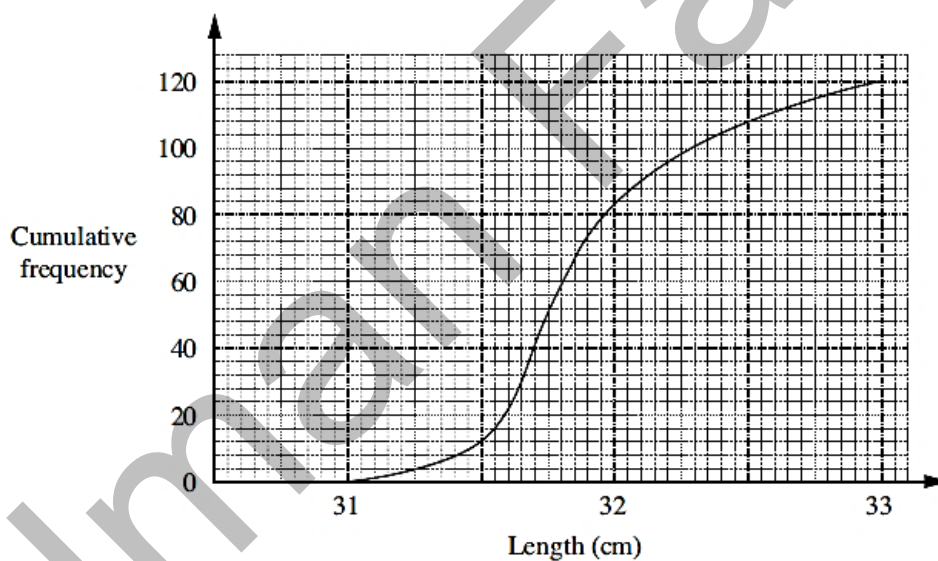
Use this graph to estimate

(i) the median,   [1]

(ii) the interquartile range,   [2]

(iii) the number of leaves whose length is more than 31.5 cm.   [1]

(b) Each member of a group of 16 children solved a puzzle.
The times they took are summarised in the table below.

Time (t minutes)	$5 < t \leqslant 10$	$10 < t \leqslant 12$	$12 < t \leqslant 14$	$14 < t \leqslant 16$	$16 < t \leqslant 20$
Frequency	2	4	6	3	1

(i) Write down an estimate of the number of children who took less than 13 minutes. [1]

(ii) Calculate an estimate of the mean time taken to solve the puzzle. [3]

(iii) Two children are chosen at random.

Calculate, as a fraction in its simplest form, the probability that one of these children took more than 10 minutes and the other took 10 minutes or less. [2]

(iv) A histogram is drawn to illustrate this information.
The height of the rectangle representing the number of children in the interval $10 < t \leqslant 12$ is 8 cm.
Calculate the height of the rectangle representing the number of children in the interval $5 < t \leqslant 10$. [2]

Answers: (a)(i) 31.8 cm, (ii) 0.45, (iii) 108; (b)(i) 9, (ii) 12.5 min, (iii) $\frac{7}{30}$, (iv) 1.6 cm.   **J06/2/Q10**

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6   **(a)** In a survey, some students were asked which of three pictures, labelled X, Y and Z, they preferred.

The results are represented in the pie chart.
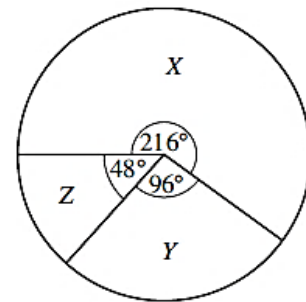
(i) Calculate the percentage of students who preferred X. [1]

(ii) Find, in its simplest form, the ratio of the number of students who preferred X to those who preferred Y.

Give your answer in the form $m : n$, where m and n are integers. [1]

(iii) Given that 44 students preferred Y, calculate the number of students who took part in the survey. [2]

Answer. (a)(i) 60, (ii) 9 : 4 (iii) 165;   **J07/2/Q4**

---

7   Emma noted the number of letters in each of the 25 words in an examination question.
The results are given in the table below.

Number of letters	2	3	4	5	6	7	8
Frequency	2	6	5	5	4	0	3

(a) For this distribution,

    (i) write down the mode, [1]

    (ii) find the median, [1]

    (iii) calculate the mean. [2]

(b) Emma chose one word, at random, from the 25 words.

    Find the probability that this word had

    (i) 5 or 6 letters, [1]

    (ii) fewer than 9 letters. [1]

(c) Peter chose one word, at random, from the 25 words.
He then chose a second word, at random, from the remaining words.
Expressing each answer as a fraction in its lowest terms, find the probability that

    (i) both words had 6 letters, [1]

    (ii) one word had 2 letters and the other had 4 letters. [2]

Answer: (a)(i) 3, (ii) 4, (iii) 4.6; (b)(i) $\dfrac{9}{25}$, (ii) 1; (c) $\dfrac{1}{50}$, $\dfrac{1}{30}$.

J07/2/Q5

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8   Paul and Sam are two athletes who have training sessions together.
On 80 sessions during 2007 they ran the same route, and their times were recorded.

(a) The cumulative frequency curve shows the distribution of Paul's times.
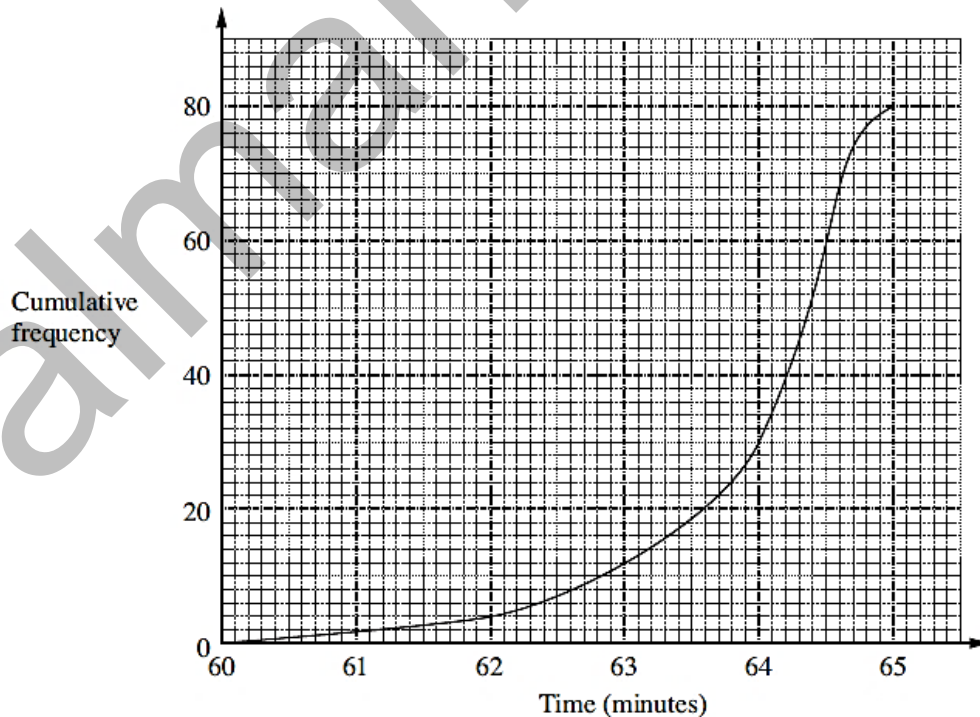
Use the curve to estimate

    (i)   the median,   [1]

    (ii)   the interquartile range,   [2]

    (iii) how often Paul took more than 64 minutes.   [1]

(b) Sam's times had a lower quartile of 62.5 minutes, a median of 63 minutes and an upper quartile of 64 minutes.

State which athlete was the more consistent runner, giving a reason for your answer.   [1]

Answers: **(a)(i)** 64.2,   **(ii)** 0.9,   **(iii)** 50,   **(b)** Paul, smaller IQR.   **J08/2/Q6**

9   **Answer THE WHOLE of this question on a sheet of graph paper.**

The waiting times of 50 people at a supermarket checkout were recorded.
The results are summarised in the table below.

Time (t minutes)	$1 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 7$	$7 < t \leqslant 9$	$9 < t \leqslant 12$
Number of people	4	10	8	14	8	6

(a) Using a scale of 1 cm to represent 1 minute, draw a horizontal axis for waiting times between 0 and 12 minutes.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 10 units.
On your axes, draw a histogram to illustrate the distribution of waiting times.   [3]

(b) In which class does the upper quartile lie?   [1]

(c) Calculate an estimate of the mean waiting time.   [3]

(d) One person is chosen, at random, from the 50 people.

Write down the probability that this person waited

    (i)   less than 1 minute,   [1]

    (ii) more than 5 minutes.   [1]

(e) A second person is now chosen, at random, from the remaining 49 people.

Expressing each answer as a fraction in its lowest terms, calculate the probability that

    (i)   both people waited more than 5 minutes,   [1]

    (ii) one person waited more than 5 minutes and the other waited 5 minutes or less.   [2]

J09/2/Q10

10 **Answer the WHOLE of this question on a sheet of graph paper.**

The table below shows the amount of time spent playing sport each week by 80 students.

Time (t hours)	$0 < t \leqslant 2$	$2 < t \leqslant 4$	$4 < t \leqslant 6$	$6 < t \leqslant 8$	$8 < t \leqslant 10$
Frequency	20	31	22	5	2

(a) Calculate an estimate of the mean time spent playing sport each week by the students. [3]

(b) Copy and complete the cumulative frequency table below.

Time (t hours)	$t \leqslant 2$	$t \leqslant 4$	$t \leqslant 6$	$t \leqslant 8$	$t \leqslant 10$
Cumulative frequency	20	51			80

[1]

(c) Using a horizontal scale of 1 cm to represent 1 hour and a vertical scale of 1 cm to represent 5 students, draw a smooth cumulative frequency curve for this data. [3]

(d) Use your graph to estimate

(i) the median, [1]

(ii) the interquartile range. [2]

J10/21/Q7

11 **Answer the WHOLE of this question on a sheet of graph paper.**

(a) The time taken by 140 children to run 200 metres was recorded.
The results are summarised in the table below.

Time (t seconds)	$22 \leqslant t < 24$	$24 \leqslant t < 26$	$26 \leqslant t < 31$	$31 \leqslant t < 36$	$36 \leqslant t < 46$
Frequency	12	18	42	28	40

(i) Using a scale of 1 cm to represent 2 seconds, draw a horizontal axis for time from 22 seconds to 46 seconds.
Using a scale of 1 cm to represent 1 unit, draw a vertical axis for frequency density from 0 to 9 units.

On your axes, draw a histogram to represent the information in the table. [3]

(ii) Estimate the number of children who took less than 25 seconds to run 200 metres. [1]

(iii) One child was chosen at random.

Calculate the probability that the time taken by this child was less than 36 seconds. Express your answer as a fraction in its lowest terms. [1]

(iv) Out of the 30 children who took less than 26 seconds, two were chosen at random.

Calculate the probability that they both took less than 24 seconds. [2]

(b) Some boys were put into five groups, A, B, C, D and E, based on the times they took to run 100 metres.
The pie chart shows the proportion of boys in each group.

Group A contains $\frac{1}{4}$ of the boys.
Group B contains 35% of the boys.
Group C is represented by a sector with an angle of 42°.
Group D contains 9 boys.
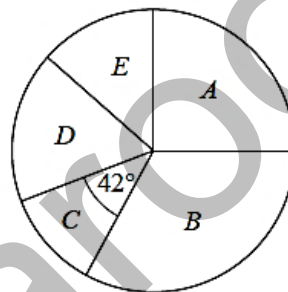
(i) Find the fraction of boys in group C.
Give your answer in its lowest terms. [1]

(ii) Given that the number of boys in group B is 21, find the total number of boys who ran the 100 metres. [2]

(iii) Calculate the number of boys in group E. [2]

Answers: (a)(ii) 20 or 21, (iii) $\frac{5}{7}$, (iv) $\frac{22}{145}$; (b) (i) $\frac{7}{60}$, (ii) 60, (iii) 8. **J10/22/Q11**

12

9 The masses of 120 potatoes were recorded.
The table below shows the distribution of their masses.

Mass (m grams)	$0 \leqslant m < 100$	$100 \leqslant m < 150$	$150 \leqslant m < 200$	$200 \leqslant m < 250$	$250 \leqslant m < 350$
Frequency	14	28	37	21	20

(a) (i) Using a scale of 2 cm to represent 50 grams, label the horizontal axis for masses from 0 to 350 grams.
Using a scale of 1 cm to represent 0.1 unit, label the vertical axis for frequency densities from 0 to 0.8 units.

Draw a histogram to represent the information in the table.

[3]
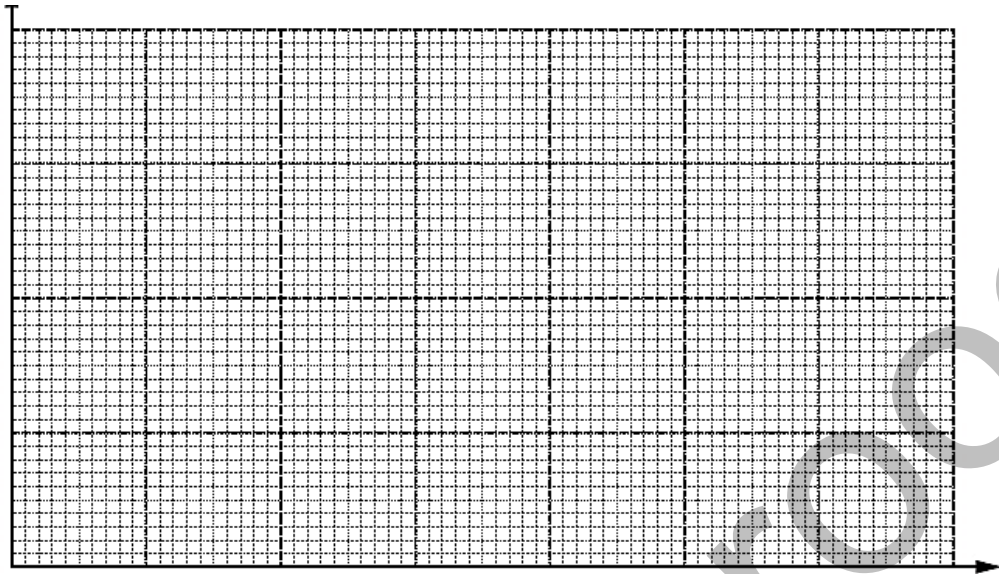
(ii) Estimate the number of potatoes with a mass greater than 270 grams.

Answer .... [1]

(iii) In which interval is the upper quartile of the distribution?

Answer .... [1]

(iv) Find the probability that a potato chosen at random has a mass less than 150 grams.
Give your answer as a fraction in its simplest form.

Answer .... [1]

The masses of some oranges were recorded.
The table below shows the distribution of their masses.

Mass (n grams)	$100 \leqslant n < 150$	$150 \leqslant n < 200$	$200 \leqslant n < 250$
Frequency	14	p	26

(b) The estimated mean mass of an orange is 183 grams.

Find the value of p. *Answer* .... [3]

(c) (i) An orange is chosen at random.

Find the probability that it has a mass less than 250 grams.

Answer .... [1]

(ii) A potato and an orange are chosen at random.

Calculate the probability that they both have a mass less than 150 grams.

Answer .... [2]

729

13

The pie chart, not drawn accurately, represents the weekly income of the five employees in a small British company in 2009.
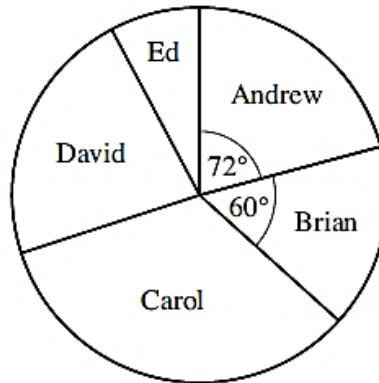
Andrew's weekly income is represented by a sector with an angle of 72°.
Brian's weekly income is represented by a sector with an angle of 60°.

(a) Andrew's weekly income was £270.

Find the total weekly income of the five employees.   Answer £ ....[1]

(b) Calculate Brian's weekly income.   Answer £ ....[1]

(c) Carol's weekly income was £405.

Calculate the angle of the sector representing Carol's weekly income.

Answer ....[1]

(d) David's weekly income was twice as much as Ed's weekly income.

Calculate David's weekly income.   Answer £ ....[2]

(e) Andrew paid 20% of his weekly income of £270 as tax.
He also paid 6% of his weekly income of £270 towards his pension.

How much of his weekly income did he have left after paying tax and pension?

Answer £ ....[2]

(f) Carol paid 20% of her weekly income of £405 as tax.
She also paid x% of her weekly income towards her pension.
She then had £287.55 of her weekly income left.

Find x.   Answer ....[3]

(g) Andrew's weekly income of £270 in 2009 was 8% more than his weekly income in 2008.

Find his weekly income in 2008.   Answer £ ....[2]

14   12   The time taken by each of 320 students taking a Physics test was recorded.
The following table shows a distribution of their times.

Time (m minutes)	$60 < m \leq 70$	$70 < m \leq 80$	$80 < m \leq 90$	$90 < m \leq 100$	$100 < m \leq 110$	$110 < m \leq 120$
Frequency	24	92	104	68	24	8

(a) Complete the cumulative frequency table below.

Time (m minutes)	$m \leq 60$	$m \leq 70$	$m \leq 80$	$m \leq 90$	$m \leq 100$	$m \leq 110$	$m \leq 120$
Cumulative frequency	0	24	116				

[1]

(b) For this part of the question use the graph paper opposite.

(i) Using a scale of 2 cm to represent 10 minutes, draw a horizontal m-axis
for $60 \leq m \leq 120$.
Using a scale of 1 cm to represent 20 students, draw a vertical axis for cumulative
frequencies from 0 to 320.
On your axes, draw a smooth cumulative frequency curve to illustrate the information.

[3]

(ii) Use your graph to estimate

(a) the median,   *Answer* .... minutes [1]

(b) the interquartile range,   *Answer* .... minutes [2]

(c) the percentage of students who took at least 95 minutes to complete the test.

Answer ....[2]

731

(iii) A group of 300 students of similar ability took an equivalent test the previous year. The following graph shows a distribution of their times.
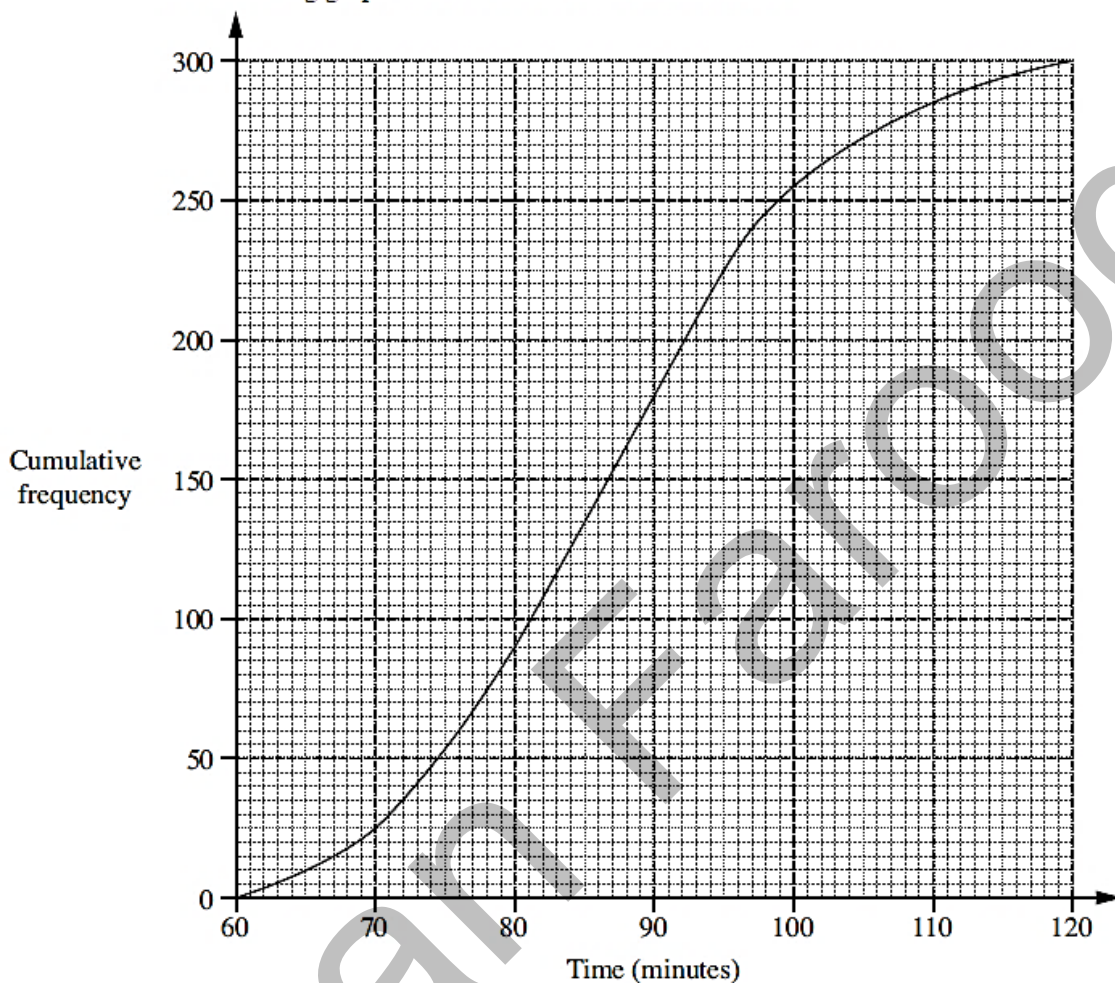
(a) Find the 20th percentile.　　　　*Answer* .... minutes [1]

(b) Find the percentage of students who took at least 95 minutes to complete the test.

Answer ....[1]

(c) Hence make a comparison between the two tests.　　....[1]

Answers: **(a)** 220, 288, 312, 320　**(b)(ii)(a)** 83 to 85　**(b)** 13.5 to 16.5　**(c)** 15 to 19%　**J11/22/Q12**
(b)(iii) (a) 76　**(b)** 25%

15　**9**　A group of 80 music students recorded the time each spent practising last week.
The results are summarised in this table.

Time (m minutes)	$0 < m \leqslant 20$	$20 < m \leqslant 40$	$40 < m \leqslant 60$	$60 < m \leqslant 80$	$80 < m \leqslant 100$	$100 < m \leqslant 120$
Frequency	6	15	29	18	9	3

(a) Calculate an estimate of the mean.　　*Answer* .... minutes [3]

(b) Complete the cumulative frequency table below.

Time (m minutes)	$m = 0$	$m \leqslant 20$	$m \leqslant 40$	$m \leqslant 60$	$m \leqslant 80$	$m \leqslant 100$	$m \leqslant 120$
Cumulative frequency	0	6	21				80

[1]

(c) For this part of the question use the graph paper opposite.

Using a scale of 1 cm to represent 10 minutes, draw a horizontal m-axis for $0 \leqslant m \leqslant 120$.
Using a scale of 2 cm to represent 10 students, draw a vertical axis for cumulative frequency from 0 to 80.
Using your axes draw a smooth cumulative frequency curve to illustrate the information. [3]

(d) Use your graph to estimate

(i) the median, *Answer* .... minutes [1]

(ii) the interquartile range, *Answer* .... minutes [2]

(iii) the probability that a student, chosen at random, practised for **more than** 75 minutes.

Answer ....[2]

Answers: (a) 54.5   (b) 50, 68, 77   (d)(i) 50 to 55   (ii) 31   (iii) $\dfrac{16}{80}$ **J12/21/Q9**

16 (b) The frequency diagram shows the distribution of the number of letters received by a family each day over a 31 day period.
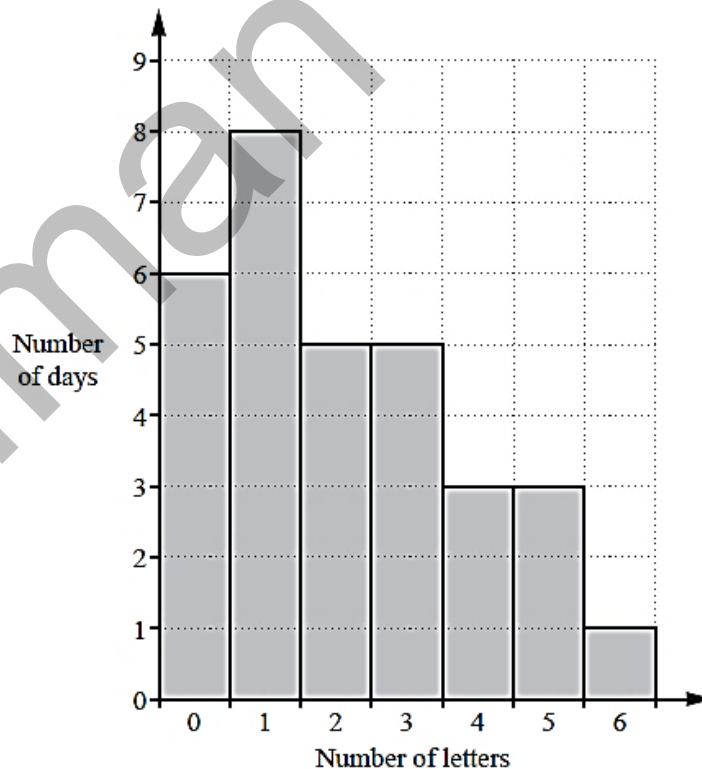
For this distribution, find

(i) the mode,   *Answer* .... [1]

(ii) the median.   *Answer* .... [1]

(b) 9/22 (b)(i) 1 (ii) 2   **J13/21/Q4b**

17   (b) The pie chart represents the distribution of the birth places of a group of 60 students.
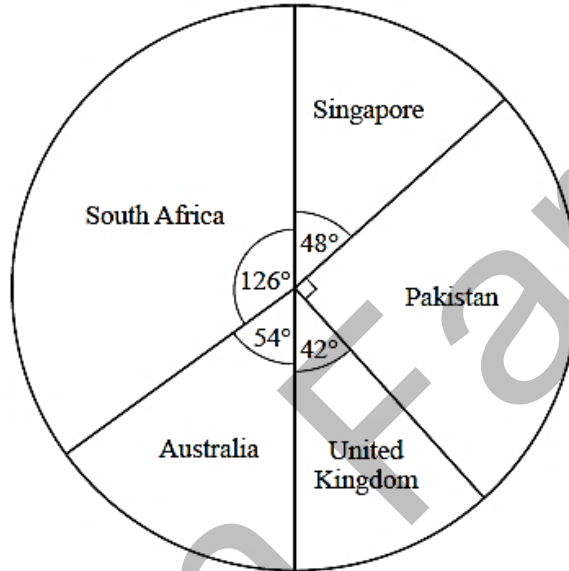
(i) Find the number of students in the group who were born in Australia.

                                 Answer .... [1]

(ii) Calculate the percentage of students in the group who were born in South Africa.

                                 Answer ....% [1]

(iii) Four more students join the group.
Of these, two students were born in Pakistan, one in Singapore and one in China.
A new pie chart is to be drawn using the information about the **whole group** of students.

For the new pie chart, calculate the angle of the sector that represents the students born in Pakistan.
Give your answer correct to the nearest degree.

                                 Answer .... [2]

(b)(i) 9 (ii) 35% (iii) 96°   **J13/21/Q12b**

18   A group of 80 students enters a science quiz.
The table shows the distribution of their scores.

Score (s)	$0 < s \leqslant 10$	$10 < s \leqslant 20$	$20 < s \leqslant 30$	$30 < s \leqslant 40$	$40 < s \leqslant 50$	$50 < s \leqslant 60$
Frequency	4	12	16	23	20	5

(a) Calculate an estimate of the mean score.   *Answer* .... [3]

(b) (i) Complete the cumulative frequency table for their scores.

Score (s)	$s \leqslant 10$	$s \leqslant 20$	$s \leqslant 30$	$s \leqslant 40$	$s \leqslant 50$	$s \leqslant 60$
Cumulative frequency	4					80

[1]

(ii) On the grid below,
draw a horizontal s-axis for $0 \leqslant s \leqslant 60$ using a scale of 2 cm to represent 10 points
and a vertical axis from 0 to 80 using a scale of 2 cm to represent 20 students.

Draw a smooth cumulative frequency curve to represent this information.

[3]
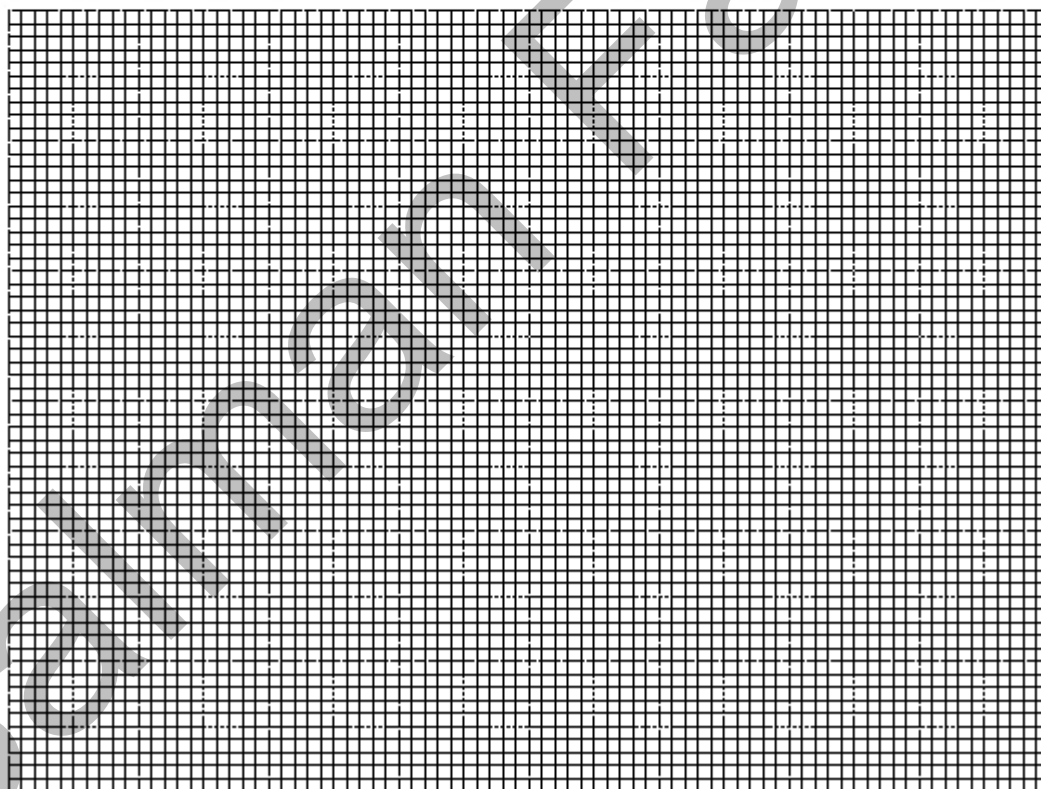
(iii) Use your graph to estimate

    (a) the median score,   *Answer* .... [1]

    (b) the interquartile range of the scores. *Answer* .... [2]

    (c) Students who scored more than 40 points can enter the next round of the quiz.
        Two of these students are selected at random.

        Work out the probability that both students scored more than 50 points.

                                                          Answer .... [2]

Answers: (a) 32.25 or 32.75 (b)(i) 4,16, 32, 55, 75, 80 (iii)(a) 33 to 35 (b) 18 to 20 (c) 1/30 or 0.0333   **J14/21/Q8**

19   One day a farmer collected 300 eggs from his chickens.
        The table below shows the distribution of the masses of the eggs.

Mass (m grams)	$42 < m \leqslant 46$	$46 < m \leqslant 48$	$48 < m \leqslant 50$	$50 < m \leqslant 54$	$54 < m \leqslant 58$	$58 < m \leqslant 66$
Frequency	60	40	48	72	56	24

    (a) (i) An egg is chosen at random.

           Calculate the probability that the mass of this egg is not greater than 48 grams.

                                          Answer .... [1]

      (ii) An egg is chosen at random from the 300 eggs.
           Another egg is chosen at random from those that remain.

           Calculate the probability that the mass of one egg is at most 46 grams, and the mass of the other is more than 58 grams.

                                          Answer .... [2]

    (b) Calculate an estimate of the mean mass of an egg. *Answer* .... g [3]

736

(c) (i) Complete the cumulative frequency table.

Mass (m grams)	m ≤ 42	m ≤ 46	m ≤ 48	m ≤ 50	m ≤ 54	m ≤ 58	m ≤ 66
Cumulative Frequency	0	60					300

[1]

(ii) On the grid, draw a smooth cumulative frequency curve to illustrate this information.
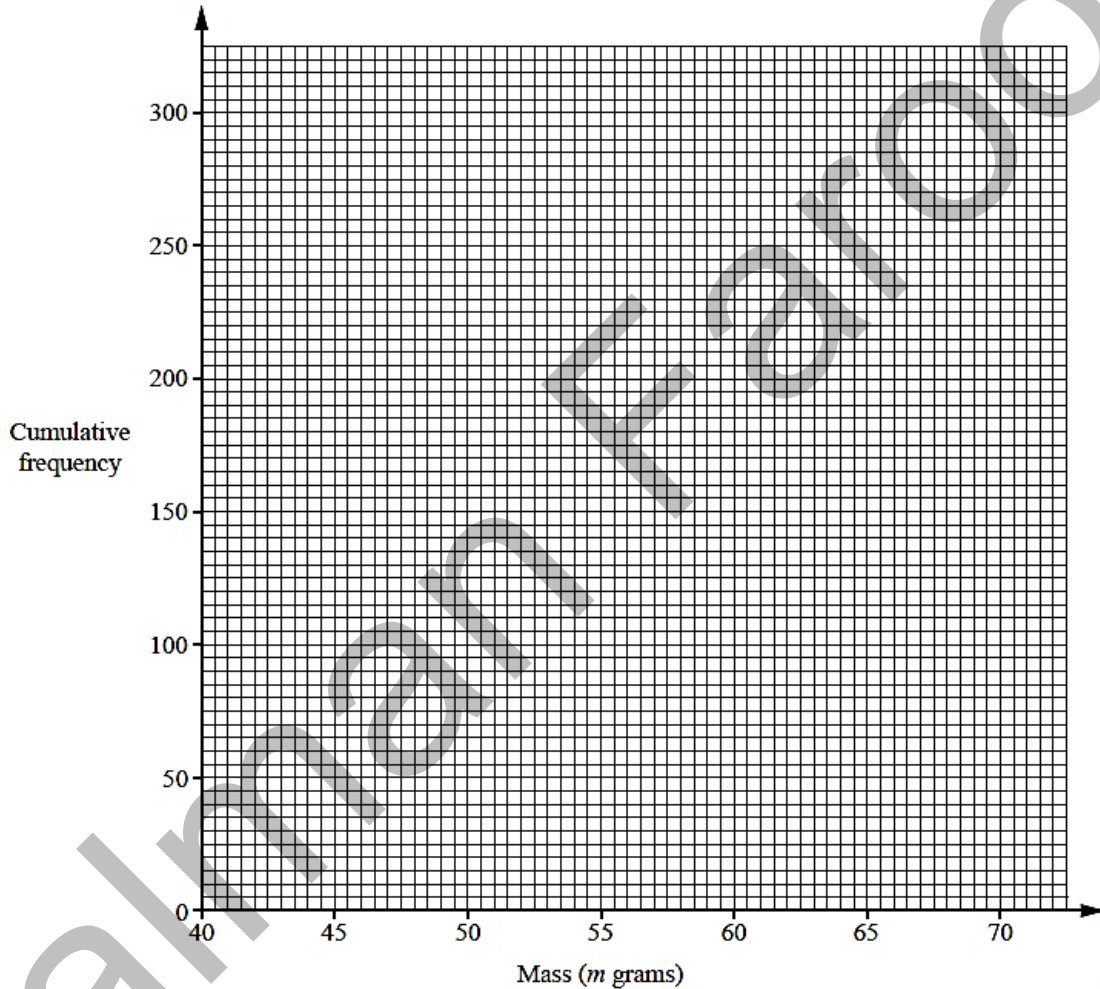
[2]

(d) (i) Use your graph to find the median mass of the eggs.

Answer .... g [1]

(ii) Use your graph to find the interquartile range.

Answer .... g [2]

Answers: (a)(i) 1/3 (ii) 48/1495 (b) 50.8 (c)(i) 100 148 220 276 (d)(i) 50 to 50.5 (ii) 7.25 to 8.00   J15/21/Q10

737

20   Answer the whole of this question on a sheet of graph paper.

The length of time taken by 80 drivers to complete a journey is given in the table below.

Time (t minutes)	$60 < t \leqslant 80$	$80 < t \leqslant 90$	$90 < t \leqslant 95$	$95 < t \leqslant 100$	$100 < t \leqslant 110$	$110 < t \leqslant 130$
Number of drivers	4	10	14	20	24	8

(a) Using a scale of 2 cm to represent 10 minutes, draw a horizontal axis for times between 60 minutes and 130 minutes.
Choose a suitable scale for the vertical axis and draw a histogram to represent the information in the table. [3]

(b) In which interval does the median of the distribution lie? [1]

(c) Calculate an estimate of the mean time taken to complete the journey. [3]

(d) One driver is chosen at random.

Expressing your answer as a fraction in its lowest terms, calculate the probability that she took 90 minutes or less for the journey. [1]

(e) Two drivers are chosen at random.
Expressing each answer as a fraction in its lowest terms, calculate the probability that

(i) both took more than 110 minutes for the journey, [2]

(ii) one took 80 minutes or less for the journey and the other took more than 110 minutes. [2]

Answers: (b) 95 to 100; (c) 98.2; (d) $\dfrac{7}{40}$; (e)(i) $\dfrac{7}{790}$, (ii) $\dfrac{4}{395}$.   N01/2/Q8

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21   (b) Answer the whole of this part of the question on a sheet of graph paper.

At a school, 300 pupils took an English test.
The table below is the cumulative frequency table for their scores.

Score (s)	$\leqslant 10$	$\leqslant 20$	$\leqslant 30$	$\leqslant 40$	$\leqslant 50$	$\leqslant 60$
Cumulative frequency	15	33	67	142	257	300

(i) Using a scale of 2 cm to represent 10 marks, draw a horizontal s-axis for $0 \leqslant s \leqslant 60$.
Using a scale of 2 cm to represent 50 pupils, draw a vertical axis for values from 0 to 300.
On your axes, draw a smooth cumulative frequency curve to illustrate this information. [2]

(ii) Use your graph to find the lower quartile score. [1]

(iii) Find the interquartile range of the scores. [1]

(iv) Given that 70% of the pupils passed the test, use your graph to find the pass mark. [2]

N02/2/Q11

22   Answer the whole of this question on a sheet of graph paper.

The speeds of 50 cars being driven along a stretch of road were recorded.
The table below shows the distribution of the speeds of the cars.

Speed (v km/h)	$20 < v \leqslant 40$	$40 < v \leqslant 50$	$50 < v \leqslant 55$	$55 < v \leqslant 60$	$60 < v \leqslant 70$	$70 < v \leqslant 110$
Frequency	4	14	10	8	10	4

(a) Using a scale of 1 cm to represent 10 km/h, draw a horizontal axis for speeds up to 110 km/h.
Using a scale of 4 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 2 units.

On your axes, draw a histogram to represent the information in the table.   [3]

(b) Write down the modal class of the distribution.   [1]

(c) In which interval is the upper quartile of the distribution?   [1]

(d) Find the probability that one car, selected at random, had a speed of

    (i)   less than 20 km/h,   [1]

    (ii) more than 60 km/h.   [1]

(e) There is a speed limit of 60 km/h on this stretch of road.
Two cars were selected at random.

Calculate the probability that one car was breaking the speed limit and the other was not breaking the limit.   [2]

Answers: (b) $50 < v \leqslant 55$; (c) $60 < v \leqslant 70$; (d)(i) 0, (ii) $\dfrac{7}{25}$; (e) $\dfrac{72}{175}$.

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23   Answer the whole of this question on a sheet of graph paper.

The table below shows the marks obtained in tests of English and Mathematics by 140 students.

Mark (x)	Number of candidates	
	English	Mathematics
$0 < x \leqslant 20$	4	10
$20 < x \leqslant 40$	26	20
$40 < x \leqslant 60$	50	30
$60 < x \leqslant 80$	56	55
$80 < x \leqslant 100$	4	25

(a) Copy and complete the cumulative frequency table below.

Mark (x)	Number of candidates	
	English	Mathematics
$x =$ 0	0	0
$x \leqslant$ 20	4	
$x \leqslant$ 40		
$x \leqslant$ 60		
$x \leqslant$ 80		
$x \leqslant$ 100	140	

[2]

(b) Using a scale of 2 cm to represent 20 marks, draw a horizontal x-axis for $0 \leqslant x \leqslant 100$.
Using a scale of 2 cm to represent 20 pupils, draw a vertical axis for values from 0 to 140.
On your axes, draw and label both smooth cumulative frequency curves to illustrate this
information. [3]

(c) Use your curves to find

   (i) the upper quartile mark for English, [1]

  (ii) the interquartile range for English, [1]

 (iii) the median mark for English and the median mark for Mathematics. [1]

(d) State, with a reason, which you think is the easier test. [1]

(e) One student is chosen at random.
It may be assumed that the marks gained in the two subjects are independent.

Expressing each answer as a fraction in its lowest terms, calculate the probability that the
student gains

   (i) more than 60 marks on both papers, [1]

  (ii) more than 80 marks on one paper, but not on the other. [2]

Answers: **(a)** English 30, 80, 136; Mathematics 10, 30, 60, 115, 140; **(c)(i)** 64 to 68, **(ii)** 20 to 26, **(iii)** 56 to 58   **N04/2/Q11**
and 63 to 65; **(e)(i)** $\frac{12}{49}$, **(ii)** $\frac{193}{980}$.

24

Answer the whole of this question on a sheet of graph paper.

The table shows the number of cars owned by each of 25 families.

2	0	3	4	1
0	1	1	2	3
2	3	6	1	0
1	2	0	3	2
3	4	1	2	1

(a) Draw a bar chart to represent the information in the table. [2]

(b) Find

    (i)   the median number of cars, [1]

    (ii) the modal number of cars, [1]

    (iii) the mean number of cars. [1]

(c) A **family** is chosen at random.

    Find the probability that it owns 3 cars. [1]

(d) Two families are chosen at random.

    Find the probability that one family owns 2 cars and the other owns 4 cars. [2]

(e) A **car** is chosen at random.

    Find the probability that it belongs to a family which owns 2 cars. [2]

Answers: **(a)** 4, 7, 6, 5, 2, 0 and 1; **(b)(i)** 2, **(ii)** 1, **(iii)** 1.92; **(c)** 1/5; **(d)** 1/25; **(e)** 1/4.   **N05/2/Q4**

25

11 **Answer the whole of this question on a sheet of graph paper.**

The diagram shows the histogram which represents the heights of the pupils in a small school.

Height (*h* cm)
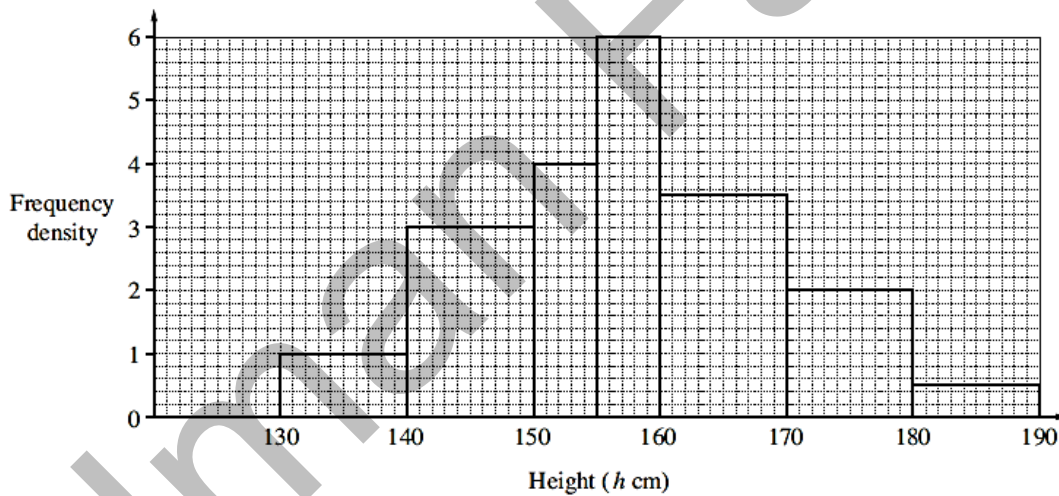
(a) (i) **On your graph paper**, copy and complete this frequency table that represents the distribution.

130 < h ≤ 140	140 < h ≤ 150	150 < h ≤ 155	155 < h ≤ 160	160 < h ≤ 170	170 < h ≤ 180	180 < h ≤ 190
10						

[2]

(ii) Hence copy and complete this cumulative frequency table that represents the distribution.

Height (h cm)	$\leqslant 130$	$\leqslant 140$	$\leqslant 150$	$\leqslant 155$	$\leqslant 160$	$\leqslant 170$	$\leqslant 180$	$\leqslant 190$
Cumulative frequency	0	10						

[1]

(b) Using a scale of 2 cm to represent 10 cm, draw a horizontal h-axis for $130 \leqslant h \leqslant 190$.
Using a scale of 1 cm to represent 10 pupils, draw a vertical axis.

On your axes, draw a smooth cumulative frequency curve to illustrate the information. [3]

(c) Use your graph to find

    (i) the median height of the pupils, [1]

    (ii) the lower quartile height, [1]

    (iii) the interquartile range. [1]

Answer: **(a)(i)** 30 20 30 35 20 5 **(ii)** 40 60 90 125 145 150   **N06/2/Q11**

(c)(i) 157 to 158 **(ii)** 149 to 149.6 **(iii)** 16.0 to 17.0 **(d)** 2/5 **(e)** 10/449

742

26   **10**   **Answer the whole of this question on a sheet of graph paper.**

Potatoes are sold in sacks.
One sack, picked at random, contained 260 potatoes.
The masses, in grams, of the potatoes in this sack are summarised in the table below.

Mass (m grams)	$50 < m \leqslant 100$	$100 < m \leqslant 150$	$150 < m \leqslant 200$	$200 < m \leqslant 250$	$250 < m \leqslant 300$	$300 < m \leqslant 350$
Frequency	4	56	84	76	36	4

(a) (i)   Calculate an estimate of the total mass of the potatoes in this sack.   [2]

   (ii)   Calculate an estimate of the mean mass, in grams, of a potato.   [1]

(b) (i)   **Copy and complete** the cumulative frequency table given below.

Mass (m grams)	$m \leqslant 50$	$m \leqslant 100$	$m \leqslant 150$	$m \leqslant 200$	$m \leqslant 250$	$m \leqslant 300$	$m \leqslant 350$
Cumulative frequency	0	4	60				260

[1]

   (ii)   Using a scale of 2 cm to represent 50 grams, draw a horizontal axis for masses between 0 and 350 grams.
   Using a scale of 2 cm to represent 50 potatoes, draw a vertical axis for values from 0 to 300.

   On your axes, draw a smooth cumulative frequency curve to illustrate this information.   [3]

   (iii)   Use your curve to find

      (a)   the median,   [1]

      (b)   the interquartile range.   [2]

(c)   The organisers of a barbecue expect to sell 500 baked potatoes.
   Each potato should have a mass greater than 200 g.

   Estimate the number of sacks of potatoes they will need.   [2]

Answers: (a)(i) 50300 g,   (ii) 193,   (b)(i) 144 220 256,   (ii) All seven points plotted and joined with a smooth curve,   (iii)(a) 190.0 to 197.5 g,   (b) 72.5 to 82.5 g,   (c) 5.   **N07/2/Q10**

27 **4** **Answer the whole of this question on a sheet of graph paper.**

The heights of 120 children were measured.
The results are summarised in the table below.

Height (h cm)	$135 < h \leqslant 140$	$140 < h \leqslant 145$	$145 < h \leqslant 150$	$150 < h \leqslant 155$	$155 < h \leqslant 160$	$160 < h \leqslant 180$
Frequency	15	20	25	30	20	10

   (a) Using a scale of 1 cm to represent 5 cm, draw a horizontal axis for heights from 135 cm to 180 cm. Using a scale of 2 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 6 units.

      On your axes, draw a histogram to represent the information in the table. [3]

   (b) Estimate how many children have heights greater than 170 cm. [1]

   (c) One child was chosen at random.

      Find the probability that the height of this child was less than or equal to 140 cm.
      Give your answer as a fraction in its lowest terms. [1]

   (d) Two children were chosen at random.

      Find the probability that they both had heights in the range $150 < h \leqslant 155$. [2]

Answers: **(a)** Histogram, correct column widths, frequency densities 3,4,5,6,4,0.5, **(b)** 5, **(c)** $\frac{1}{8}$,   **N08/2/Q4**

     (d) $\frac{870}{14280}$ or equivalent.

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28 **10** **Answer the whole of this question on a sheet of graph paper.**

80 electric light bulbs of brand A were tested to find how long each bulb lasted.
The results are summarised in the table below.

Time (t hours)	$t \leqslant 50$	$50 < t \leqslant 100$	$100 < t \leqslant 150$	$150 < t \leqslant 200$	$200 < t \leqslant 250$	$250 < t \leqslant 300$	$300 < t \leqslant 350$	$350 < t \leqslant 400$
Number of bulbs	1	2	6	34	26	8	2	1

   (a) Copy and complete the following cumulative frequency table.

Time (t hours)	$t \leqslant 50$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$	$t \leqslant 250$	$t \leqslant 300$	$t \leqslant 350$	$t \leqslant 400$
Number of bulbs	1	3						80

      [1]

   (b) Using a horizontal scale of 2 cm to represent 50 hours and a vertical scale of 2 cm to represent 10 bulbs, draw a smooth cumulative frequency curve for these brand A bulbs. [3]

   (c) Use your graph to estimate

(i) the median, [1]

(ii) the 10th percentile. [1]

(d) 80 brand B bulbs were also tested and a report on the test gave the following information.

3 bulbs lasted 50 hours or less.
No bulbs lasted more than 350 hours.
The median time was 250 hours.
The upper quartile was 275 hours.
The interquartile range was 75 hours.

On the same axes, draw a smooth cumulative frequency curve for the brand B bulbs. [3]

(e) Use your graphs to estimate the number of bulbs that lasted 260 hours or less

(i) for brand A, [1]

(ii) for brand B. [1]

(f) Which brand of bulb is more likely to last longer than 250 hours?
Justify your answer. [1]

N09/2/Q10

29 (a) The results of a survey of the number of televisions in 40 households are given in the table below.

Number of televisions	1	2	3	4
Number of households	8	17	12	3

(i) Calculate the mean number of televisions per household. [2]

(ii) The survey was extended to include 5 more households.
It was found that none of these had a television.

For the 45 households, calculate the mean number of televisions per household. [1]

(b) 120 children were asked which colour they liked best.
59 said red, 21 said blue and the rest said yellow.

(i) Using a radius of 4 cm, draw an accurate pie chart to show these results. [3]

(ii) Out of the children who liked blue best, $\frac{5}{7}$ were wearing blue socks.

How many of the children who liked blue best were not wearing blue socks? [1]

N10/21/Q6

30

5   The table shows the distribution of the masses of 90 apples.

Mass (m grams)	$60 < m \leqslant 80$	$80 < m \leqslant 90$	$90 < m \leqslant 95$	$95 < m \leqslant 100$	$100 < m \leqslant 110$	$110 < m \leqslant 130$
Frequency	10	16	20	21	22	1

(a)   In which interval does the median lie?   [1]

(b)   Calculate an estimate of the mean.   [3]

(c)   A histogram is drawn to represent this information.

   (i)   Calculate the frequency density of the interval $90 < m \leqslant 95$.   [1]

   (ii)   The rectangle representing the apples with masses in the interval $80 < m \leqslant 90$ has width 2 cm and height 4 cm.

      Find the width and height of the rectangle representing the apples with masses in the interval $90 < m \leqslant 95$.   [2]

Answers: (a) $90 < m \leqslant 95$,   (b) 93.2 g   (c)(i) 4   (ii) 1 cm and 10 cm   **N10/22/Q5**

10 The distribution of the masses of 140 eggs is given in the table below.

Mass (m grams)	$35 < m \leq 40$	$40 < m \leq 45$	$45 < m \leq 50$	$50 < m \leq 55$	$55 < m \leq 60$	$60 < m \leq 70$
Number of eggs	15	20	30	35	28	12

(a) Using a scale of 1 cm to represent 5 grams, draw a horizontal axis for $30 \leq m \leq 70$.
Using a scale of 1 cm to 1 unit, draw a vertical axis to represent frequency density.

Draw a histogram to represent the information in the table.

[3]
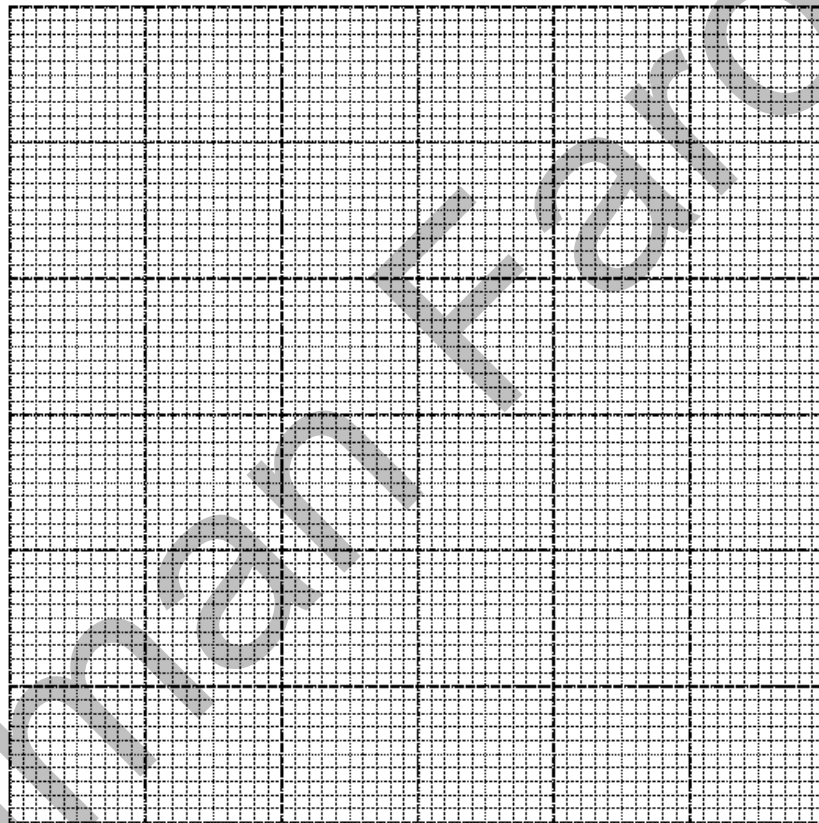
(b) (i) Complete the cumulative frequency table below.

Mass (m grams)	$m \leq 35$	$m \leq 40$	$m \leq 45$	$m \leq 50$	$m \leq 55$	$m \leq 60$	$m \leq 70$
Cumulative frequency	0	15					140

[1]

(ii) On the grid on the next page, draw a smooth cumulative frequency curve to represent this information.
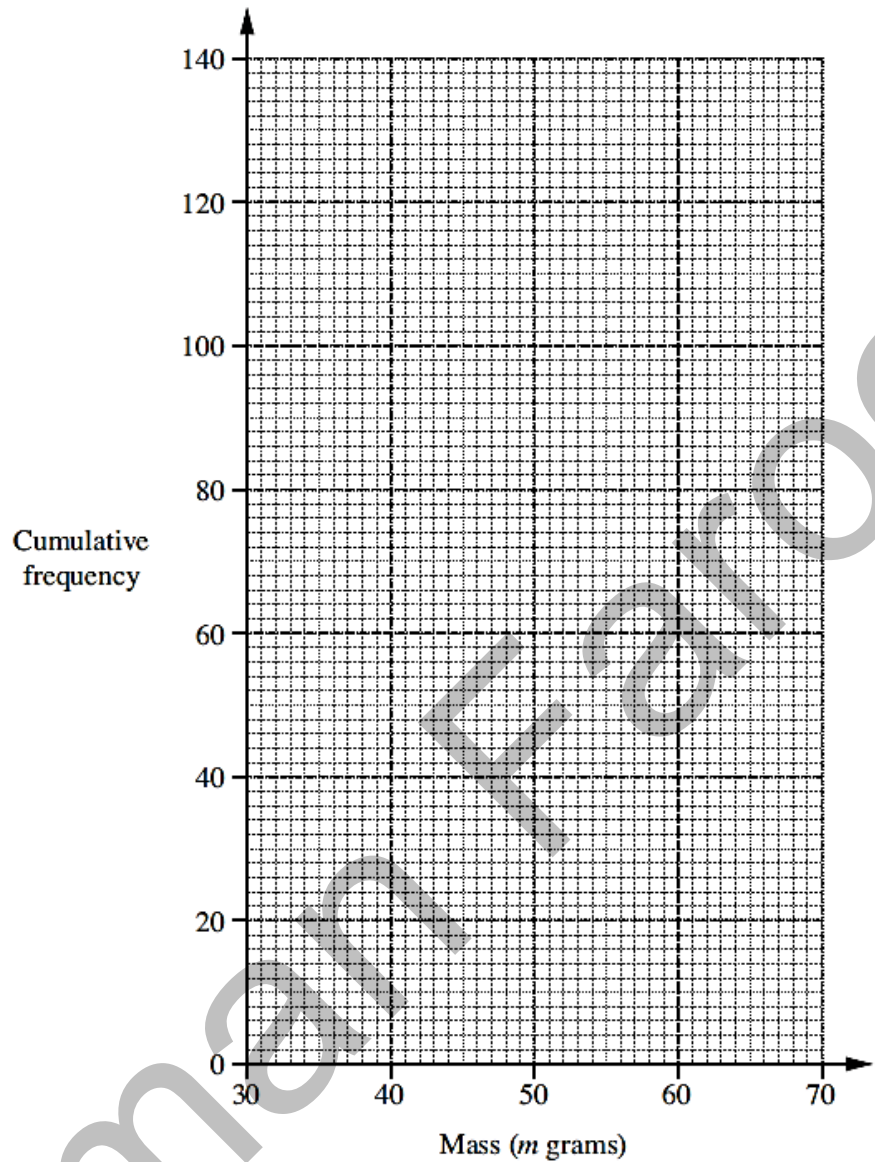
(c) Use your graph to find

    (i) the median mass of the eggs,   *Answer* .... g [1]

    (ii) the interquartile range.   *Answer* .... g [2]

(d) The 12 eggs with the greatest mass are classed as extra large.
The 30 eggs with the least mass are classed as small.

    Use your graph to find an estimate of the smallest difference in mass between an extra large egg and a small egg.   *Answer* .... g [2]

Answers: **(b)(i)** 35, 65, 100, 128 **(c)(i)** 50.5 **(ii)** 10.5 **(d)** 16.5   **N11/21/Q10**

11 **(a)** A sports club has 120 members.
The cumulative frequency table for their ages is shown below.

Age (x years)	$x \leqslant 5$	$x \leqslant 15$	$x \leqslant 25$	$x \leqslant 35$	$x \leqslant 45$	$x \leqslant 55$	$x \leqslant 65$
Cumulative frequency	0	12	30	60	96	114	120

(i) On the grid on the next page
draw a horizontal x-axis for $0 \leqslant x \leqslant 70$, using a scale of 2 cm to represent 10 years
and a vertical axis from 0 to 120, using a scale of 2 cm to represent 20 members.

On your axes draw a smooth cumulative frequency curve to illustrate the information
in the table. [3]

(ii) Find the upper quartile age. *Answer* ....years [1]

(iii) Find the interquartile range of the ages. *Answer* ....years [1]

(iv) Members who are not more than 15, and members who are over 50, pay reduced fees.
Use your graph to find an estimate of the number of members who pay reduced fees.

Answer .... [1]
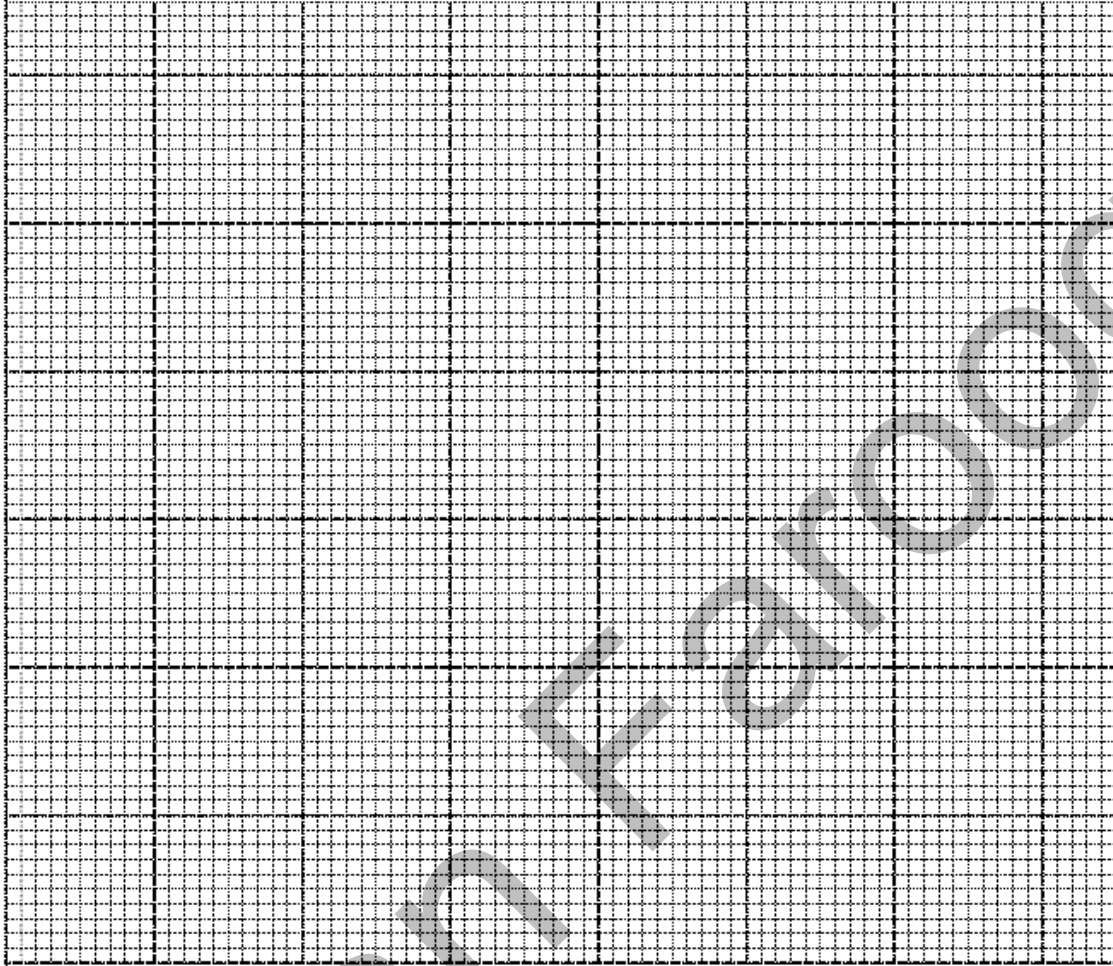
Answers: (a)(ii) 43 (iii) 18 (iv) 26 (b)(i) $\frac{8}{12}$ and $\frac{4}{12}$ for the first disc, $\frac{7}{11}, \frac{4}{11}$ and $\frac{8}{11}$ for the second disc **N11/22/Q11**

(ii)(a) $\frac{1}{11}$ (b) $\frac{10}{11}$ (iii) $\frac{1}{55}$

33

6 The journey times of 80 drivers are summarised in the table.

Time (t minutes)	$60 < t \leqslant 80$	$80 < t \leqslant 90$	$90 < t \leqslant 95$	$95 < t \leqslant 100$	$100 < t \leqslant 110$	$110 < t \leqslant 130$
Number of drivers	4	10	14	20	24	8

(a) Calculate an estimate of the mean journey time.

Answer .... minutes [3]

(b) (i) A driver is chosen at random.

Find the probability that the journey time for this driver is 95 minutes or less.

Answer ....[1]

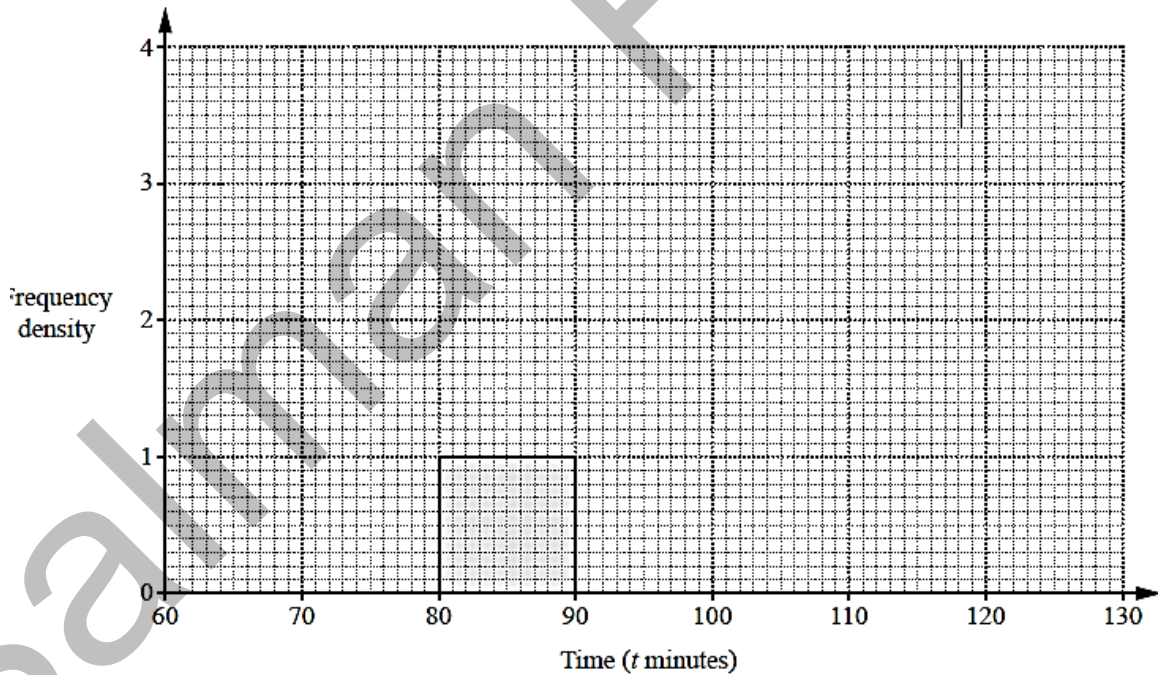**(ii)** Two drivers are chosen at random without replacement.

Calculate the probability that both their journey times are more than 100 minutes.

13 *Answer* ....[2]

(c) Complete the histogram to represent the information in the table.

Frequency density

Time (t minutes)

[3]

Answer: (a) 98.2 (b)(i) $\dfrac{28}{80}$ (ii) $\dfrac{992}{6320}$

N12/21/Q6

751

34   **4**   The table shows the distribution of the masses of 100 babies at birth.

Mass (x kg)	$1.5 < x \leqslant 2$	$2 < x \leqslant 2.5$	$2.5 < x \leqslant 3$	$3 < x \leqslant 3.5$	$3.5 < x \leqslant 4$	$4 < x \leqslant 4.5$	$4.5 < x \leqslant 5$
Number of babies	3	12	20	24	25	14	2

(a) Write down the modal class.

Answer .... [1]

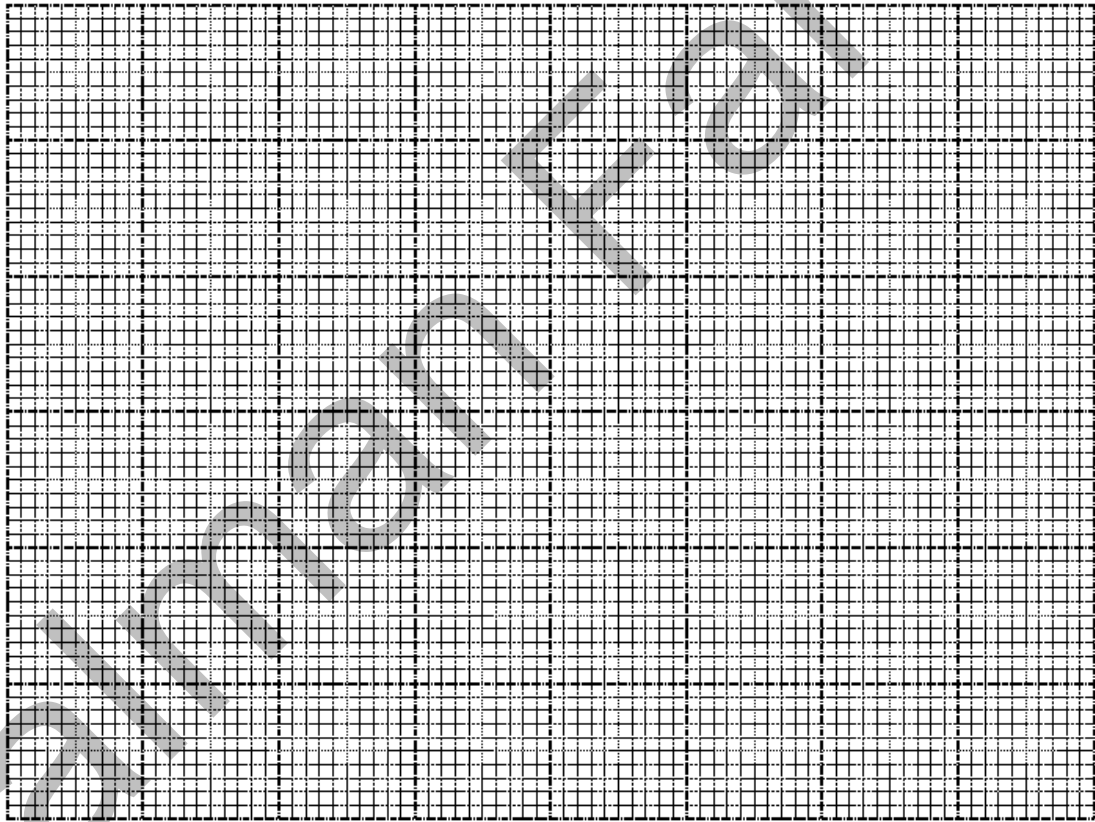**(b)** For this part of the question use the grid below.
Using a scale of 4 cm to represent 1 kg, draw a horizontal *x*-axis for $1 \leqslant x \leqslant 5$.
Using a scale of 2 cm to represent 5 babies, draw a vertical axis for frequency from 0 to 30.

Using your axes, draw a frequency polygon to represent these results.

[2]

752

(c) (i) Complete the cumulative frequency table below.

Mass (x kg)	$x \leqslant 2$	$x \leqslant 2.5$	$x \leqslant 3$	$x \leqslant 3.5$	$x \leqslant 4$	$x \leqslant 4.5$	$x \leqslant 5$
Cumulative frequency	3	15					100

[1]

(ii) On the grid below draw a smooth cumulative frequency curve to represent these results.
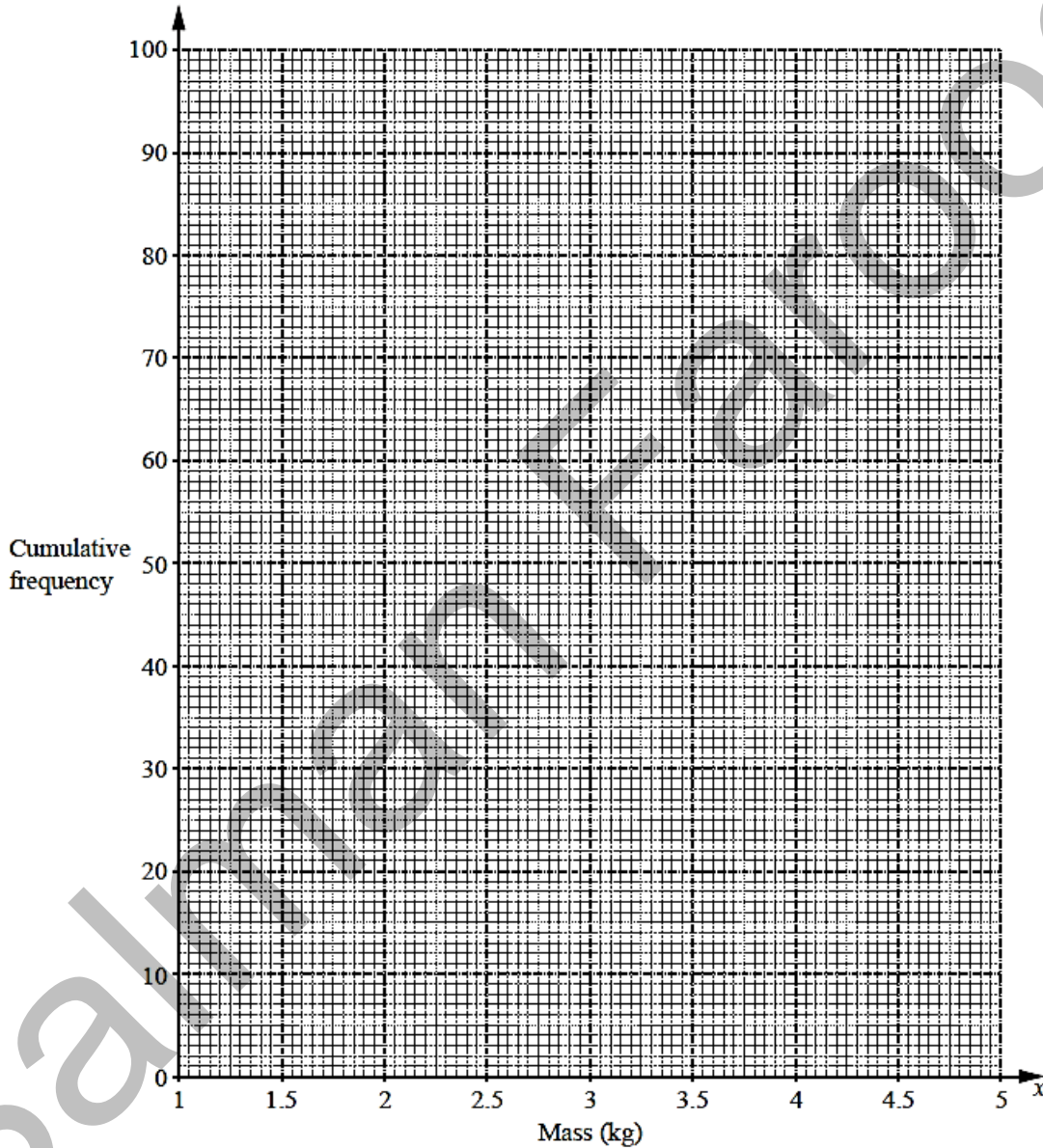
[2]

(d) Use your curve to estimate

(i) the median mass,

Answer ....kg [1]

(ii) the 10th percentile.

Answer ....kg [1]

753

35   **4**   (a) The histogram represents the distribution of the masses, in grams, of individual apples in a box.
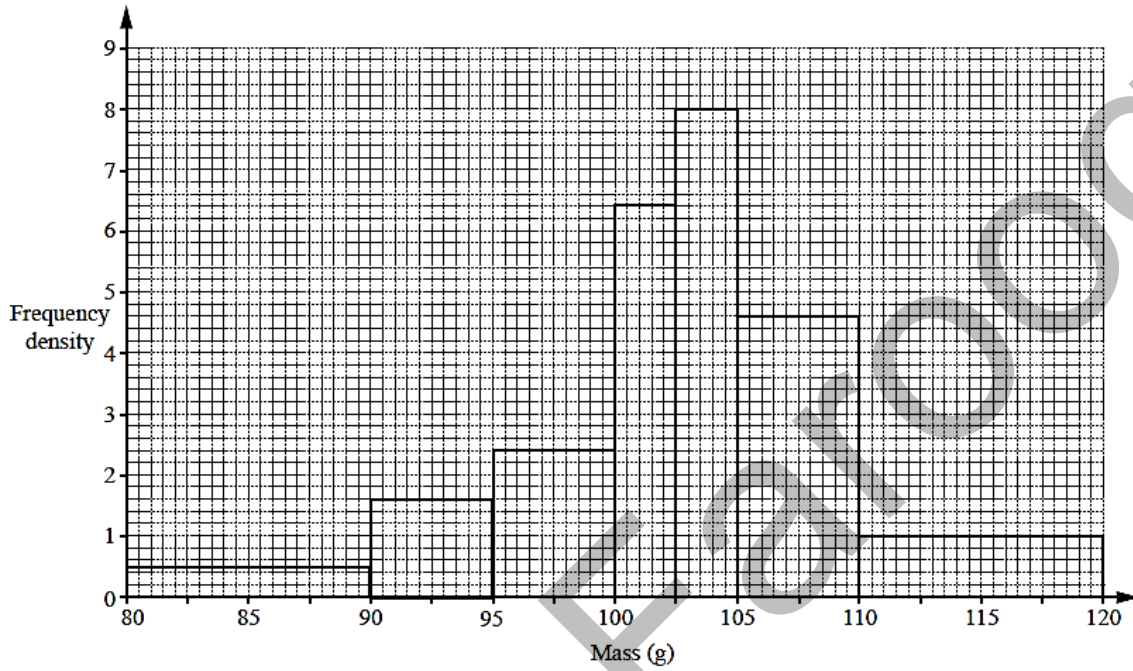
This information is summarised in the table below.

Mass (m g)	Frequency
80 < m ≤ 90	5
90 < m ≤ 95	8
95 < m ≤ 100	p
100 < m ≤ 102.5	q
102.5 < m ≤ 105	20
105 < m ≤ 110	23
110 < m ≤ 120	10

Calculate p and q.   *Answer* p = ... q = ... [2]

(b) The mass of each plum in a box is recorded correct to the nearest 5 grams.

Mass (to the nearest 5 g)	Frequency
10 – 15	6
20 – 25	18
30 – 35	25
40 – 45	10
50 – 55	1

(i) Calculate an estimate of the mean mass of a plum.

Answer .... g [3]

(ii) Calculate the upper bound for the total mass of plums in the box.

Answer .... g [2]

Answers: (a) $p = 12$ $q = 16$; (b)(i) 29.5; (ii) 2070.   **N14/21/Q4**

36 ⟩ The cumulative frequency graph for the lengths of the 50 tracks on Abi's MP3 player is shown below.

Length of track (minutes : seconds)
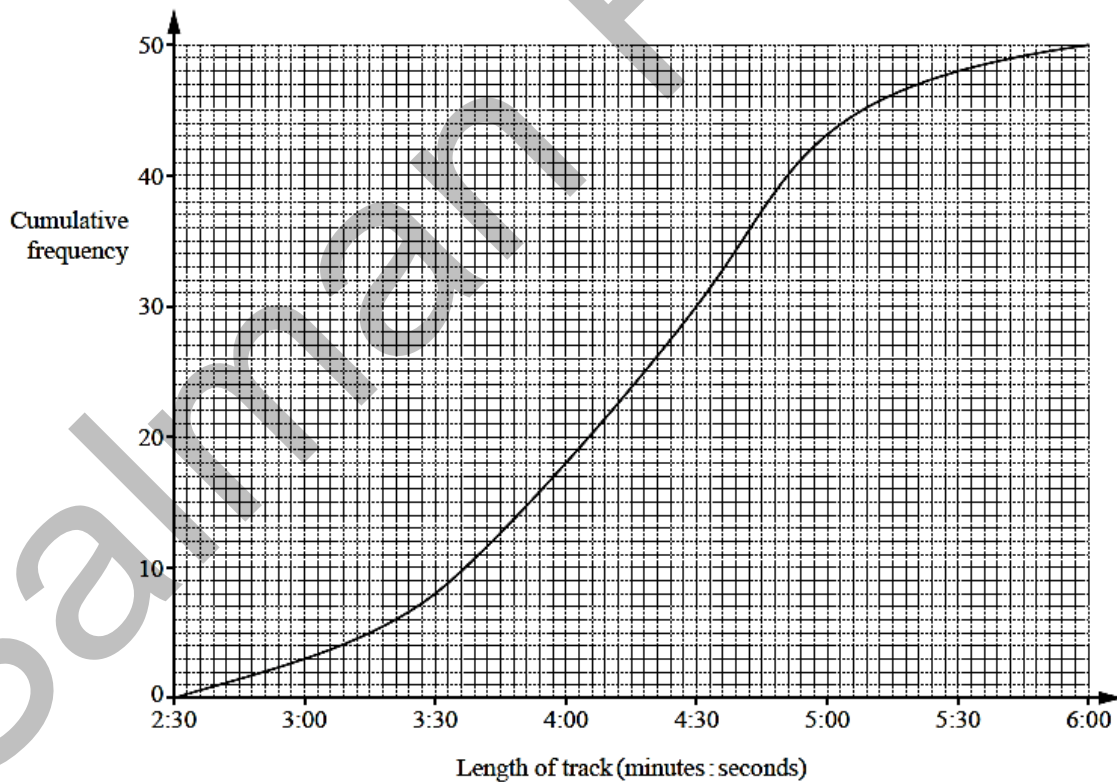
(a) Use the graph to find

    (i) the median,

                                                                   Answer .... minutes .... seconds [1]

    (ii) the interquartile range.

                                                                   Answer .... minutes .... seconds [2]

(b) Use the information on the graph to complete the frequency table for the length of the tracks.

Length (minutes : seconds)	Frequency
2:30 < length ⩽ 3:00	3
3:00 < length ⩽ 3:30	5
3:30 < length ⩽ 4:00	
4:00 < length ⩽ 4:30	
4:30 < length ⩽ 5:00	
5:00 < length ⩽ 5:30	
5:30 < length ⩽ 6:00	

                                                                            [2]

(c) Abi plays three tracks from her MP3 player with no break between them.

Given that no track is repeated, what is the maximum possible length of time taken to play these tracks?

                                                     Answer .... minutes .... seconds [2]

(d) Abi travels on a train from station A to station F.
The exact times the train arrives at and leaves stations A to F are shown below.

Station	A	B	C	D	E	F
Arrive	–	10 03	10 06	10 11	10 15	10 21
Depart	09 58	10 04	10 07	10 12	10 16	–

    (i) How many minutes did her journey take?   *Answer* .... [1]

    (ii) Abi starts playing tracks at random from her MP3 player as she leaves station A.

    What is the probability that the first track is still playing when she arrives at station B?

                                                        Answer .... [2]

(e) Abi plays two different tracks at random from her MP3 player.

What is the probability that neither track is longer than 3 minutes 30 seconds?

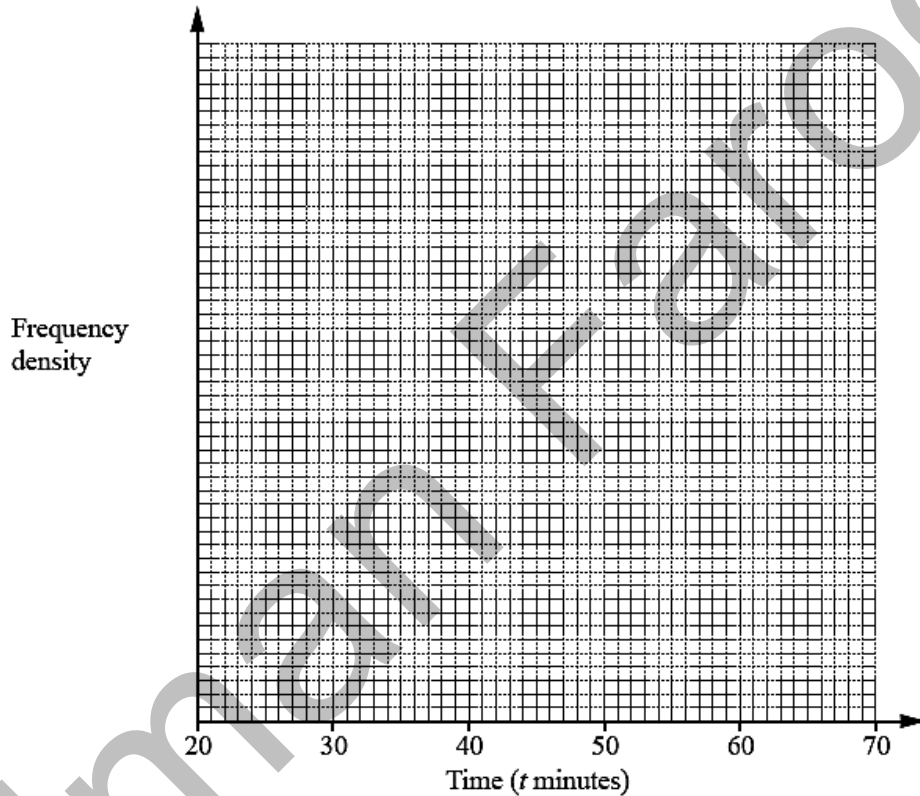   *Answer* .... [2]

Answers: (a)(i) 4 minutes 18 seconds; (ii) 1 minute 0 seconds; (b) 10, 12, 13, 5, 2;
(c) 17 minutes 30 seconds; (d)(i) 23; (ii) $\frac{7}{50}$; (e) $\frac{4}{175}$

37   **(a)**   The times taken by 135 runners to complete a cross-country course were recorded.
The results are summarised in the table.

Time (t minutes)	$20 < t \leqslant 30$	$30 < t \leqslant 35$	$35 < t \leqslant 40$	$40 < t \leqslant 50$	$50 < t \leqslant 70$
Number of runners	15	30	40	35	15

(i)   On the grid, draw a histogram to represent this information.

Frequency density

Time (t minutes)

[3]

(ii)   Calculate an estimate of the mean time.

Answer .... minutes [3]

Answers: (a)(i) correct histogram (ii) 39.4 (b)(i) $\frac{33}{95}$ (ii) $\frac{48}{95}$ (iii) 12 (iv) $\frac{91}{190}$
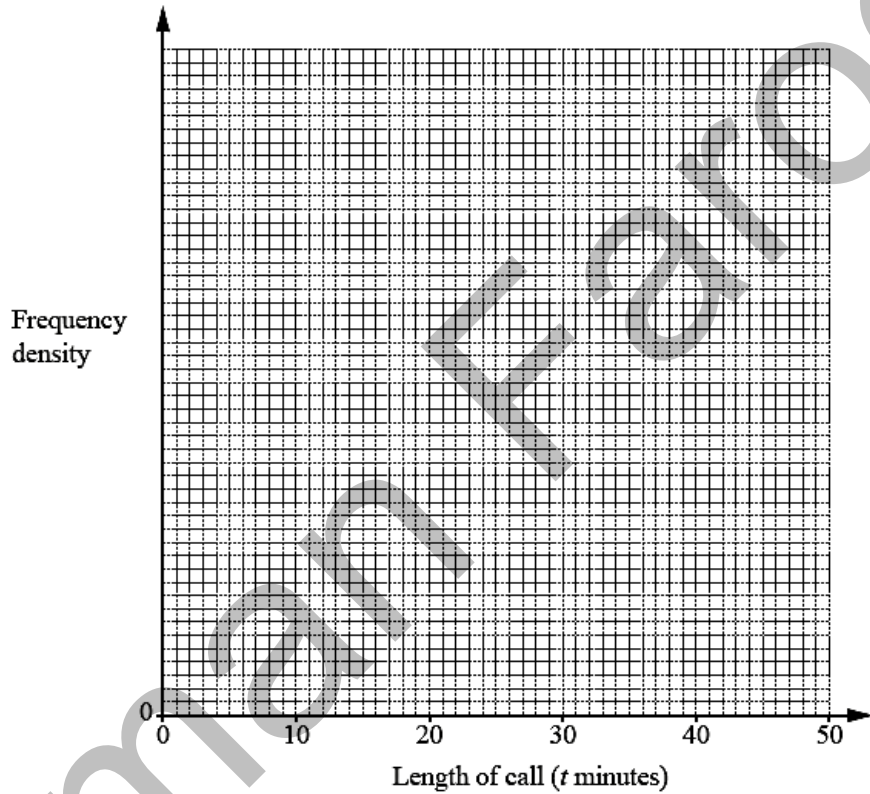
38 Sunil recorded the lengths, in minutes, of the 150 phone calls he made one month.
His results are summarised in the table.

Length of call (t minutes)	$0 < t \leqslant 5$	$5 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 50$
Frequency	35	42	30	28	15

(a) Calculate an estimate of the mean length of a call.   *Answer* .... minutes [3]

(b) On the grid below, draw a histogram to represent this data.

Frequency density

Length of call (t minutes)

[3]

(c) Find an estimate for the percentage of Sunil's calls that were longer than 25 minutes.

Answer .... % [2]

Answers: (a) 14.35 (b) Correct histogram (c) 19.3   **N17/21/Q2**

758

Vectors Paper 1

1

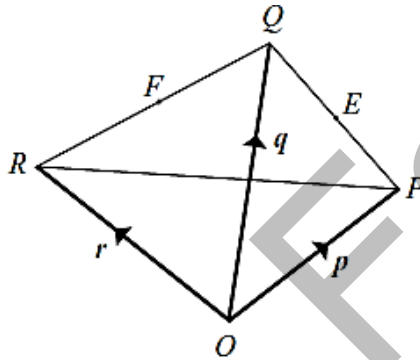$$a = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \qquad b = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \qquad c = \begin{pmatrix} u \\ 10 \end{pmatrix}.$$

(a) Express $2a + b$ as a column vector.   *Answer (a)* .... [1]

(b) Given that the vector **c** is parallel to the vector **a**, calculate the value of u.

                                                    (b) $u = $ .... [1]

Answers: (a) $\begin{pmatrix} -7 \\ 10 \end{pmatrix}$; (b) -5.

2

In the diagram,

$\overrightarrow{OP} = p$, $\overrightarrow{OQ} = q$ and $\overrightarrow{OR} = r$.

The midpoints of PQ and QR are E and F, respectively.

(a) Express, as simply as possible, in terms of **p** and/or **q**,

   (i) $\overrightarrow{PE}$,   *Answer (a)*(i) $\overrightarrow{PE} = $ .... [1]

   (ii) $\overrightarrow{OE}$.   (ii) $\overrightarrow{OE} = $ .... [1]

(b) Hence write down $\overrightarrow{OF}$.   (b) $\overrightarrow{OF} = $ .... [1]

(c) Find $\overrightarrow{EF}$.   (c) $\overrightarrow{EF} = $ .... [1]

(d) Write down two facts about EF and PR.

   (d) ....[1]

Answers: (a)(i) $\frac{1}{2}(q - p)$, (ii) $\frac{1}{2}(p + q)$; (b) $\frac{1}{2}(q + r)$; (c) $\frac{1}{2}(r - p)$; (d) The length of EF is half that of PR
and the lines are parallel

3 On the grid in the answer space, $\overrightarrow{OP} = \text{p}$ and $\overrightarrow{OQ} = \text{q}$.

(a) Given that $\overrightarrow{OR} = \text{p} - \text{q}$, mark the point R clearly on the grid.

(b) The point S is shown on the grid.

Given that $\overrightarrow{OS} = \text{q} + h\text{p}$, find h.
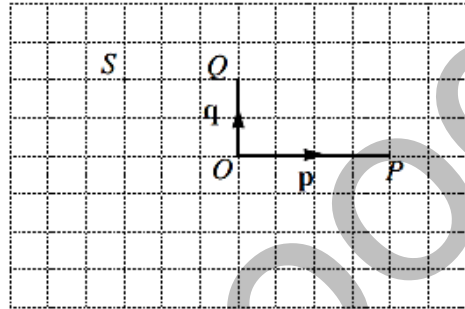
Answer (a)

[1]

(b) h =...[1]

Answer: (a) R marked 2 squares below P   (b) - 0.75

4
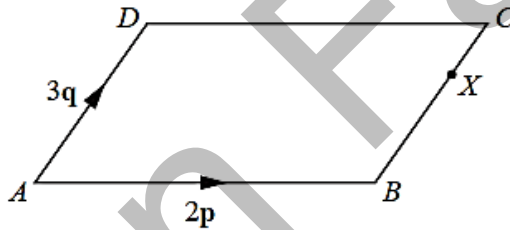
$ABCD$ is a parallelogram.
X is the point on BC such that $BX : XC = 2 : 1$.
$\overrightarrow{AB} = 2\text{p}$ and $\overrightarrow{AD} = 3\text{q}$.

Find, in terms of **p** and **q**,

(a) $\overrightarrow{AC}$,

Answer (a) $\overrightarrow{AC}$ =...........................[1]

(b) $\overrightarrow{AX}$,

Answer (b) $\overrightarrow{AX}$ = ... [1]

(c) $\overrightarrow{XD}$.

Answer (c) $\overrightarrow{XD}$ = ... [1]

Answer: (a) 2p +3q   (b) 2p + 2q   (c) - 2p + q

5

$\text{c} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\text{d} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

(a) Calculate $2\text{c} - \text{d}$.

Answer $\begin{pmatrix} \\ \end{pmatrix}$ [1]

(b) Calculate $|\text{d}|$.

Answer ....[1]

Answer. (a) $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$ (b) 10

6
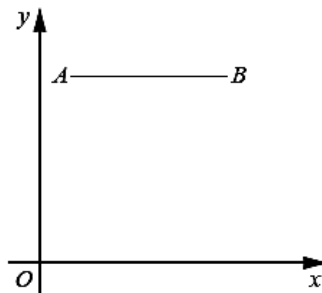
A is the point $(1, 7)$ B is the point $(6, 7)$
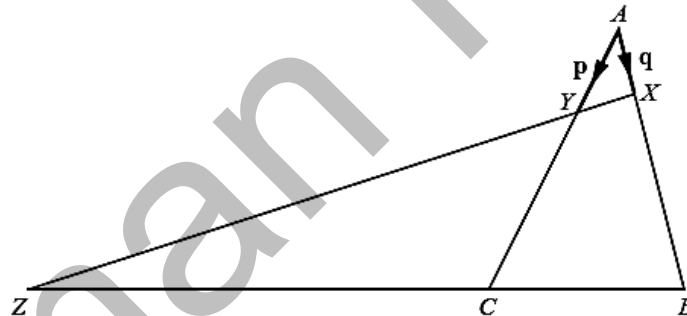
The line AB is mapped onto the line PQ by the translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$.

(a) Find the coordinates of Q.   *Answer* (................. , ...) [1]

(b) What special type of quadrilateral is $ABQP$?   *Answer* .... [1]

(c) Find the area of the quadrilateral $ABQP$.   *Answer* .... units^2 [1]

Answers: **(a)** $(6, 2)$ **(b)** Square **(c)** 25

7

In the diagram,

X is the point on AB where $AX = \frac{1}{4}AB$,

Y is the point on AC where $AY = \frac{1}{3}AC$,

Z is the point on BC produced where $CZ = 2BC$.

$\overrightarrow{AY} = \mathbf{p}$ and $\overrightarrow{AX} = \mathbf{q}$.

(a) Express, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\overrightarrow{XY}$,   *Answer* $\overrightarrow{XY}$ = ... [1]

(ii) $\overrightarrow{BC}$,   *Answer* $\overrightarrow{BC}$ = ... [1]

(iii) $\overrightarrow{XZ}$.

Answer $\overrightarrow{\quad}$ = ... [2]

(b) Hence find *XY* : *YZ*.

Answer .... : .... [1]

8
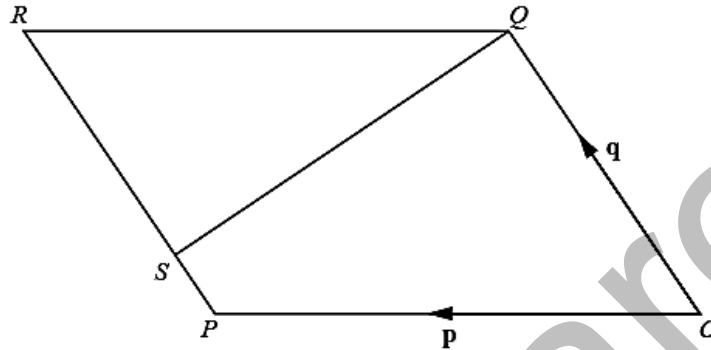
OPRQ is a parallelogram and *S* is a point on *PR* such that *PS* : *SR* = 1 : 3.

$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.

(a) (i) Express $\overrightarrow{PQ}$ in terms of **p** and/or **q**.   *Answer* .... [1]

(ii) Express $\overrightarrow{QS}$, as simply as possible, in terms of **p** and/or **q**.

Answer .... [1]

(b) *T* is a point on *QS* extended such that $\overrightarrow{QT} = \frac{4}{3}\overrightarrow{QS}$.

(i) Express $\overrightarrow{PT}$, as simply as possible, in terms of **p** and/or **q**.

Answer .... [2]

(ii) What can you conclude about the points *O*, *P* and *T* ?

.... [1]

9   In the diagram, $OABC$ is a parallelogram, $\vec{OA} = 4\mathbf{p} - \mathbf{q}$ and $\vec{OC} = \mathbf{p} + 5\mathbf{q}$.

(a) Express, as simply as possible, in terms of **p** and **q**,

   (i) $\vec{BC}$,
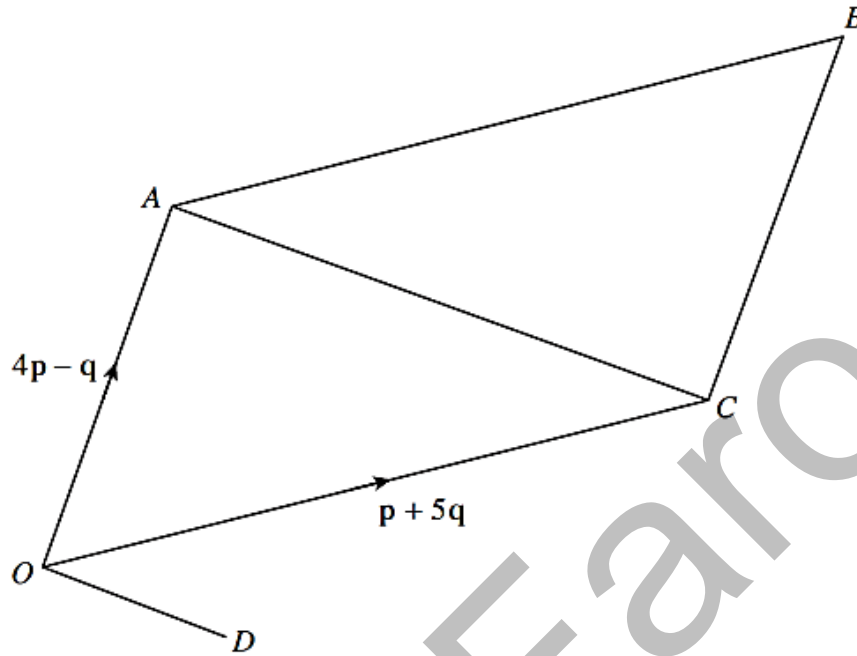
   (ii) $\vec{AC}$.

(b) D is the point such that $\vec{OD} = -\mathbf{p} + 2\mathbf{q}$.

   (i) Explain why $\vec{AC}$ is parallel to $\vec{OD}$.

   (ii) Given that the area of triangle OAC is 18 square units, find the area of triangle OCD.

Answer (a)(i) $\vec{BC} =$ .... [1]

(ii) $\vec{AC} =$ .... [1]

Answer (b)(i) ....

.... [1]

(ii) .... units^2 [1]

N03/Q20

10

$\overrightarrow{AB} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$.

(a) Express $\overrightarrow{AC}$ as a column vector.

Answer (a) $\overrightarrow{AC} = \begin{pmatrix} \\ \end{pmatrix}$ [1]

(b) It is given that $\overrightarrow{CD} = \begin{pmatrix} -11 \\ h \end{pmatrix}$.

Find the two possible values of h which will make $ABCD$ a trapezium.

You may use the grid below to help you with your investigation.

A
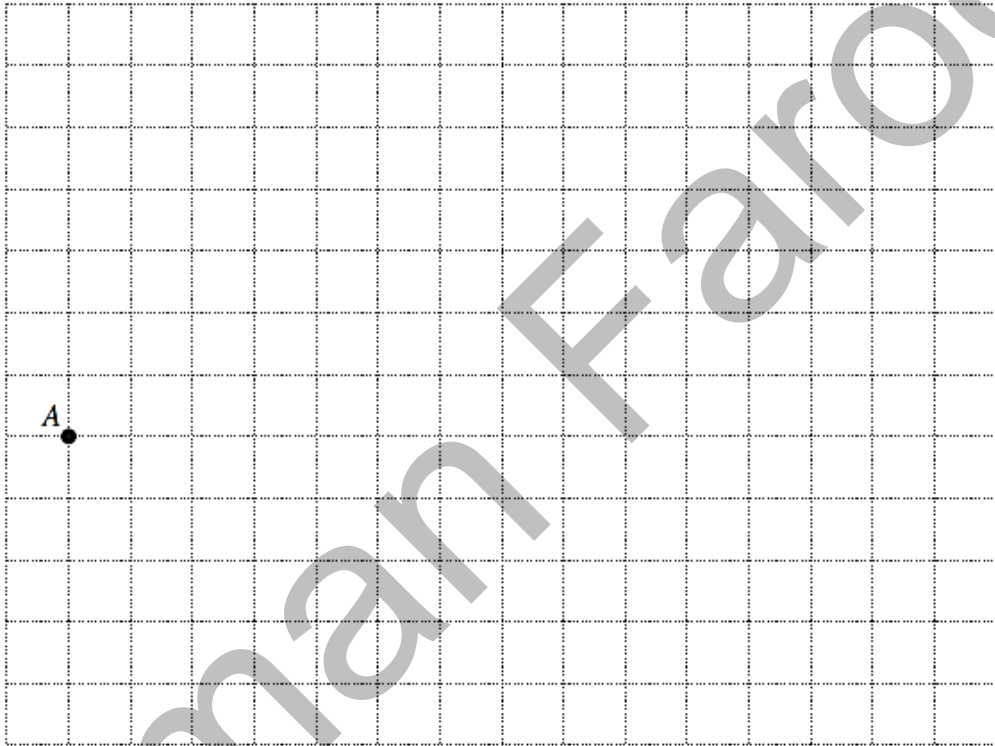
Answer (b) h = ... and ....[2]

N04/1/Q13

11

In the diagram, $\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OC} = 2\mathbf{c}$ and $\overrightarrow{CB} = \mathbf{a}$.

(a) Express $\overrightarrow{BA}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

(b) $\overrightarrow{OP} = 2\mathbf{a} - \frac{4}{3}\mathbf{c}$.

Explain why $\overrightarrow{OP}$ is parallel to $\overrightarrow{BA}$.

(c) Find $\dfrac{\text{area of triangle } OBA}{\text{area of triangle } OPA}$.

Answer: (a) $3a - 2c$; (b) Establishing $k\,\overrightarrow{OP} = \ell\,\overrightarrow{BA}$; (c) $\dfrac{3}{2}$. **N06/1/Q12**

12

$$\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$
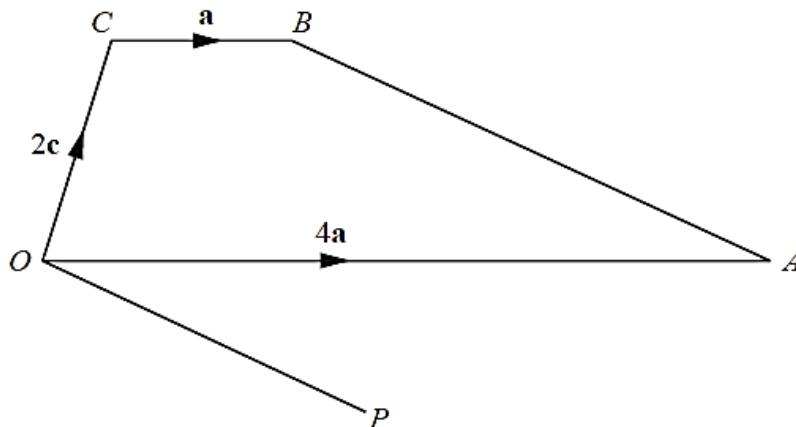
(a) Express $\mathbf{a} + 2\mathbf{b}$ as a column vector.

Answer (a) $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} \ \\ \ \end{pmatrix}$ [1]

(b) (i) Find $|\mathbf{a}|$.

Answer (b)(i) $|\mathbf{a}| = $ ....[1]

(ii) Given that $\dfrac{|\mathbf{b}|}{|\mathbf{a}|} = \sqrt{n}$, where n is an integer, find the value of n.

Answer (b)(ii) $n = $ ....[1]

Answers: (a) $\begin{pmatrix} 1 \\ 10 \end{pmatrix}$ (b)(i) 5   (ii) 2 **N09/1/Q15**

13

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

(a) Find $\left| \overrightarrow{AB} \right|$.   *Answer* (a) .... [1]

(b) A is the point $(0, 2)$.

    (i) The equation of the line AB may be written $3y + 4x = k$.
        Find the value of k.   *Answer* (b)(i) $k =$ .... [1]

    (ii) Find the coordinates of the midpoint of AB.   *Answer* (b)(ii) (............ , ...) [1]

Answer: **(a)** (±) 5 **(b)(i)** 6 **(ii)** (1.5, 0)   **N10/12/Q9**

14

In the diagram,

$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{CA} = \mathbf{q}$

and $\overrightarrow{DC} = 3\overrightarrow{AB}$.

(a) Express $\overrightarrow{DA}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

    Answer (a) $\overrightarrow{DA} =$ .... [1]
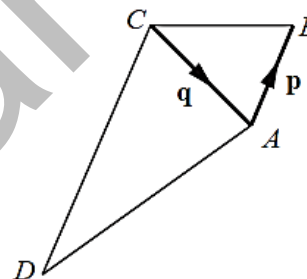
(b) E is the point such that $\overrightarrow{BE} = k\mathbf{q}$.   *Answer* (b)(i) .... [1]

    (i) Write down the name given to the special quadrilateral $ACBE$.

    (ii) Express $\overrightarrow{AE}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and k.   *Answer* (b)(ii) $\overrightarrow{AE} =$ .... [1]

    (iii) Given that D, A and E lie on a straight line, find the value of k.

              Answer (b)(iii) $k =$ .... [1]

Answer: **(a)** 3p + q **(b)(i)** trapezium **(ii)** p + kq **(iii)** $\frac{1}{3}$ **N10/12/Q24**
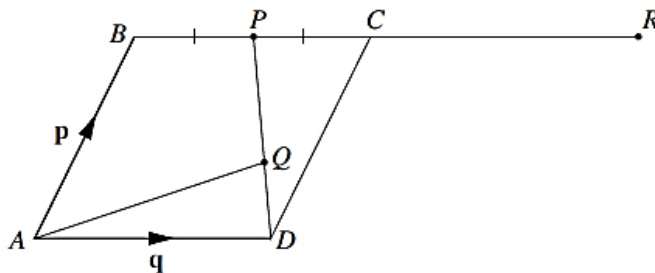
In the diagram, $ABCD$ is a parallelogram.
P is the midpoint of BC.
$DQ : QP = 1 : 2$.
$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$.

(a) Express $\overrightarrow{DP}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer .... [1]

(b) Express $\overrightarrow{DQ}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer .... [1]

(c) Express $\overrightarrow{AQ}$ in terms of $\mathbf{p}$ and $\mathbf{q}$, giving your answer in its simplest form.

Answer .... [1]

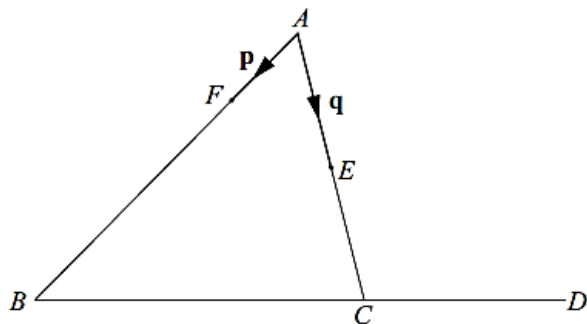(d) R is the point on BC produced such that $\overrightarrow{BR} = k\overrightarrow{BP}$.

(i) Express $\overrightarrow{AR}$ in terms of $\mathbf{p}$ and $\mathbf{q}$ and k.

Answer .... [1]

(ii) Given that A, Q and R lie on a straight line, find the value of k.

Answer $k =$ .... [1]

N11/11/Q24

Answers: (a) $\mathbf{p} - \dfrac{1}{2}\mathbf{q}$ (b) $\dfrac{1}{3}\mathbf{p} - \dfrac{1}{6}\mathbf{q}$ (c) $\dfrac{1}{3}\mathbf{p} + \dfrac{5}{6}\mathbf{q}$ (d)(i) $\mathbf{p} + \dfrac{k}{2}\mathbf{q}$ (ii) 5

16

In the diagram, F is the point on AB where $AF = \frac{1}{4} AB$.
E is the midpoint of AC.
$\overrightarrow{AF} = \mathbf{p}$ and $\overrightarrow{AE} = \mathbf{q}$.

(a) Express, in terms of $\mathbf{p}$ and $\mathbf{q}$,

   (i) $\overrightarrow{FE}$, *Answer* .... [1]

   (ii) $\overrightarrow{BC}$. *Answer* .... [1]

(b) D is the point on BC produced such that $BD = kBC$.

   (i) Express $\overrightarrow{FD}$ in terms of k, $\mathbf{p}$ and $\mathbf{q}$. *Answer* .... [1]

   (ii) Given that F, E and D are collinear, find the value of k.

   Answer $k = $ .... [2]

Answer: (a)(i) -p + q (ii) -4p + 2q (b)(i) (3 – 4k)p + 2kq (ii) 1.5

N11/12/Q28

17   The diagram shows the points
$A(-1, 8)$ and $B(7, 2)$.
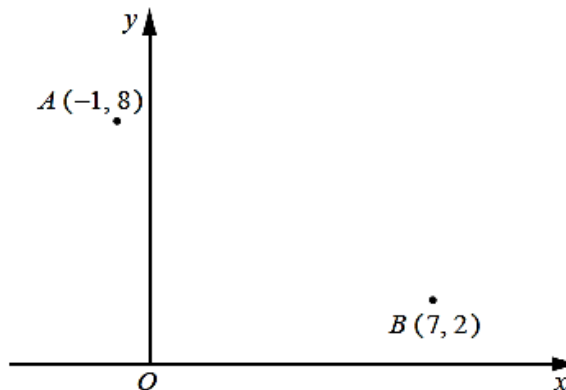
(a) Find the coordinates of the midpoint of *AB*.   *Answer* (.............. , ...) [1]

(b) $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

    (i) Find the coordinates of *C*.

        Answer (.............. , ...) [1]

    (ii) Given that $|\overrightarrow{AB} + \overrightarrow{BC}| = \sqrt{k}$, find *k*.   *Answer* *k* = ...[2]

Answers: **(a)** (3, 5) **(b)(i)** (4, 6) **(ii)** 29

18

In the diagram, $\overrightarrow{BC} = 2\mathbf{p} + \mathbf{q}$, $\overrightarrow{CD} = 2\mathbf{q} - \mathbf{p}$ and *D* is the midpoint of *CE*.

(a) Express, in its simplest form, in terms of **p** and/or **q**

    (i) $\overrightarrow{CE}$,   *Answer* ....[1]

    (ii) $\overrightarrow{\ \ \ }$.   *Answer* ....[1]

(b) Given that $\overrightarrow{\ \ \ } = k\mathbf{p}$, express $\overrightarrow{\ \ \ }$ in terms of *k*, **p** and **q**.

        Answer ....[1]

(c) Given that *AE* is parallel to *BC*, find *k*.   *Answer* *k* =..[1]

Answers: **(a) (i)** 4q – 2p **(ii)** 5q or ft their **(i)** + 2p + q simplified **(b)** kp + their **(ii)** **(c)** 10

19 **(a)**
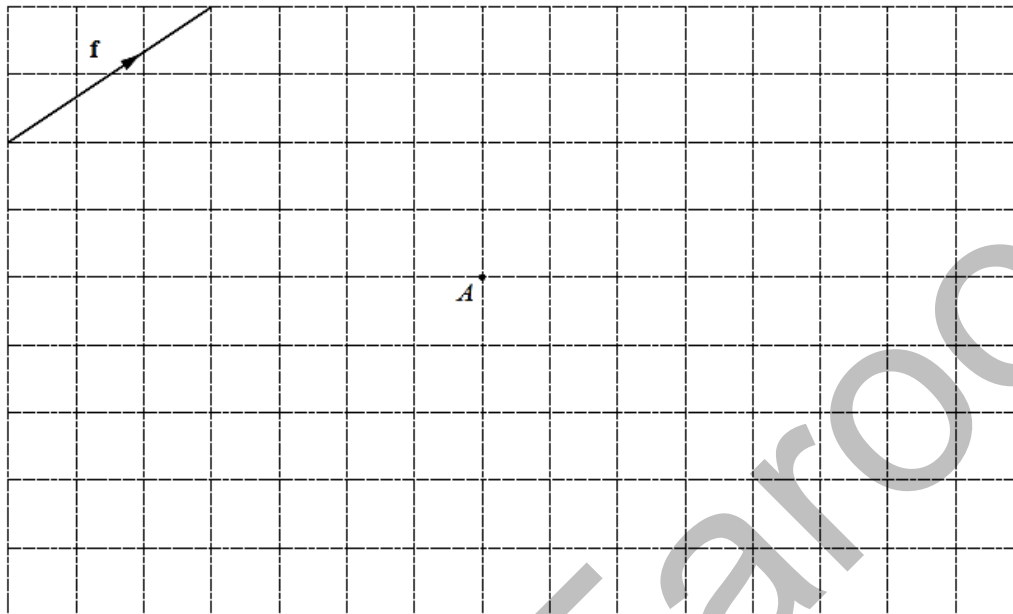
$$\mathbf{f} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \qquad \mathbf{g} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad \mathbf{h} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

The vector **f** and the point A are shown on the grid.

On the grid, mark and label

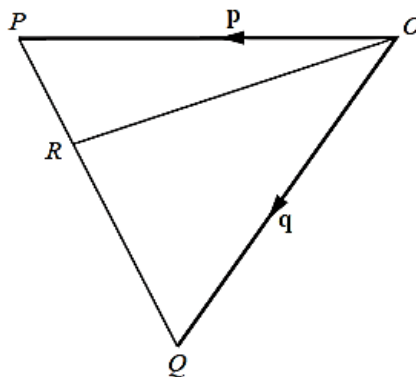**(i)** the point B when $\overrightarrow{AB} = \mathbf{f} + \mathbf{g}$, [1]

(ii) the point C when $\overrightarrow{AC} = -2\mathbf{h}$, [1]

(iii) the point D when $\overrightarrow{AD} = 2\mathbf{f} - 3\mathbf{g}$. [1]

(b)

In the diagram, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
R is the point on PQ such that $PR : RQ = 1 : 2$.

(i) Express $\overrightarrow{PQ}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer .... [1]

(ii) Express $\overrightarrow{OR}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer .... [1]

(iii) T is a point such that $\overrightarrow{TR} = 2\overrightarrow{OP}$.

Express $\overrightarrow{}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$.

Answer .... [2]

Answers: **(a)** Points B, C and D marked correctly; **(b)(i)** $\mathbf{q} - \mathbf{p}$, **(ii)** $\dfrac{2}{3}\mathbf{p} + \dfrac{1}{3}\mathbf{q}$, **(iii)** $\dfrac{1}{3}\mathbf{q} - \dfrac{4}{3}\mathbf{p}$.

20
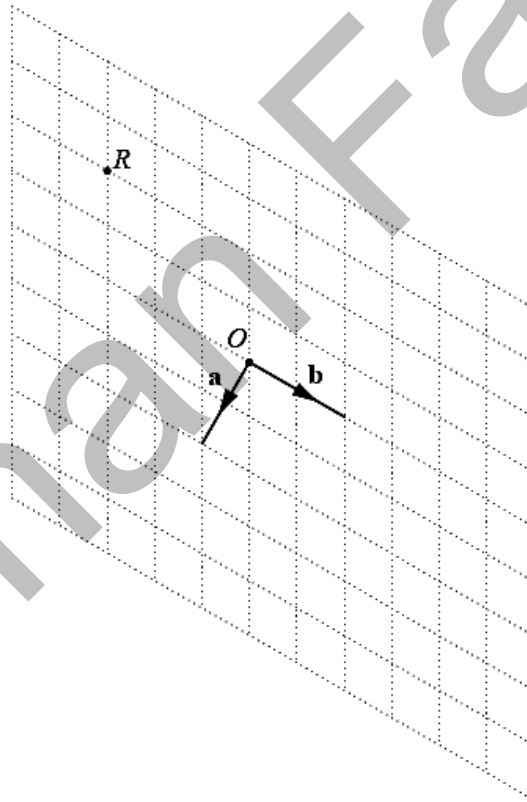
The diagram shows the points O and R and the vectors $\mathbf{a}$ and $\mathbf{b}$.

(a) Given that $\overrightarrow{OP} = 2\mathbf{a}$, mark and label the position of P on the grid. [1]

(b) Given that $\overrightarrow{OQ} = 2\mathbf{b} - \mathbf{a}$, mark and label the position of Q on the grid. [1]

(c) Express $\overrightarrow{OR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. *Answer* $OR = $ .... [2]

21
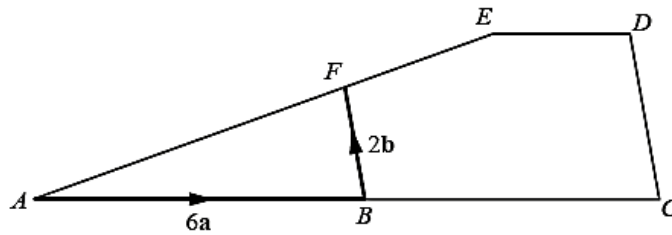
In the diagram, *ABC* and *AFE* are straight lines.

$\overrightarrow{AB} = 6\mathbf{a}$ and $\overrightarrow{BF} = 2\mathbf{b}$.

(a) Express $\overrightarrow{AF}$ in terms of **a** and **b**.   *Answer* .... [1]

(b) $\overrightarrow{AE} = 9\mathbf{a} + k\mathbf{b}$.

  (i) Find *k*.   *Answer* *k* = ... [1]

  (ii) *ED* is parallel to *BC*, *CD* is parallel to *BF* and *BC* = *AB*.

  Find, in terms of **a** and/or **b**,

   (a) $\overrightarrow{CD}$,   *Answer* .... [1]

   (b) $\overrightarrow{DE}$.   *Answer* .... [1]

Vectors Paper 2:

1

(a) $\overrightarrow{OP} = \begin{pmatrix} -9 \\ 40 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 3 \\ -16 \end{pmatrix}$.

Find

   (i) $|\overrightarrow{OP}|$,   **[2]**

   (ii) $\overrightarrow{PQ}$.   **[1]**

(b) In the diagram, *ABC* and *EBD* are two straight lines.

Angle *EAB* = angle *CDB*.

AB = 2 cm, *BC* = 6 cm and *BD* = 4 cm.
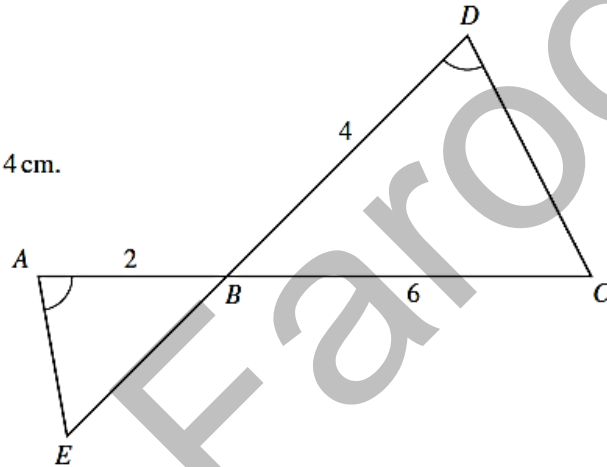
   (i) Explain why triangle *ABE* is similar to triangle *DBC*.   **[1]**

   (ii) Explaining your working fully, show that *BE* = 3 cm.   **[2]**

   (iii) Write down, as a fraction in its lowest terms, the value of

$$\frac{\text{area of triangle } ABE}{\text{area of triangle } DBC}.$$ **[1]**

   (iv) It is given that $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{DB} = \mathbf{q}$.
Express each of the following in terms of **p** and/or **q**

      (a) $\overrightarrow{BC}$,   **[1]**

      (b) $\overrightarrow{BE}$,   **[1]**

      (c) $\overrightarrow{AE}$,   **[1]**

      (d) $\overrightarrow{DC}$.   **[1]**

   (v) Use your answers to parts **(iv)(c)** and **(d)** to explain why *AE* is not parallel to *DC*.   **[1]**

J02/2/Q11

2

(b) In the diagram,

$\overrightarrow{OP} = \mathbf{p}$,
$\overrightarrow{OQ} = \mathbf{q}$,
$\overrightarrow{PY} = k\mathbf{q}$,
$\overrightarrow{PX} = \frac{1}{3}\overrightarrow{PQ}$.
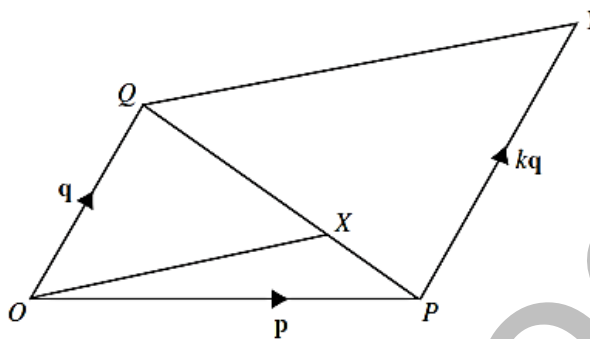
(i)   Express $\overrightarrow{PX}$ in terms of **p** and **q**.   [1]

(ii)   Express $\overrightarrow{OX}$ in terms of **p** and **q**.   [1]

(iii)   Express $\overrightarrow{QY}$ in terms of k, **p** and **q**.   [1]

(iv)   Given that OX is parallel to QY, find the value of k.   [2]

(v)   The line OX, when produced, meets PY at Z.

Express $\overrightarrow{PZ}$ in terms of **q**.   [2]

(b)(i) $-\frac{1}{3}p+\frac{1}{3}q$, **(ii)** $\frac{2}{3}p+\frac{1}{3}q$, **(iii)** $p+(k-1)q$, **(iv)** $\frac{3}{2}$, **(v)** $\frac{1}{2}q$ **J04/2/Q11b**

3

(b) $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -9 \end{pmatrix}$ $\overrightarrow{PR} = \begin{pmatrix} h \\ -6 \end{pmatrix}$ $\overrightarrow{QU} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$ $\overrightarrow{PS} = \begin{pmatrix} 17 \\ k \end{pmatrix}$

(i)   Given that R lies on PQ, find the value of h.   [1]

(ii)   Express $\overrightarrow{PU}$ as a column vector.   [1]

(iii)   Given that U is the midpoint of QS, find the value of k.   [2]

J06/2/Q11b

(b)(i) 2, **(ii)** $\begin{pmatrix} 10 \\ -7 \end{pmatrix}$, ,   (ii) –5

4
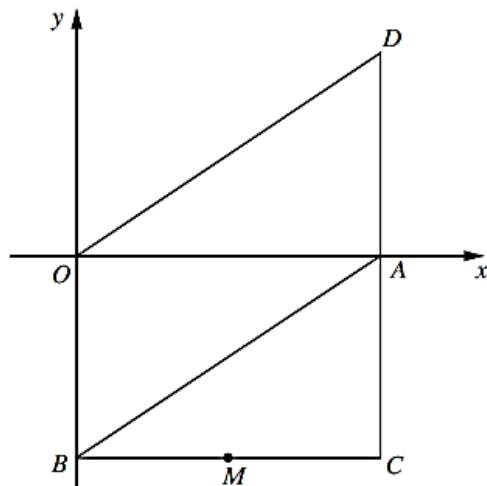
In the diagram, A is the point $(6, 0)$ and B is the point $(0, -4)$.
$OACB$ is a rectangle and M is the midpoint of BC.
$\overrightarrow{CA} = \overrightarrow{AD}$.

(a) Describe, fully, the symmetry of quadrilateral $ODAB$.   [2]

(b) Express as column vectors

  (i) $\overrightarrow{CD}$,   [1]

  (ii) $\overrightarrow{OC}$,   [1]

  (iii) $\overrightarrow{DO}$.   [1]

(c) What type of triangle is OCD?   [1]

(d) The transformation P maps the rectangle $OBCA$ onto the quadrilateral $OBAD$.
     It also maps M onto M'.

  (i) Write down the coordinates of M'.   [1]

  (ii) Identify the transformation P.   [1]

Answer: **(a)** Rotational symmetry of order 2, centre (3, 0);   **(b)(i)** $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$;   **J07/2/Q6**

(c) Isosceles;   **(d)(i)** (3, −2),   **(ii)** Shear.

5 **(b)** In the diagram,

$OT = 3OP$, $RS = \frac{1}{6}RT$ and
Q is the midpoint of PR.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{PQ} = \mathbf{q}$.
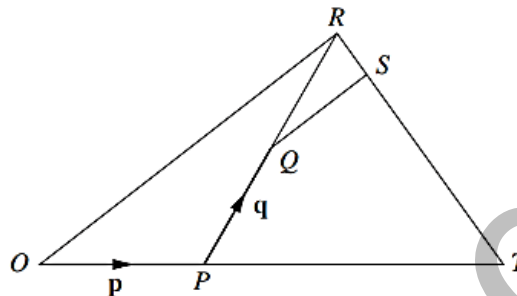
(i) Express, as simply as possible, in terms of **p** and **q**,

    (a) $\overrightarrow{OR}$, [1]

    (b) $\overrightarrow{RT}$, [1]

    (c) $\overrightarrow{QS}$. [2]

(ii) Write down the value of $\dfrac{QS}{OR}$. [1]

(b) (i) (a) p + 2q, **(b)** 2p − 2q, **(c)** $\frac{1}{3}$p + $\frac{2}{3}$q; **(ii)** $\frac{1}{3}$.	**J08/2/Q11b**

6 **(b)**
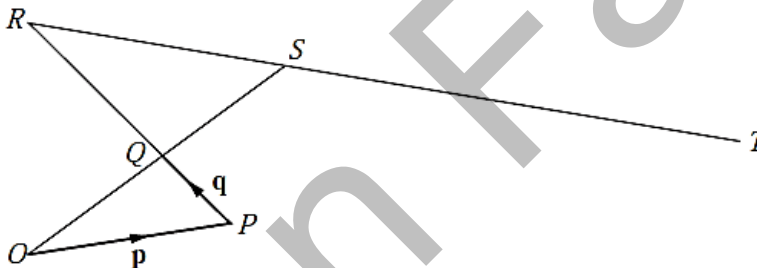
In the diagram, $\overrightarrow{OQ} = \overrightarrow{QS}$, $\overrightarrow{QR} = 2\overrightarrow{PQ}$ and $\overrightarrow{ST} = 2\overrightarrow{RS}$.

$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{PQ} = \mathbf{q}$.

(i) Express, as simply as possible, in terms of **p** and/or **q**,

    (a) $\overrightarrow{OQ}$, [1]

    (b) $\overrightarrow{RS}$, [1]

    (c) $\overrightarrow{OS}$, [1]

    (d) $\overrightarrow{OT}$. [1]

(ii) Hence write down two facts about O, P and T. [2]

(b)(i)(a) p + q, **(i)(b)** p − q; **(i)(c)** 2p + 2q; **(i)(d)** 4p, **(ii)** collinear and $OT = 4OP$ or equivalent.	**J09/2/Q9b**

7   (a)
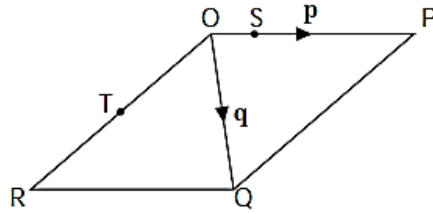
In the diagram, *OPQR* is a parallelogram.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
S is the point on *OP* such that *OS* : *SP* is 1 : 3.
T is the midpoint of *OR*.

Giving your answers in their simplest form, find, in terms of **p** and **q**,

(i) $\overrightarrow{QP}$,   [1]

(ii) $\overrightarrow{TS}$.   [2]

Answers: (a)(i) p − q   (ii) $\frac{3}{4}$p − $\frac{1}{2}$q   J10/21/Q12a

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8

(a) $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 2 \\ -3 & -1 \end{pmatrix}$.

   (i)   Find **AB**.   *Answer*   [2]

   (ii)   Find $\mathbf{B}^{-1}$.   *Answer*   [2]

(b) $\overrightarrow{PQ} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$.

   (i)   Calculate $|\overrightarrow{PQ}|$.   *Answer* .... [2]

   (ii)   Find $\overrightarrow{PR}$.   *Answer*   [1]

(c) You may use the grid below to help you answer this question.
T is the point $(13, 7)$ and U is the point $(8, 9)$.

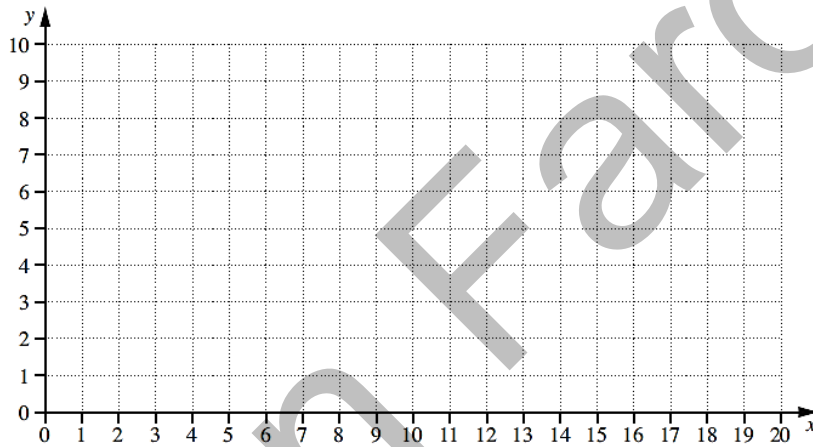(i) Find $\overrightarrow{TU}$. *Answer* .... [1]

(ii) TUV is an isosceles triangle with $TU = TV$.
The y-coordinates of the points U and V are equal.
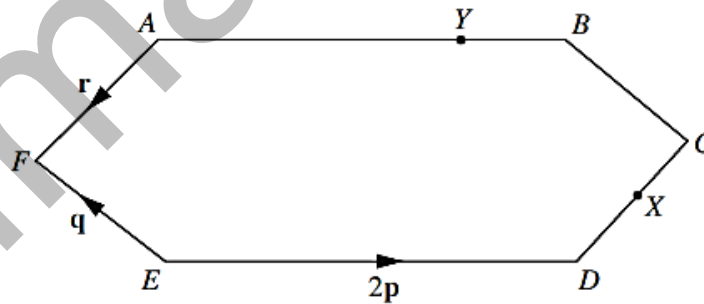
Find the coordinates of V.

Answer (.........., ...) [1]

(iii) W is the point $(1, 3)$.

Calculate the area of triangle TUW. *Answer* .... units^2 [3]

Answers: (a)(i) $\begin{pmatrix} -10 & -4 \\ 15 & 7 \end{pmatrix}$ (ii) $\begin{pmatrix} -0.5 & -1 \\ 1.5 & 2 \end{pmatrix}$ (b)(i) 13 (ii) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ (c)(i) $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ (ii) (18, 9) (iii) 22   **J11/21/Q5**

---

9   (a)

In the diagram, $ABCDEF$ is a hexagon with rotational symmetry of order 2.
$\overrightarrow{ED} = 2\mathbf{p}$, $\overrightarrow{EF} = \mathbf{q}$ and $\overrightarrow{AF} = \mathbf{r}$.
X is the midpoint of CD and Y is the point on AB such that $AY : YB$ is $3 : 1$.

(i) How many lines of symmetry does $ABCDEF$ have?

(ii) Express, as simply as possible, in terms of one or more of the vectors **p**, **q** and **r**,

(a) $\overrightarrow{EA}$,

Answer ....[1]

(b) $\overrightarrow{FC}$,

Answer ....[1]

(c) $\overrightarrow{FY}$,

Answer ....[1]

(d) $\overrightarrow{YX}$.

(b)
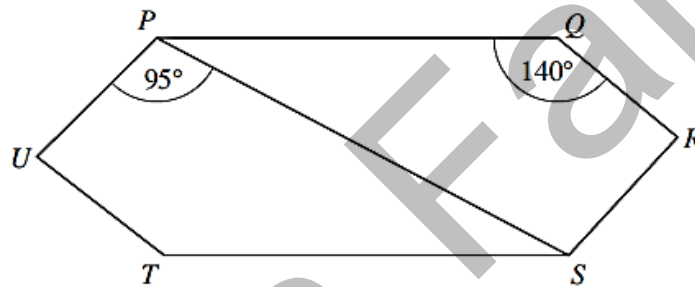
PQRSTU is a similar hexagon to *ABCDEF*.
$U\hat{P}S = 95°$ and $P\hat{Q}R = 140°$.

Find

(i) $Q\hat{P}S$,

Answer ....[1]

(ii) $P\hat{S}R$,

Answer ....[1]

(iii) $P\hat{U}T$.

Answer ....[1]

Answers: **(a)(i)** 2 **(ii)(a)** q − r **(b)** 2p − q − r **(c)** $\frac{3}{2}$p − r **(d)** $\frac{1}{2}$p − q + $\frac{1}{2}$r **J11/22/Q7**
(b)(i) 45° **(ii)** 95° **(iii)** 80°

10   *ABCDEF* is a regular hexagon with centre *O*.
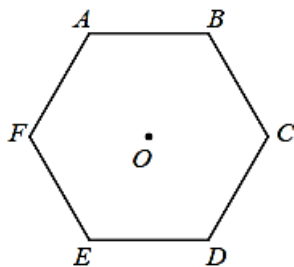
(a) (i)   Find *AÔB*.   *Answer* ....

(ii)   Explain why *AO = BC*.   *Answer* ....[1]

(b) $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

   G is the point on *AB* such that *AG : GB* is 1 : 3.
   H is the midpoint of *BC*.
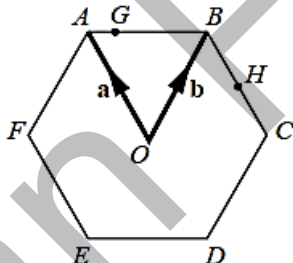
Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{AB}$,   *Answer* ....[

(ii) $\overrightarrow{FB}$,   *Answer* ....[1]

(iii) $\overrightarrow{OG}$,   *Answer* ....[2]

(iv) $\overrightarrow{OH}$,   *Answer* ....[1]

(v) $\overrightarrow{GH}$.   *Answer* ....[2]

Answers: (a)(i) 60°   (b)(i) b − a   (ii) 2b − a   (iii) $\frac{1}{4}$(3a + b)   (iv) b − $\frac{1}{2}$a   (v) $\frac{1}{4}$(3b − 5a)   **J12/21/Q7**

11 **(b)**
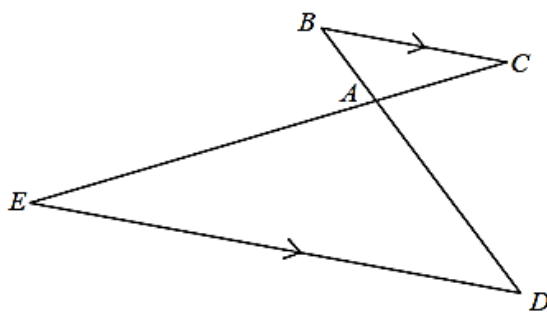
BAD and *CAE* are straight lines and *BC* is parallel to *ED*.

$\overrightarrow{BA} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\overrightarrow{ED} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$ and $\overrightarrow{BA} = \frac{1}{4}\overrightarrow{BD}$.

(i) Describe fully the **single** transformation that maps triangle *ABC* onto triangle *ADE*.

Answer ....

....[2]

(ii) Calculate $|\overrightarrow{BA}|$.   *Answer* .... [1]

(iii) Find *CD*.

                          Answer $\begin{pmatrix} \\ \end{pmatrix}$ [2]

(iv) *F* is the midpoint of *BD*.
Find $\overrightarrow{EF}$.   *Answer* $\begin{pmatrix} \\ \end{pmatrix}$ [2]

(b)(i) Enlargement, scale factor –3, centre *A*   **(ii)** √5 or 2.24
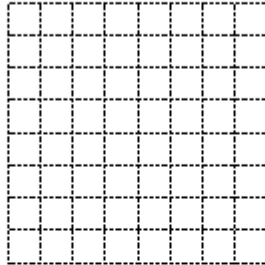
(iii) vector CD = $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$ **(iv)** vector EF = $\begin{pmatrix} 10 \\ 1 \end{pmatrix}$

12

(a) $p = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ $q = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

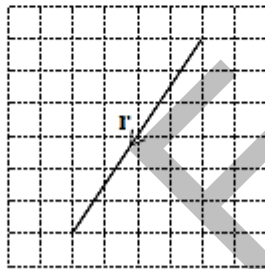(i) Find $|p|$.

Answer .... [1]

(ii) On the unit grid below, draw and label the vector $p - q$.

[2]

(iii) The vector r is shown on the unit grid below.

It is given that $r = ap + bq$.
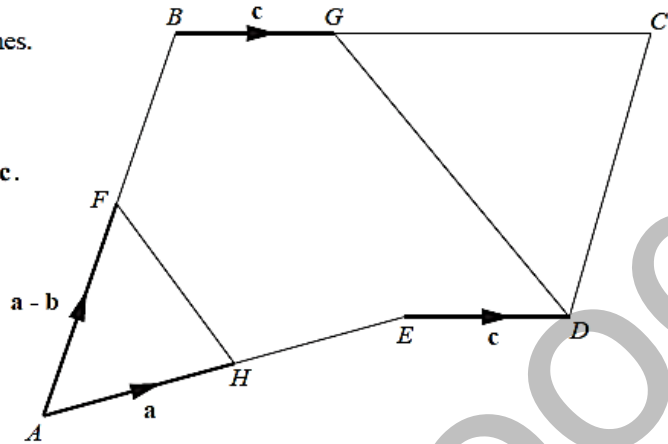
Answer $a =$ ....

Find the values of a and b.

$b =$ .... [2]

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13 **(a)** *ABCDE* is a pentagon.
AFB, *AHE* and *BGC* are straight lines.
F is the midpoint of *AB*.
H is the midpoint of *AE*.
G divides *BC* in the ratio 1 : 2.
$\overrightarrow{AH} = \mathbf{a}$, $\overrightarrow{AF} = \mathbf{a} - \mathbf{b}$, $\overrightarrow{BG} = \overrightarrow{ED} = \mathbf{c}$.

(i) Find $\overrightarrow{FH}$.　　　　　　　　　*Answer* ....[1]

(ii) Using vectors, show that *GD* is parallel to *FH*.　　　　　　　　　[2]

(iii) It is given that $\mathbf{c} = \dfrac{4}{5}\mathbf{a} + \dfrac{1}{5}\mathbf{b}$.

(a) Express $\overrightarrow{DC}$ in terms of **a** and **b**.　　*Answer* ....[2]

(b) Find $|\overrightarrow{}| : |\overrightarrow{}|$.　　*Answer* .... : ....[1]

Answers: (a)(i) b (ii) 2b (iii)(a) $\dfrac{8}{5}$ a $-\dfrac{8}{5}$ b (b) 1: $\dfrac{8}{5}$ (b)(i)(a) reflection in $y = x$ (b) matrix $\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ **J15/21/Q11**

(ii) (−3, 6) (− 3, 0) (0, −2) (iii) 90°

14
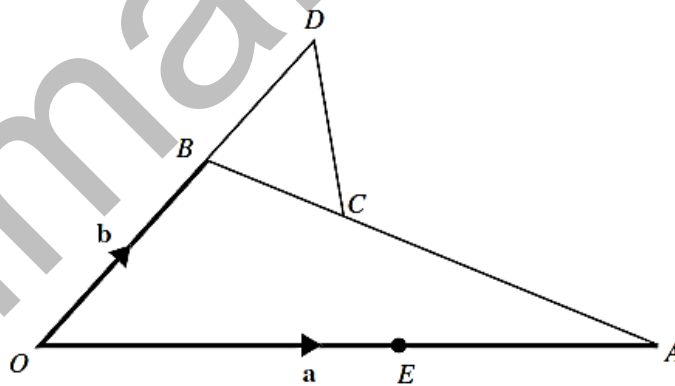
In the diagram, *OAB* is a triangle.
C is the point on *AB* such that *AC* : *CB* = 2 : 1.
The side *OB* is produced to the point *D* such that *OB* : *BD* = 3 : 2.
It is given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{AB}$,　　　　　　　　　　　　　　　　　　　[1]

(ii) $\overrightarrow{AC}$, [1]

(iii) $\overrightarrow{OC}$, [1]

(iv) $\overrightarrow{OD}$. [1]

(b) Show that $\overrightarrow{CD} = \mathbf{b} - \dfrac{1}{3}\,\mathbf{a}$. [2]

(c) It is given that E is the point on OA such that $\overrightarrow{OE} = \dfrac{5}{9}\mathbf{a}$.

Express, as simply as possible, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector $\overrightarrow{ED}$. [1]

(d) (i) Show that $\overrightarrow{ED} = k\,\overrightarrow{CD}$, where k is a constant. [1]

(ii) Write down two facts about ED and CD. [2]

(e) Calculate $\dfrac{\text{the area of triangle } AEC}{\text{the area of triangle } OEC}$. [2]

Answers: (a)(i) $\mathbf{b} - \mathbf{a}$, (ii) $\dfrac{2(\mathbf{b}-\mathbf{a})}{3}$, (iii) $\dfrac{(\mathbf{a}+2\mathbf{b})}{3}$, (iv) $\dfrac{5\mathbf{b}}{3}$; (c) $\dfrac{5(3\mathbf{b}-\mathbf{a})}{9}$; (d)(i) $k = \dfrac{5}{3}$; (e) $\dfrac{4}{5}$. N01/2/Q11

15
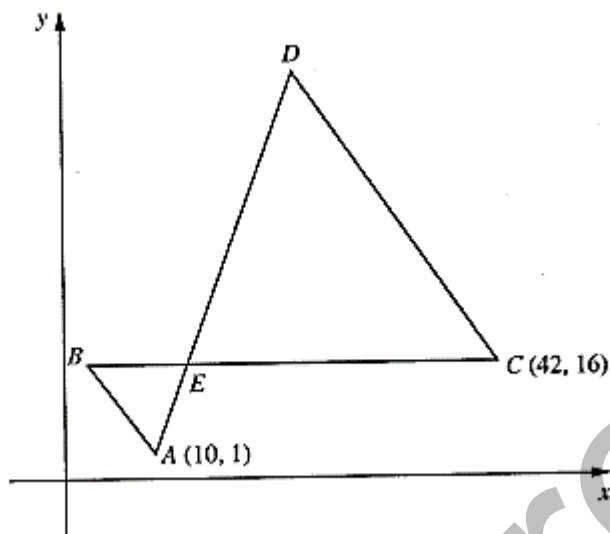
In the diagram, A is the point $(10, 1)$ and $\overrightarrow{AB} = \begin{pmatrix} -8 \\ 15 \end{pmatrix}$.

(a) Find

    (i) $|\overrightarrow{AB}|$,   [2]

    (ii) the coordinates of B.   [1]

The point C is $(42, 16)$ and $\overrightarrow{CD} = 3\overrightarrow{AB}$.

(b) Find

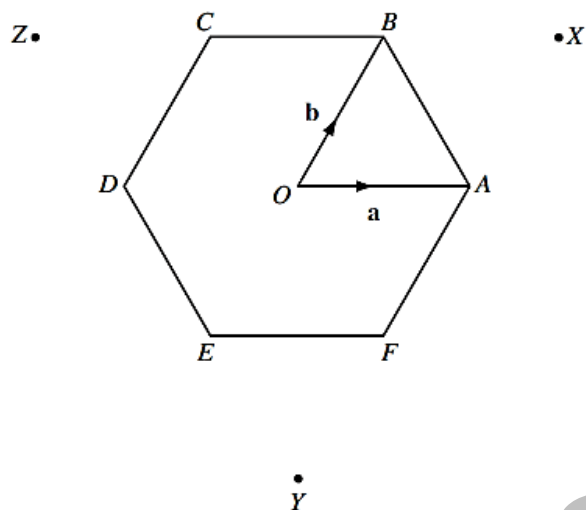   (i) the coordinates of D,   [2]

    (ii) the vector $\overrightarrow{AD}$.   [1]

The point E is $(k, 16)$.

(c) (i) Find, in terms of k, the vector $\overrightarrow{AE}$.   [1]

    (ii) Given that AED is a straight line, find k.   [2]

(d) Find $\dfrac{\text{Area of triangle } ABE}{\text{Area of triangle } CDE}$.   [2]

Answers: **(a)(i)** 17, **(ii)** (2, 16); **(b)(i)** (18, 61), **(ii)** $\begin{pmatrix} 8 \\ 60 \end{pmatrix}$; **(c)(i)** $\begin{pmatrix} k-10 \\ 15 \end{pmatrix}$, **(ii)** 12; **(d)** $\dfrac{1}{9}$.   **N02/2/Q5**

16

A regular hexagon, *ABCDEF*, has centre *O*.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express, as simply as possible, in terms of **a** and/or **b**,

    (i) $\overrightarrow{DO}$,   [1]

    (ii) $\overrightarrow{AB}$,   [1]

    (iii) $\overrightarrow{DB}$.   [1]

(b) Explain why $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{b} - \mathbf{a}|$.   [1]

(c) The points *X*, *Y* and *Z* are such that

    $\overrightarrow{OX} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{OY} = \mathbf{a} - 2\mathbf{b}$ and $\overrightarrow{OZ} = \mathbf{b} - 2\mathbf{a}$.

    (i) Express, as simply as possible, in terms of **a** and/or **b**,

        (a) $\overrightarrow{AX}$,   [1]

        (b) $\overrightarrow{YX}$.   [1]

    (ii) What can be deduced about *Y*, *A* and *X*?   [1]

(d) Express, as simply as possible, in terms of **a** and/or **b**, the vector $\overrightarrow{XZ}$.   [1]

(e) Show that triangle *XYZ* is equilateral.   [2]

(f) Calculate $\dfrac{\text{Area of triangle } OAB}{\text{Area of triangle } XYZ}$.   [2]

Answers: (a)(i) *a*, (ii) *b* − *a*, (iii) *a* + *b*; (c)(i)(a) *b*, (b) 3*b*, (ii) Collinear; (d) −3*a*; (e) 1/9.   **N05/2/Q11**

17 The points A and B are $(5, 3)$ and $(13, 9)$ respectively.

   (a) Find

       (i) the midpoint of AB,   [1]

       (ii) the gradient of the line through A and B,   [1]

       (iii) the length of the line AB.   [1]
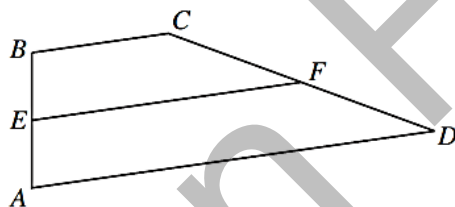
   (b) C is the point $(-8, 5)$.

       The point D is such that $\overrightarrow{DC} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

       (i) Find the coordinates of D.   [2]

       (ii) What type of quadrilateral is $ABCD$?   [1]

Answer: **(a)(i)** $(9, 6)$ **(ii)** ¾   **(iii)** 10   **(b)(i)** $(-12, 2)$ **(ii)** Trapezium

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18   **(a)** Given that $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$, find $\overrightarrow{PS}$.   [1]

   (b)

   In the diagram, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{AD} = 3\mathbf{a}$ and $\overrightarrow{DF} = \mathbf{b} - \mathbf{a}$.
   E is the midpoint of AB and F is the midpoint of DC.

      (i) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

         (a) $\overrightarrow{EA}$,   [1]

         (b) $\overrightarrow{DC}$,   [1]

         (c) $\overrightarrow{EF}$,   [1]

         (d) $\overrightarrow{BC}$.   [1]

      (ii) **(a)** Give the special name of the quadrilateral $ABCD$.

            Give your reason.   [2]

         (b) Find the ratio $\left|\overrightarrow{BC}\right| : \left|\overrightarrow{EF}\right| : \left|\overrightarrow{AD}\right|$.   [1]

Answers: **(a)** $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, **(b)(i)(a)** $-\mathbf{b}$, **(b)** $2\mathbf{b} - 2\mathbf{a}$, **(c)** $2\mathbf{a}$, **(d)** $\mathbf{a}$, **(ii)(a)** Trapezium, $AD//BC$, **(b)** 1:2:3,

(c)(i) 146°, angles in the same segment are equal, **(ii)** 73°, angle at the centre of a circle is twice the angle at the circumference, **(iii)** 34°, angles in opposite segments are supplementary, **(iv)** 73°, angle sum of a triangle.

19 (a) $\overrightarrow{PQ} = \begin{pmatrix} 12 \\ -35 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$.

    (i) Find

        (a) $|\overrightarrow{PQ}|$,   [1]

        (b) $\overrightarrow{PR}$.   [1]

    (ii) Given that T is the midpoint of QR, find $\overrightarrow{PT}$.   [2]

    (iii) $PQRS$ is a parallelogram.
         The coordinates of R are (6 , 16).
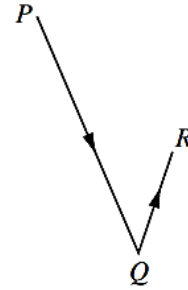
         Find the coordinates of S.   [2]

Answers: **(a)(i)(a)** 37,   **(b)** $\begin{pmatrix} 16 \\ -21 \end{pmatrix}$,   **(ii)** $\begin{pmatrix} 14 \\ -28 \end{pmatrix}$,   **(iii)** (−6,51)   **(b)(i)** 2,   **(ii)(a)** (−2,3),   **N08/2/Q11**
      (b) 32,   **(iii)(a)** (3,1),   **(b)** 2.

20 **(a)**
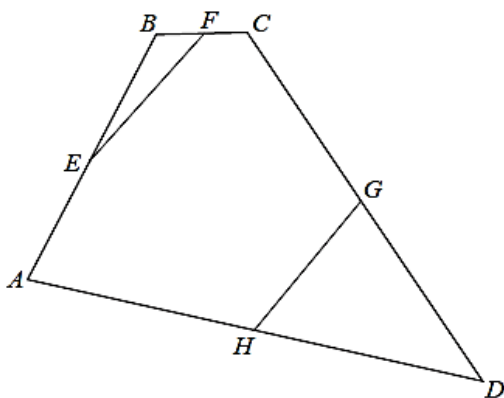
In the diagram, $ABCD$ is a quadrilateral where $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 8 \\ -12 \end{pmatrix}$.

E, F, G and H are the midpoints of AB, BC, CD and DA respectively.

(i) Find $\overrightarrow{AD}$. [1]

(ii) Calculate $|\overrightarrow{AD}|$. [2]

(iii) Show that EF and HG are opposite sides of a parallelogram. [2]

(b) The diagram shows triangle P.
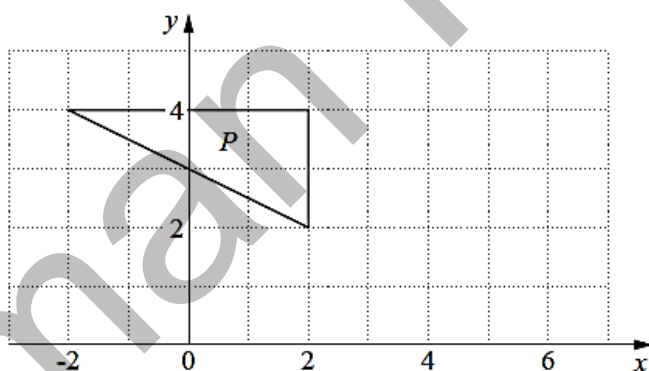
Triangle Q has vertices $(-2, 4)$, $(6, 0)$ and $(6, 4)$.

Describe fully the **single** transformation that maps triangle P onto triangle Q. [3]

Answers: **(a)(i)** $\begin{pmatrix} 14 \\ -4 \end{pmatrix}$ **(ii)** 14.6 **(b)** Enlargement, centre $(-2, 4)$, scale factor 2 **(c)(i)** $(5,0)$ $(7,3)$ $(2,3)$ **(ii)** $\frac{1}{15}\begin{pmatrix} 3 & -2 \\ 0 & 5 \end{pmatrix}$

21 The point A is $(0, 7)$, and the point B is $(6, 9)$.

    (a) Express $\overrightarrow{AB}$ as a column vector.   *Answer*   [1]

    (b) Find the gradient of AB.   *Answer*   .... [1]

    (c) The equation of the line AB is $x + Py + Q = 0$.   *Answer* $P =$ ....

        Find P and Q. $Q =$ .... [2]

    (d) The point C is $(12, 2)$.

        (i) Given that C is the midpoint of BM, find the coordinates of M.

                                         Answer   (.......... , ...)   [1]

        (ii) Calculate AC.   *Answer*   .... units [1]

        (iii) The point D lies on the line AB.
             The line CD is parallel to the y-axis.

            (a) Find the coordinates of D.   *Answer*   (.......... , ...)   [2]

            (b) Express $\overrightarrow{AD}$ in terms of $\overrightarrow{AB}$.   *Answer* $\overrightarrow{AD} =$ .... [1]
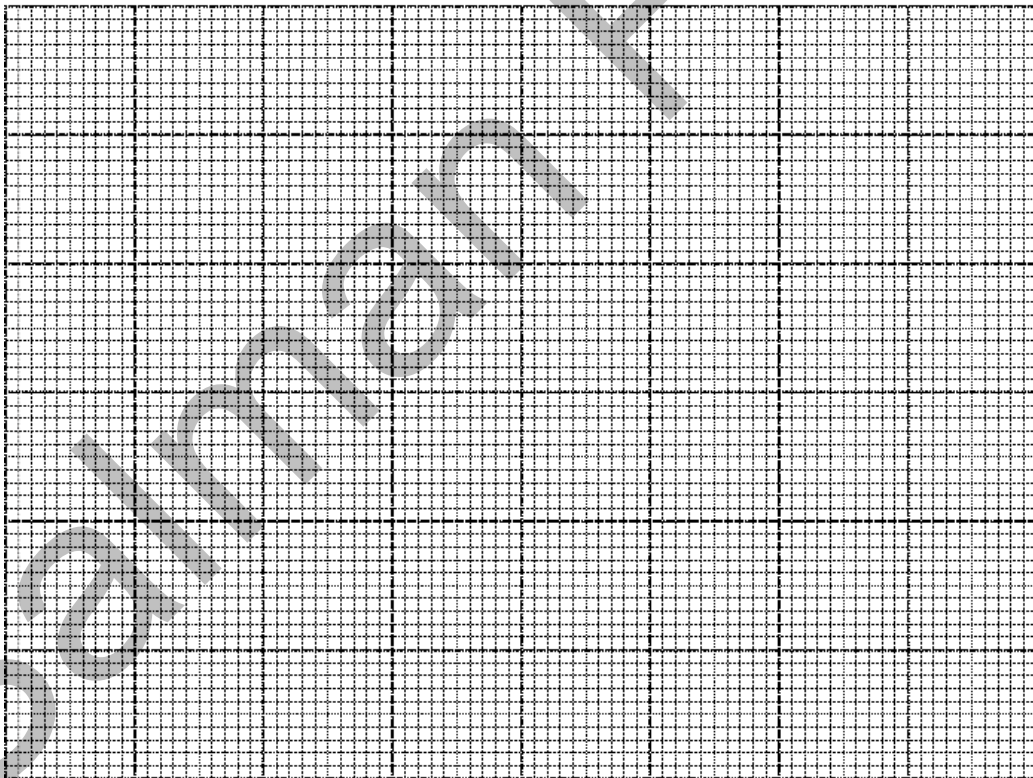
Answers: **(a)** $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ **(b)** $\frac{1}{3}$ **(c)** $P = -3$ $Q = 21$ **(d)(i)** $(18, -5)$ **(ii)** 13 **(iii)(a)** $(12,11)$ **(b)** $2\overrightarrow{AB}$ **N11/22/Q6**

22 (a) *E, F, G* and *H* are the midpoints of $\overrightarrow{AB}, BC, CD$ and *DA* respectively. $\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{BC} = \mathbf{r}$.
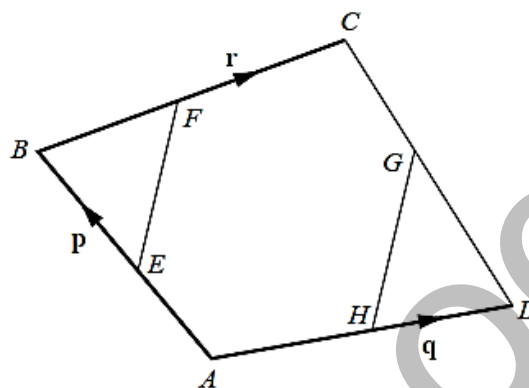
   (i) Find, in terms of **p**, **q** and **r** as appropriate

   (a) $\overrightarrow{EF}$,
   Answer ....[1]

   (b) $\overrightarrow{DC}$,
   Answer ....[1]

   (c) $\overrightarrow{HG}$, expressing the vector as simply as possible.

   Answer ....[2]

   (ii) What conclusions can be drawn about the lines *EF* and *HG*?

   ....[1]

Answer: (a)(i)(a) $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$ (b) $\mathbf{r} + \mathbf{p} - \mathbf{q}$ (c) $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$ (ii) Equal and Parallel (b)(i) Triangle with vertices (-2,0), (0,6), (0,7) (ii) Triangle with vertices (-2,0), (0,0),(0,-1) (iii) Rotation, 90° anticlockwise centre (0,3)   **N12/21/Q11**
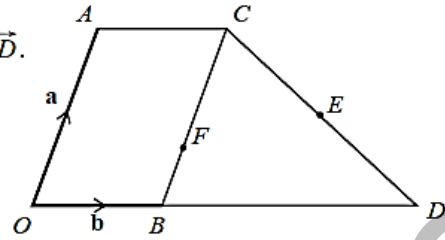
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23 (a) The diagram shows the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} a \\ b \end{pmatrix}$.
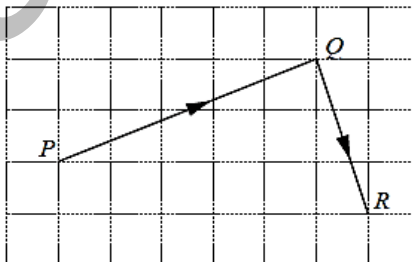
   (i) Find *a* and *b*.

   Answer   *a* = ... *b* = ... [2]

   (ii) Calculate $|\overrightarrow{PQ}|$.
   Answer .... [2]

(b) *OACB* is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and *D* is the point such that $2\overrightarrow{OB} = \overrightarrow{BD}$.
E is the midpoint of *CD*.

(i) Express $\overrightarrow{CE}$, as simply as possible, in terms of **a** and **b**.

Answer ....[1]

(ii) Express $\overrightarrow{OE}$, as simply as possible, in terms of **a** and **b**.

Answer ....[1]

(iii) *F* is a point on *BC* such that $\overrightarrow{OF} = k\overrightarrow{OE}$.
Find *BF* : *FC*.

Answer ....:.................[2]

Answers: (a)(i) $a = 1$, $b = -3$; (ii) 5.39; (b)(i) $b - \dfrac{1}{2}a$; (ii) $2b + \dfrac{1}{2}a$; (iii) 1 : 3

24 **(a)**
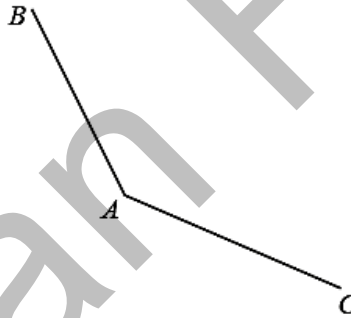
In the diagram, $\overrightarrow{AB} = \begin{pmatrix} -6 \\ 11 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$.

(i) Find $|\overrightarrow{AC}|$.

Answer .... [2]

(ii) *D* is the point such that $\overrightarrow{AD} = \begin{pmatrix} 0 \\ k \end{pmatrix}$, where $k > 0$.

BD is parallel to *AC*.

(a) Show that $\overrightarrow{BD} = \begin{pmatrix} 6 \\ k - 11 \end{pmatrix}$. [1]

(b) Find *k*. Answer $k =$ .... [2]

(c) Find the difference between the lengths of *AD* and *AC*.

Answer .... [1]

25   **(b)** The diagram shows triangle *PRS*.
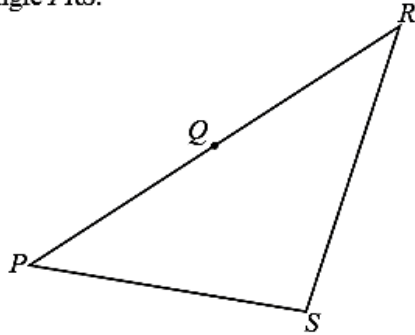
Q is the midpoint of *PR*.

$\overrightarrow{PQ} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ and $\overrightarrow{PS} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$.

(i) Find $\overrightarrow{SR}$.

Answer $\begin{pmatrix} \\ \end{pmatrix}$ [2]

(ii) *T* is the point on *SR* such that *ST* : *TR* = 1 : 3.

Find $\overrightarrow{PT}$.

Answer $\begin{pmatrix} \\ \end{pmatrix}$ [2]